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Application of Collateralized Debt Obligation Approach for Managing Inventory Risk in Classical Newsboy Problem

by

Rina Isogai, Satoshi Ohashi and Ushio Sumita

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Rina Isogai
Morgan Stanley Japan Securities Co., Ltd
Yebisu Garden Place Tower 4-20-3 Ebisu, Shibuya-ku, Tokyo, 150-6008, Japan
rina.isogai@morganstanley.com

Satoshi Ohashi and Ushio Sumita
Graduate School of Systems and Information Engineering,
University of Tsukuba,
1-1-1 Tennoudai, Tsukuba, Ibaraki, 305-8573, Japan
sumita@sk.tsukuba.ac.jp

ABSTRACT

In the midst of the ongoing world financial crisis, the Collateralized Debt Obligation (CDO) became notorious. However, the fact that the misuse of the CDO resulted in collapse of the world economy does not necessarily imply that the CDO itself would be hazardous. The purpose of this paper is to explore the potential of the CDO approach for controlling general risks, by applying it to the classical Newsboy Problem (NBP). The underlying opportunity loss of NBP replaces the credit risk of CDO. For VaR (Value at Risk) problems formulated without or with CDO, extensive numerical experiments reveal that the overall effect of CDO is rather limited. It could be effective, however, if (i) the underlying risk is high in that the variability of the stochastic demand D is substantially large; (ii) the expected profit should be held above a high level; (iii) the probability of having a huge loss should be contained; and (iv) the detachment point K_d should be held relatively low.

Keyword: Collateralized Debt Obligation, Risk Control, Newsboy Problem, Value at Risk

1. Introduction

It is widely believed that the Collateralized Debt Obligations (CDOs) played a major role in the ongoing worldwide financial crisis. Naturally, the CDO became notorious for its role in destructing the world economy. However, the fact that the misuse of the CDO resulted in collapse of the world economy does not necessarily imply that the CDO itself would be useless or even hazardous. It is still worth asking whether or not the CDO would be a genuine financial tool for managing risks. The purpose of this paper is to answer this question by exploring the potential of the CDO for controlling general risks. In order to examine the essential structure of the CDO in a neutral manner, we stay away from the problem of controlling financial risks and apply the CDO approach to the classical Newsboy Problem (NBP), where the optimal solution for a VaR (Value at Risk) problem without CDO would be compared against that with CDO.

In the literature, the CDO has been studied largely from the perspective of how to deal with possible dependencies among defaults, see e.g. Li (2000), Duffie and Garlean (2001) and Schonbucher and Schubert (2001), Takada and Sumita (2009), and how to compute the risk-neutral unit premium, see e.g. Landor (2004), Kock, Kraft and Steffensen (2007) and Takada, Sumita and Takahashi (2008). In this paper, we focus on one-term CDO applied to the classical Newsboy Problem (NBP) in order to investigate the effectiveness of the CDO as a means for managing general risks.

The classical NBP is concerned with how to determine the optimal order quantity of a product, whose value drops substantially over one period, so as to maximize the expected profit, given the probability distribution of the demand of the product. Instead of maximizing the expected profit, one often deals with the loss function which can be expressed as the difference between the maximum possible profit and the actual profit. This loss function consists of the loss due to the reduced residual value when the order quantity is larger than the actual demand and the opportunity loss when the order quantity is less than the actual demand. Clearly maximizing the expected profit is equivalent to minimizing the expected loss function. The reader is referred to Khouja (1999) for further details. Recently the classical NBP has been analyzed from the perspective of a conditional VaR problem by Gotoh and Takano (2007).

In our analysis, the loss function of the classical NBP replaces the credit risk in the original CDO context. The risk-neutral unit premium is formally introduced so as to assure no-arbitrage. A VaR problem, which is different from that of Gotoh and Takano (2007), is then formulated without or with CDO. Computational algorithms are developed for evaluating the optimal solutions for the two respective cases. By comparing the optimal solutions without CDO against those with CDO for a broad range of underlying parameter values, the effectiveness of the CDO for controlling general risks is examined.

The structure of this paper is as follows. In Section 2, a general multi-term CDO model is formally described and the risk-neutral unit premium is introduced for assuring no-arbitrage. By incorporating the revenue and cost structure within the framework of the CDO, it is also shown that the CDO would not affect the expected profit. Section 3 is devoted to the classical NBP, providing a succinct summary of the fundamental structure. In Section 4, the associated VaR problem is formulated. In order to solve the VaR problem, the distribution function of the profit is derived explicitly. In Section 5, the CDO approach for the classical NBP is developed and the VaR problem is reformulated with CDO. The distribution function of the profit with CDO is then obtained in a closed form. Numerical examples with uniformly distributed demand are given in Section 6 for illustrating the merits of the CDO approach under certain conditions. Some concluding remarks are given in Section 7.

