Between Language and Mathematics: In the Case of Inclusive OR*

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1. Introduction

In this paper, it is shown that inclusive OR is sometimes used in everyday language. Inclusive OR is the operator for weak disjunction in mathematics or logic.

In mathematics or logic, mere disjunction, that is, unmarked disjunction is weak disjunction. Disjunction cannot mean strong disjunction, so if one intends to express strong disjunction, one has to say strong disjunction, not disjunction. In mathematics or logic, weak disjunction, namely, inclusive OR is unmarked, whereas strong disjunction, namely, exclusive OR is marked.

In everyday language, however, the reverse is the case. Inclusive OR is usually marked and exclusive OR unmarked. (1) is one of the examples that show this fact.

(1) and/or

This English expression has the meaning (2).[1]

(2) both or either

The OR in (1) can only be exclusive, as in (2). If not so, the AND in (1) is unnecessary. In everyday language, mere OR, whose meaning is "either", is exclusive and unmarked, whereas AND/OR, whose meaning is "both or either", is inclusive and marked.[2,3]

(3) is a concrete example which shows that mere OR is exclusive, whereas AND/OR is inclusive.

(3) Money and/or clothes are welcome.

The meaning of (3) is (4), where OR is used exclusively.

(4) Both money and clothes are welcome, or either money or clothes are welcome.

(4) shows that the AND/OR in (3) is used inclusively, and that the OR in the AND/OR in (3) is used exclusively. The existence of the expression AND/OR in English is a piece of evidence that the OR in everyday language is originally exclusive.

One cannot help saying, "Inclusive OR is usually marked and exclusive OR is usually unmarked in everyday language," or "The OR in everyday language is originally exclusive." Such words as usually, originally and so on cannot be omitted, because the OR in everyday language is also inclusive, not exclusive, under certain conditions. Under marked conditions, OR is inclusive even in everyday language. Such conditions will be examined in Section 4.

The case where OR is inclusive in everyday language is one of the contact points of language and mathematics. What are the conditions under which inclusive OR is used in everyday language? The answer to this question is one of the contact points of language and mathematics.

2. Inclusive OR and exclusive OR

Inclusive OR corresponds to (weak) disjunction, and exclusive to strong disjunction. The truth table of weak disjunction and strong disjunction is (5).[4]

(5)

	A	B	A v B	A ⓥ B
a.	T	T	T	F
b.	T	F	T	T
c.	F	T	T	T
d.	F	F	F	F

The difference between v and ⓥ is only (5a). While weak disjunction admits the case where both A and B are true, strong disjunction does not. This sole difference between v and ⓥ is subtle, but not trivial. It is crucial rather than peripheral.

Kanno(1980) argues that (6) follows from (5), especially from (5a).[5]

(6) Implication cannot be translated by strong disjunction, but only by weak disjunction.

The truth table of implication[6] and its translated form[7] is as follows.

(7)

	A	B	A → B	∼A v B
a.	T	T	T	T
b.	T	F	F	F
c.	F	T	T	T
d.	F	F	T	T

As is shown in (5) and (7), the number of the true cases of implication and weak disjunction is three, whereas that of strong disjunction is two. This is the reason for (6).

It cannot be ignored whether implication can be translated or not, because implication plays a vastly important

role in mathematics or logic. So one disjunction should be distinguished from the other. In other words, inclusive OR and exclusive OR should be distinguished.

The history of mathematics or logic has the movement from strong disjunction to weak disjunction. Why did mathematicians and logicians begin to use inclusive OR in place of exclusive OR? It might be reasonable to suppose that the translatability of implication is one of the reasons for this movement. This supposition, however, awaits further investigation from various points of view.

3. Difference between language and mathematics

In this section, one of the differences between everyday language and mathematics is made explicit. This difference is another side of one of the contact points of everyday language and mathematics.

As is discussed in Section 1., OR is used inclusively, that is, as the operator for weak disjunction in mathematics or logic, whereas OR is usually used exclusively, that is, like the operator for strong disjunction in everyday language. In everyday language, OR is exclusive as far as there are no special conditions.

To be more precise, further explanations of OR in both mathematics and language are given. As for the OR in mathematics or logic, it was used not inclusively, but exclusively a long time ago. This fact is not very tough to understand, since exclusive OR is more cocrete and more intuitive than inclusive OR. Very roughly speaking, the initial state of

almost all studies is concrete and intuitive, and then the latter state is more abstract and more theoretical than the former state. Neither mathematics nor logic was exceptional.

The movement from exclusive OR to inclusive OR is not very difficult to understand, either. The development of mathematics or logic needed abstraction. Since inclusive OR is more abstract than exclusive OR, the latter was replaced by the former in mathematics or logic. This replacement was necessary for mathematical or logical abstraction or theorization.

On the other hand, the OR in everyday language has been used exclusively for a long time under no special conditions.[8] While the abstraction or theorization in mathematics and logic preferred inclusive OR to exclusive OR, everyday language has been favoring exclusive OR over inclusive OR, because one of the main functions of everyday language is communicativity. Exclusive OR, which is more concrete, is more communicative than inclusive OR,[9] which is less concrete. As far as exclusive OR is considered concrete, OR will continue to be used exclusively.

The sentence (8) is an example in which OR is interpreted as exclusive.

(8) John or Mary will visit you tomorrow.

The most natural interpretation of (8) may be (9).

(9) One of John and Mary will visit you tomorrow.

(9) is almost the same as (10).

(10) Either John or Mary will visit you tomorrow.