2. General CDO Model

We consider a financial institution which provides loans to a reference portfolio, i.e. a group of corporations or consumers. Naturally, the financial institution faces the credit risk. The CDO is a structured financial product to control this credit risk by exchanging premium payments from the financial institution to the investors, and certain protections from the investors to the financial institution. More specifically, in the CDO scheme, the credit risk is divided into tranches of increasing seniority, where a tranche is defined by a pair of an attachment point K_a and a detachment point K_d of the cumulative aggregate loss of the reference portfolio. Here, the attachment point K_a means that the protection buyer, which is the financial institution issuing the CDO, is fully responsible for the portfolio loss up to K_a. In principle, the protection seller, which is the tranche investor buying the CDO, compensates the portfolio loss beyond K_a up to K_d for the protection buyer during a contracted period. In exchange, predetermined premiums are paid to the protection seller by the protection buyer according to a predetermined schedule up to the maturity year in such a way that no-arbitrage condition of the credit derivative market is satisfied. These relationships are depicted in Figure 2.1. In what follows, we analyze a mathematical model for the CDO scheme, providing procedural details and certain basic properties.

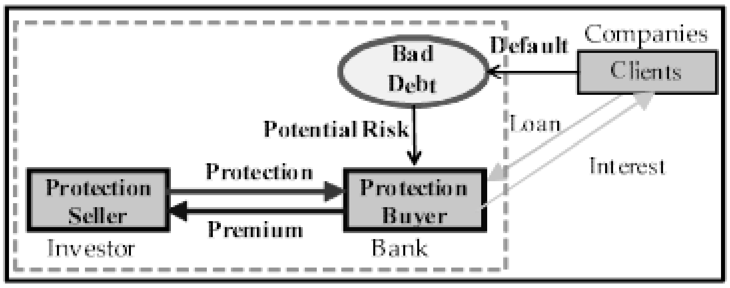


Figure 2.1 Relationship between the Protection Buyer and the Protection Seller

Given a tranche $[K_a, K_d]$, the associated CDO contract consists of a predetermined premium per monetary unit, denoted by ξ, and a predetermined settlement schedule $\underline{\tau} = [\tau_0, \tau_1, \cdots, \tau_K]$ where $\tau_0 = 0 < \tau_1 < \cdots < \tau_K$. Let $\tilde{l}(t)$ be the cumulative aggregate loss of the reference portfolio valued at time t. We note that, throughout the paper, financial values with ~ mean that those values are evaluated at a point in time, and financial values without ~ represent their discounted present values at time τ_0. The protection seller taking the credit exposure to the tranche with K_a and K_d would bear losses occurring in portfolio in excess of K_a but up to K_d. In order to capture such transactions, we introduce $\tilde{L}_{[K_a,K_d]}(t)$ as

$$\tilde{L}_{[K_a,K_d]}(t) \stackrel{\text{def}}{=} \begin{cases} 0 & if \quad \tilde{l}(t) \in [0, K_a] \\ \tilde{l}(t) - K_a & if \quad \tilde{l}(t) \in [K_a, K_d] \\ K_d - K_a & if \quad \tilde{l}(t) \in [K_d, \infty] \end{cases} . \tag{1}$$

If we define $[a]^+ = \max\{0, a\}$, $\tilde{L}_{[K_a,K_d]}(t)$ in (1) can be rewritten as

$$\tilde{L}_{[K_a,K_d]}(t) = [\tilde{l}(t) - K_a]^+ - [\tilde{l}(t) - K_d]^+. \tag{2}$$

In terms of $\tilde{L}_{[K_a,K_d]}(t)$ in (2), the payment to the protection buyer from the protection seller at time τ_k, denoted by $P\tilde{A}Y_{sell\to buy}(\tau_k)$, can be described, for k = 1, 2, 3, $\cdots$, K as

$$P\tilde{A}Y_{sell\to buy}(\tau_0) = 0 \ ; \ P\tilde{A}Y_{sell\to buy}(\tau_k) = \Delta\tilde{L}_{[K_a,K_d]}(\tau_k) \ , \tag{3}$$

where Δ denotes the first difference of a sequence, i.e. given b_k for k = 0, 1 ,⋯, K, we define $\Delta b_k = b_k - b_{k-1}$, k = 1, 2, ⋯, K. In return, the protection buyer pays the premium ξ per monetary unit applied to the remaining protected amount at time τ_{k-1}. More specifically, one sees that, for k = 1, 2, ⋯, K,

$$P\tilde{A}Y_{buy \to sell}(\tau_{k-1}, \xi) \stackrel{\text{def}}{=} \xi \times [(K_d - K_a) - \sum_{i=1}^{k} P\tilde{A}Y_{sell \to buy}(\tau_{i-1})]. \tag{4}$$

By substituting (3) into (4) and working out the summation, it follows that

$$P\tilde{A}Y_{buy \to sell}(\tau_{k-1}, \xi) \stackrel{\text{def}}{=} \xi \times [K_d - K_a - \tilde{L}_{[K_a, K_d]}(\tau_{k-1})] \ , \ k = 1, 2, \cdots, K \ . \tag{5}$$

In order to satisfy the no-arbitrage condition of the credit derivative market, the risk-neutral premium should be determined in such a way that the present value of the expected total payment over the contract period from the protection seller to the protection buyer is equal to that from the protection buyer to the protection seller. For this purpose, let r_f be the risk free rate to be employed for assessing the present value. Based on (3) and (5), the present values $PAY_{sell \to buy}$ and $PAY_{buy \to sell}(\xi)$, describing the present value of the total payment over the contract period from the protection seller to the protection buyer and that from the protection buyer to the protection seller respectively, can be given by

$$PAY_{sell \to buy} = \sum_{k=1}^{K} e^{-r_f \tau_k} P\tilde{A}Y_{sell \to buy}(\tau_k) \ ; PAY_{buy \to sell}(\xi) = \sum_{k=1}^{K} e^{-r_f \tau_{k-1}} P\tilde{A}Y_{buy \to sell}(\tau_{k-1}, \xi) \ . \tag{6}$$

As shown in (2.7) of Takada et al. (2008), the risk-neutral premium ξ_{RN} can be obtained by setting $E[PAY_{sell \to buy}] = E[PAY_{buy \to sell}(\xi_{RN})]$, which we transcribe in the following theorem.

Theorem 2.l

Given a tranche $[K_a, K_d]$ and a contract period $\underline{\tau} = [\tau_0, \tau_1, \cdots, \tau_K]$, the risk-neutral premium ξ_{RN} per monetary unit for the associated CDO is given by

$$\xi_{RN} = \frac{\sum_{k=1}^{K} e^{-r_f \tau_k} E[\Delta \tilde{L}_{[K_a, K_d]}(\tau_k)]}{\sum_{k=1}^{K} e^{-r_f \tau_{k-1}} E[(K_d - K_a) - \tilde{L}_{[K_a, K_d]}(\tau_{k-1})]} \ .$$

When the risk-neutral premium ξ_{RN} is employed, the CDO would not affect the expected profit. In order to observe this point explicitly, let $\tilde{R}V(\tau_k)$ be the cumulative revenue of the protection buyer evaluated at time τ_k. Furthermore, let $\tilde{P}R(\tau_k)$ and $\tilde{P}R_{CDO}(\tau_k)$ be the profit of the protection buyer over the k-th period evaluated at time τ_k without and with CDO respectively. One then sees, for k=0, 1, ⋯, K, that

$$\tilde{PR}(\tau_k) = \Delta[\tilde{RV}(\tau_k) - \tilde{l}(\tau_k)] \tag{7}$$

and

$$\tilde{PR}_{CDO}(\tau_k, \xi) = \tilde{PR}(\tau_k) - e^{r_f \Delta\tau_k} \tilde{PAY}_{buy \to sell}(\tau_{k-1}, \xi) + \tilde{PAY}_{sell \to buy}(\tau_k). \tag{8}$$

As before, from (7) and (8), the present value of the profit of the protection buyer without or with CDO can be obtained as

$$PR = \sum_{k=1}^{K} e^{-r_f \tau_k} \tilde{PR}(\tau_k) \ , \ PR_{CDO}(\xi) = \sum_{k=1}^{K} e^{-r_f \tau_k} \tilde{PR}_{CDO}(\tau_k, \xi) \ . \tag{9}$$

We are now in a position to prove the following theorem.