Although (8) is not perfectly the same as (10), the natural interpretation is (9); in other words, the OR in (8) is used exclusively as that of (10). (8) can be interpreted as (11) if the speaker who is coward hesitates to say (10), if the speaker observes mathematical or logical way of inference strictly even in relaxed everyday life,[10] and so on.

(11) John and/or Mary will visit you tomorrow.

The interpretation of (8) as (11) is quite marked, and that of (8) as (9) is unmarked.

Although there are no examples but those of the (8) type in which OR is used exclusively, such examples as (8) are numerous in everyday language.[11]

4. Inclusive OR in everyday language

In Section 4., I examine the examples of that type which is different from that of (8), that is, which is marked and exceptional. (12) is one of them.

(12) I won't visit John or Mary tomorrow.

In (12) OR must be inclusive and cannot be exclusive at all. The OR in (12) must be inclusive obligatorily, because (12) must be interpreted as (13) uniquely.[12]

(13) I will visit neither John nor Mary tomorrow.

Both (12) and (13) are false if the speaker visits at least one of John and Mary.

The sentence (14) is another example.

(14) Will you visit John or Mary tomorrow?

The OR in (14) may be either exclusive or inclusive. If it is exclusive, John is pronounced with rising intonation and Mary with falling intonation, and its answer is either (15a) or (15b).

(15) a. I will visit John.
b. I will visit Mary.

If it is inclusive, (14) is uttered in the same manner as that of usual yes-no questions, and its answer is either (16a) or (16b).

(16) a. Yes, I will.
b. No, I won't.

(16a) means that the person who answers (14) will visit John and/or Mary, whereas (16b) means that he will visit neither John nor Mary. (16b) is exactly the same as (12). While the OR in (12) is interpreted as inclusive obligatorily, that in (14) is interpreted as inclusive optionally.

The OR in (17) is interpreted as inclusive optionally as in the case of (14).

(17) If you visit John or Mary tomorrow, I would like to do so with you.

The meaning of (17) is either (18) or (19).

(18) If you visit either John or Mary tomorrow, I would like to do so with you.
(19) If you visit John and/or Mary tomorrow, I would like to do so with you.

The OR in (18) is exclusive and that in (19) is inclusive.

To sum up, the semantic interpretation of OR in such cases as (12) and that in such cases as (14) and (17) are (20a) and (20b), respectively.

(20) a. In a negative scope, OR is inclusive obligatorily.

b. In an interrogative scope or in a conditional scope, OR is inclusive optionally.

NEGATIVE, INTERROGATIVE and CONDITIONAL[13] are conditions under which OR in everyday language may be inclusive.

5. Conclusion

The conclusion of this paper is (21).

(21) OR is used in the same way as mathematics or logic, namely, inclusively under such conditions as (20a) or (20b) in everyday language.

(21) means that mathematical or logical OR in everyday language is one of the marked cases of everyday language.

APPENDIX

Some linguists are apt to apply something mathematical or logical to linguistics directly. They must not forget that linguistics is not perfectly the same as mathematics or logic. Gazdar(1980) and Ladusaw(1980) are typical results brought about by the confusion of mathematics (or logic) and linguistics. According to Gazdar(1980), (22) as well as (23) has the meaning (24).

(22) Either John or Mary will succeed.

(23) John or Mary will succeed.

(24) John and/or Mary will succeed.

Since it is reasonable to suppose that even (23) does not mean (24) unmarkedly in everyday language, (22) can never mean (24). In this point, Gazdar(1980) is not appropriate.

Neither is Ladusaw(1980), in which it is assumed that (23) means (24) originally and that (23) means (22) by the conversational postulate (25).

(25) Interpret "A or B" as "'A or B' and $\sim$'A and B'".

Ladusaw(1980) is wrong in the point that even in everyday language OR is inclusive unmarkedly. It is the case that OR is exclusive unmarkedly in everyday language.

NOTES

* This paper is a revised version of the paper which I could not read through by accident at Tohoku University on March 30, 1982.

I am grateful to Dr. Nakau and Y. Hirose for their revision of my faulty English, and to K. Iwabe and H. Tada for their suggestion. Needless to say, remaining errors are my own.

[1] The OR in (2) is not inclusive, but exclusive.

[2] This fact is true of Ross's concept "myopia", because the shorter expression, namely, mere OR is unmarked, whereas the longer expression, namely, AND/OR is marked.

[3] AND/OR in everyday language is the true counterpart of

inclusive OR in mathematics or logic.

[4] v expresses weak disjunction and Ⓥ strong disjunction.

[5] (6) has an exception, which H. Tada pointed out to me. It is "'$\sim$A Ⓥ B' Ⓥ '$\sim$A$\wedge$B'".

This exception can be ignored because it must use two Ⓥ 's.

[6] $\rightarrow$ expresses implication.

[7] $\sim$ expresses negation.

[8] Exclusiveness of OR and the counterparts in other languages might be one of the linguistic universals, though under certain special conditions they are used inclusively.

[9] AND/OR and inclusive OR are the same in cognitive meaning, but the former may be more concrete than the latter.

[10] Gazdar should interpret (8) as (11), not as (9), because according to Gazadr(1980) even the meaning of (10) is the same as that of (11).

[11] The OR in "Hold up,or you'll be killed," is exclusive uniquely. This example was pointed out to me by K. Iwabe.

[12] If the OR in (12) is exclusive, the interpretation of (12) is "I will visit neither John nor Mary, or both John and Mary." It seems that "neither A nor B" and "both A and B" cannot be incorporated in language, but the explanation has not been found out.

[13] There might be other scopes in which OR is inclusive, but the present author has not found any of them. If any, they may not be like NEGATIVE, but like INTERROGATIVE and CONDITIONAL.

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