Theorem 2.2

Let PR and $PR_{CDO}(\xi)$ be as in (9) and define π = E[PR] and $\pi_{CDO}(\xi)$ = E[$PR_{CDO}(\xi)$]. One then has $\pi = \pi_{CDO}(\xi_{RN})$.

Proof

From the definition of $\pi_{CDO}(\xi_{RN})$ together with (7), (8) and (9), it can be seen that

$$\pi_{CDO}(\xi_{RN}) = \pi - E[PAY_{buy \to sell}(\xi_{RN})] + E[PAY_{sell \to buy}] \ , \tag{10}$$

where the last two terms on the right hand side of the above equation cancel each other from Theorem 2.1, completing the proof.

Theorem 2.2 states that the CDO scheme has no impact on the expected profit. In order to explore the effectiveness of the CDO for risk management, it is therefore necessary to introduce an objective function which involves the probability distribution of the profit beyond its expectation. In this regard, we consider the following optimization problems:

[VaR] $\quad$ $minimize\ \eta \quad subject\ to\ P[PR \le v_0] \le \eta\ ,\ \pi \ge v_1$

[VaR-CDO] $\quad$ $minimize\ \eta_{CDO} \quad subject\ to\ P[PR_{CDO}(\xi_{RN}) \le v_0] \le \eta_{CDO}\ ,\ \pi_{CDO}(\xi_{RN}) \ge v_1$

The question concerning the effectiveness of CDO for risk management can be answered by comparing the optimal solution η_{CDO}^{**} for VaR-CDO against the optimal solution η^{**} for VaR.

3. Classical Newsboy Problem: Expected Profit Maximization Approach

We consider a product whose value drops substantially after a fixed point in time, say τ. The demand for the product over the period [0, τ] is given as a non-negative random variable D. Throughout the paper, it is assumed that the distribution function of D is absolutely continuous with $F_D(x) \overset{\text{def}}{=} P[D \le x] = \int_0^x f_D(y)dy$ having the mean $\mu_D \overset{\text{def}}{=} E[D]$. The corresponding survival function is given by $\bar{F}_D(x) \overset{\text{def}}{=} P[D > x] = 1 - F_D(x) = \int_x^\infty f_D(y)dy$.

Let $\tilde{c}$ and $\tilde{p}$ be the procurement cost and the sales price per one product respectively. Given that the order quantity is Q, if D < Q, each unsold product has the residual value $\tilde{r}$. It is natural to assume that

$$0 < \tilde{r} < \tilde{c} < \tilde{p} \ . \tag{11}$$

If D > Q, each of the lost opportunities would cost $\tilde{s}$. Assuming that the payment would be made and the revenue would be received at time τ, the profit $\tilde{P}R(Q,D)$ can then be described as

$$\tilde{PR}_{NBP}(Q,D) = (\tilde{p}-\tilde{c})Q - (\tilde{p}-\tilde{r})\,[Q-D]^+ - \tilde{s}\,[D-Q]^+ \ . \tag{12}$$

Let the expectation of $\tilde{P}R_{NBP}(Q,D)$ be denoted by

$$\tilde{\pi}_{NBP}(Q) \overset{\text{def}}{=} E[\tilde{PR}_{NBP}(Q,D)] \ . \tag{13}$$

The classical NBP is then to determine the optimal order quantity Q^*_{NBP} so as to maximize $\tilde{\pi}_{NBP}(Q)$. For notational convenience, we write

$$Q^*_{NBP} = arg\ \underset{Q}{max}\ \tilde{\pi}_{NBP}(Q) \ . \tag{14}$$

From (12), the maximum profit that one can expect is $(\tilde{p}-\tilde{c}) \times D$ which occurs if Q happens to be D. The difference between this maximum profit and the actual profit may then be interpreted as the opportunity loss. More formally, we define

$$\tilde{l}_{NBP}(Q,D) \overset{\text{def}}{=} (\tilde{p}-\tilde{c})D - \tilde{PR}_{NBP}(Q,D) \ . \tag{15}$$

If we introduce $\tilde{c}_O$ and $\tilde{c}_U$ as

$$\tilde{c}_O \overset{\text{def}}{=} \tilde{c}-\tilde{r}\ ;\ \tilde{c}_U \overset{\text{def}}{=} \tilde{p}-\tilde{c}+\tilde{s}\ , \tag{16}$$

one sees from (12) and (15) that

$$\tilde{l}_{NBP}(Q,D) = \tilde{c}_O\,[Q-D]^+ + \tilde{c}_U\,[D-Q]^+ \ . \tag{17}$$

Let the expectation of $\tilde{l}_{NBP}(Q,D)$ be denoted by

$$\mu_{\tilde{l}:NBP}(Q) = E[\tilde{l}_{NBP}(Q,D)] \ . \tag{18}$$

It can be readily seen from (13) through (18) that maximizing $\tilde{\pi}_{NBP}(Q)$ is equivalent to minimizing $\mu_{\tilde{l}:NBP}(Q)$. It then follows that

$$Q^*_{NBP} = arg \min_{Q} \mu_{\tilde{l}:NBP}(Q) \ . \tag{19}$$

From (17), it can be shown that

$$\bar{H}_{\tilde{l}:NBP}(Q,x) = P[\tilde{l}_{NBP}(Q,D) > x] = F_D(Q - \frac{x}{\tilde{c}_O}) + \bar{F}_D(Q + \frac{x}{\tilde{c}_U}) \ . \tag{20}$$

Since $\mu_{\tilde{l}:NBP}(Q) = \int_0^\infty \bar{H}_{\tilde{l}:NBP}(Q,x)dx$, it then follows that

$$\mu_{\tilde{l}:NBP}(Q) = \tilde{c}_O \int_0^Q F_D(x) + \tilde{c}_U \int_Q^\infty \bar{F}_D(x)dx \ . \tag{21}$$

By differentiating $\mu_{\tilde{l}:NBP}(Q)$ with respect to Q twice, one finds that

$$\frac{\partial}{\partial Q}\mu_{\tilde{l}:NBP}(Q) = (\tilde{c}_O + \tilde{c}_U)F_D(Q) - \tilde{c}_U \ , \tag{22}$$

and

$$(\frac{\partial}{\partial Q})^2\mu_{\tilde{l}:NBP}(Q) = (\tilde{c}_O + \tilde{c}_U)f_D(Q) > 0 \ . \tag{23}$$

Hence, $\mu_{\tilde{l}:NBP}(Q)$ is strictly convex in Q and has the unique minimum point Q^*_{NBP} at which the first order derivative in (22) vanishes. Accordingly, it follows that

$$Q^*_{NBP} = F_D^{-1}(\frac{\tilde{c}_U}{\tilde{c}_O + \tilde{c}_U}) \ . \tag{24}$$

For incorporating the one-term CDO approach in the context of the NBP, it is necessary to convert the monetary values evaluated at time τ, where such values are highlighted by ~ in the above discussions, into the corresponding present values. This can be accomplished by discounting the monetary values evaluated at time τ by $e^{-r_f\tau}$ where r_f is the risk free rate as introduced in Section 2. The present value of a monetary value evaluated at time τ is denoted by dropping ~ in the notation. One can confirm the following conversions.

$$p = e^{-r_f\tau}\tilde{p} \ ; \ c = e^{-r_f\tau}\tilde{c} \ ; \ r = e^{-r_f\tau}\tilde{r} \ ; \ s = e^{-r_f\tau}\tilde{s} \ ; \ PR_{NBP}(Q,D) = e^{-r_f\tau}\tilde{PR}_{NBP}(Q,D) \ ; \tag{25}$$

$$\pi_{NBP}(Q) = e^{-r_f\tau}\tilde{\pi}_{NBP}(Q) \ ; \ l_{NBP}(Q,D) = e^{-r_f\tau}\tilde{l}_{NBP}(Q,D) \ ;$$

$$c_O = e^{-r_f\tau}\tilde{c}_O \ ; \ c_U = e^{-r_f\tau}\tilde{c}_U$$

From (25), it can be readily seen that $\pi_{NBP}(Q)$ achieves the maximum also at Q^*_{NBP} and one has

$$Q^*_{NBP} = arg\ \max_Q\ \pi_{NBP}(Q) = F_D^{-1}(\frac{c_U}{c_O + c_U})\ . \tag{26}$$

It should be noted from (13), (15) and (25) that

$$\pi_{NBP}(Q) = (p - c)\mu_D - \mu_{l:NBP}(Q)\ , \tag{27}$$

where $\mu_{l:NBP}(Q)$ can be obtained from (21) and (25) as

$$\mu_{l:NBP}(Q) = (c_O + c_U)\int_0^Q F_D(x)dx + c_U(\mu_D - Q)\ . \tag{28}$$

The next theorem provides a necessary and sufficient condition for the maximum expected profit $\pi_{NBP}(Q^*_{NBP})$ to be positive.

Theorem 3.1

$\pi_{NBP}(Q^*_{NBP}) > 0$ if and only if $\frac{s}{c_O + c_U} < \frac{1}{\mu_D}\int_0^{Q^*_{NBP}} s dF_D(x)$.

Proof

From (21), (25) and (26), after a little algebra, one finds that

$$\mu_{l:NBP}(Q^*_{NBP}) = (c_O + c_U)\int_{Q^*_{NBP}}^{\infty} x dF_D(x) - c_O\mu_D\ . \tag{29}$$

Since p – c = c_U – s from (16) and (25), substituting (29) into $\pi_{NBP}(Q^*_{NBP})$ in (27) yields

$$\pi_{NBP}(Q^*_{NBP}) = (c_O + c_U)\left\{\mu_D - \int_{Q^*_{NBP}}^{\infty} x dF_D(x)\right\} - s\mu_D$$

$$= (c_O + c_U)\int_0^{Q^*_{NBP}} x dF_D(x) - s\mu_D\ ,$$

and the theorem follows.

Throughout the paper, we assume that the condition of Theorem 3.1 is satisfied and $\pi_{NBP}(Q^*_{NBP}) > 0$.

4. Classical Newsboy Problem: Value at Risk Approach

We now consider the VaR problem for the classical NBP as specified below.

[VaR-NBP]

$$\underset{Q}{minimize}\ \eta_{NBP} \quad subject\ to\ P[PR_{NBP}(Q, D) \le v_0] \le \eta_{NBP};\ \pi_{NBP}(Q) \ge v_1$$

In order to solve this problem numerically, it is necessary to evaluate the distribution function of $PR_{NBP}(Q,D)$.

Theorem 4.1

Let $\tilde{W}_{NBP}(Q,x) = P\left[\tilde{P}R_{NBP}(Q,D) \le x\right]$ and $W_{NBP}(Q,x) = P\left[PR_{NBP}(Q,D) \le x\right]$. One then has the followings.

$$(a)\quad \tilde{W}_{NBP}(Q,x) = \begin{cases} F_D(\frac{\tilde{c}_O Q + x}{\tilde{p}-\tilde{r}}) + \bar{F}_D(\frac{\tilde{c}_U Q - x}{\tilde{s}}) & if\ x \le (\tilde{p}-\tilde{c})Q \\ 1 & otherwise \end{cases}$$
$$(b)\quad W_{NBP}(Q,x) = \begin{cases} F_D(\frac{c_O Q + x}{p-r}) + \bar{F}_D(\frac{c_U Q - x}{s}) & if\ \ x \le (p-c)Q \\ 1 & otherwise \end{cases}$$

Proof

We first define

$$\tilde{\kappa}_{NBP}(Q,D) \stackrel{\text{def}}{=} (\tilde{p}-\tilde{r})[Q-D]^+ + \tilde{s}[D-Q]^+ \tag{30}$$

so that one sees from (12) that

$$\tilde{PR}_{NBP}(Q,D) = (\tilde{p}-\tilde{c})Q - \tilde{\kappa}_{NBP}(Q,D)\ . \tag{31}$$

From the law of total probability, it can be seen that

$$P[\tilde{\kappa}_{NBP}(Q,D) > x] = P[\tilde{\kappa}_{NBP}(Q,D) > x, 0 \le D \le Q] \tag{32}$$
$$+P[\tilde{\kappa}_{NBP}(Q,D) > x, D > Q]\ .$$

From the definition of $\tilde{\kappa}_{NBP}(Q,D)$ in (30), the right hand side of Equation (32) can be rewritten as

$$P[0 \le D \le min\left\{Q, Q - \frac{x}{\tilde{p}-\tilde{r}}\right\}] + P[D > max\left\{Q, Q + \frac{x}{\tilde{s}}\right\}]\ .$$

It then follows that

$$P[\tilde{\kappa}_{NBP}(Q,D) > x] = \begin{cases} F_D(Q - \frac{x}{\tilde{p}-\tilde{r}}) + \bar{F}_D(Q + \frac{x}{\tilde{s}}) & if\ \ x \ge 0 \\ 1 & otherwise \end{cases}\ . \tag{33}$$

From (31), this then leads to

$$\tilde{W}_{NBP}(Q,x) = P[\tilde{\kappa}_{NBP}(Q,D) > (\tilde{p}-\tilde{c})Q - x]\ ,$$

and part (a) follows from (33). Part (b) is immediate from (25) since

$$W_{NBP}(Q,x) = \tilde{W}_{NBP}(Q, e^{-r_f \tau} x) \ ,$$

completing the proof.

Under the condition of Theorem 3.1, the range of Q satisfying $\pi_{NBP}(Q) \geq v_1$ can be found as a connected interval since $\pi_{NBP}(Q)$ is strictly concave from (23) and (27). Given v_0 and v_1, the optimal solution η^*_{NBP} can then be computed from Theorem 4.1 (b) by applying the bi-section method with respect to Q in this interval.

5. Application of CDO Approach to Classical Newsboy Problem

The risk structure of the classical NBP is contained in the opportunity loss $\tilde{l}_{NBP}(Q,D)$ given in (15) where the discrepancy between the actual order quantity Q and the actual occurrence of the stochastic demand D dictates its magnitude. In order to see the potential of the CDO approach for risk management in general by applying it to the classical NBP, it is then natural to replace the cumulative aggregate loss $\tilde{l}(t)$ of the reference portfolio in the original CDO model by $\tilde{l}_{NBP}(Q,D)$.

Let $\tilde{L}_{[K_a,K_d]}(Q,D)$ be defined as in (1) where $\tilde{l}(t)$ is replaced by $\tilde{l}_{NBP}(Q,D)$ given in (17). Then, the mean of $\tilde{L}_{[K_a,K_d]}(Q,D)$ and the risk-neutral premium for the NBP with CDO can be obtained from Theorem 2.1 with K=1, (17) and (18), as stated in the next theorem.

Theorem 5.1

$$(a) \quad \mu_{\tilde{L}[K_a,K_d]}(Q) = \int_{K_a}^{K_d} \left\{ F_D(Q - \frac{x}{\tilde{c}_O}) + \bar{F}_D(Q + \frac{x}{\tilde{c}_U}) \right\} dx$$

$$(b) \quad \xi^{**}_{RN:NBP}(Q) = \frac{e^{-r_f \tau} \mu_{\tilde{L}[K_a,K_d]}(Q)}{K_d - K_a}$$

With this risk-neutral premium $\xi^{**}_{RN:NBP}(Q)$, the total payment form the protection buyer to the protection seller paid at time 0 is deterministic for the NBP with CDO and is given by

$$PAY_{buy \to sell[K_a,K_d]}(Q,D) = \xi^{**}_{RN:NBP}(Q) \times (K_d - K_a) = e^{-r_f \tau} \mu_{\tilde{L}[K_a,K_d]}(Q) \ . \quad (34)$$

Accordingly, the net profit $PR_{CDO}(Q,D)$ can be described as

$$PR_{CDO}(Q,D) = PR_{NBP}(Q,D) - e^{-r_f \tau} \mu_{\tilde{L}[K_a,K_d]}(Q) + e^{-r_f \tau} \tilde{L}_{[K_a,K_d]}(Q,D) \ . \quad (35)$$

Of particular importance for further study is the distribution function of $PR_{CDO}(Q,D)$ defined as

$$W_{CDO}(Q,x) = P[PR_{CDO}(Q,D) \le x] \ . \tag{36}$$

In the next theorem, we derive $W_{CDO}(Q,x)$ in terms of demand probabilities under various conditions.

Theorem 5.2

Let $W_{CDO}(Q,x)$ be as defined in (36). One then has

$$W_{CDO}(Q,x) = \sum_{i=1}^{6} G_i(Q,x) \ ,$$

where

$$G_1(Q,x) = P[Q - \frac{K_a}{\tilde{c}_O} \le D < min\left\{Q, \frac{x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) + c_O Q}{p-r}\right\}] \ ;$$

$$G_2(Q,x) = P[max\left\{Q, \frac{x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) - c_U Q}{-s}\right\} \le D < Q + \frac{K_a}{\tilde{c}_U}] \ ;$$

$$G_3(Q,x) = P[Q - \frac{K_d}{\tilde{c}_O} \le D < min\left\{\frac{x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) + e^{-r_f\tau}K_a}{p-c}, Q - \frac{K_a}{\tilde{c}_O}\right\}] \ ;$$

$$G_4(Q,x) = P[Q + \frac{K_a}{\tilde{c}_U} \le D < min\left\{\frac{x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) + e^{-r_f\tau}K_a}{p-c}, Q + \frac{K_d}{\tilde{c}_U}\right\}] \ ;$$

$$G_5(Q,x) = P[0 \le D < min\left\{\frac{x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) - e^{-r_f\tau}(K_d - K_a) + c_O Q}{p-r}, Q - \frac{K_d}{\tilde{c}_O}\right\}] \ ;$$

$$G_6(Q,x) = P[max\left\{Q + \frac{K_d}{\tilde{c}_U}, \frac{x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) - e^{-r_f\tau}(K_d - K_a) - c_U Q}{-s}\right\} \le D] \ .$$

Proof

From (35), one sees that

$$W_{CDO}(Q,x) = P[PR_{NBP}(Q,D) + e^{-r_f\tau}\tilde{L}_{[K_a,K_d]}(Q,D) \le x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q)] \ .$$

For notational convenience, we denote the condition inside this probability by

$$COND(Q,D,x)$$

$$\Leftrightarrow PR_{NBP}(Q,D) + e^{-r_f\tau}\tilde{L}_{[K_a,K_d]}(Q,D) \le x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) \ . \tag{37}$$

With this notation, the law of total probability implies that

$$W_{CDO}(Q,x) \tag{38}$$

$$= P[COND(Q,D,x), \tilde{l}_{NBP}(Q,D) \in [0,K_a]]$$

$$+P[COND(Q,D,x), \tilde{l}_{NBP}(Q,D) \in [K_a,K_d]]$$

$$+P[COND(Q,D,x), \tilde{l}_{NBP}(Q,D) \in [K_d,\infty]] \ .$$

We now apply the law of total probability again with the conditions $0 \le D \le Q$ and $Q < D$ to each term on the right hand side of the above equation. For the first term, under the condition $\tilde{l}_{NBP}(Q,D) \in [0,K_a]$, one has $\tilde{L}_{[K_a,K_d]}(Q) = 0$ from (1) so that COND(Q, D, x) is simplified to

$$COND_1(Q,D,x) \Leftrightarrow PR_{NBP}(Q,D) \le x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) \ . \tag{39}$$

Combined with the condition $0 \le D \le Q$, $COND_1$ (Q, D, x) can be further reduced from (15), (17) and (25) to

$$COND_{11}(Q,D,x) \Leftrightarrow (p-c)D - c_O(Q-D) < x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) \tag{40}$$

$$\Leftrightarrow D < \frac{x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) + c_O Q}{p-r} \ .$$

Under $0 \le D \le Q$, the condition $\tilde{l}_{NBP}(Q,D) \in [0,K_a]$ becomes equivalent to

$$\tilde{l}_{NBP}(Q,D) \in [0,K_a] \Leftrightarrow Q - \frac{K_a}{\tilde{c}_O} < D < Q \ . \tag{41}$$

The probability of satisfying both (40) and (41) is then equal to $G_1(Q, x)$.

Similarly, when $Q < D$ is satisfied, $COND_1(Q, D, x)$ can be reduced from (15), (17) and (25) to

$$COND_{12}(Q,D,x) \Leftrightarrow (p-c)D - c_U(D-Q) < x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) \tag{42}$$

$$\Leftrightarrow \frac{x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) - c_U Q}{-s} < D \ .$$

We also note that, under the condition $Q < D$, one has

$$\tilde{l}_{NBP}(Q,D) \in [0,K_a] \Leftrightarrow Q < D < Q + \frac{K_a}{\tilde{c}_U} \ . \tag{43}$$

Hence the probability of satisfying both (42) and (43) is given by $G_2(Q, x)$. Consequently, we have shown that the first term on the right hand side of (38) is equal to $G_1(Q, x) + G_2(Q, x)$. It can be shown in a similar manner that the second term becomes $G_3(Q, x) + G_4(Q, x)$, and the third term is equal to $G_5(Q, x) + G_6(Q, x)$, completing the proof.

The expressions for $G_i(Q, x)$ $(i = 1,\cdots, 6)$ in Theorem 5.2 are somewhat awkward and may not be suitable for computing $W_{CDO}(Q,x)$ repeatedly for different values of Q and x. In order to facilitate repeated computations of $W_{CDO}(Q,x)$ better, alternative expressions for $G_i(Q, x)$ $(i = 1,\cdots, 6)$ are given in Appendix where proofs are omitted. Based on Theorem 5.2 together with those theorems in Appendix, $W_{CDO}(Q,x)$ can be computed repeatedly for different values of Q and x with speed and accuracy. Accordingly, we are now in a position to numerically explore the VaR for the classical NBP with CDO. More specifically, of our main concern is the following problem.

[VaR-NBP-CDO]

$$\underset{Q}{minimize}\ \eta_{CDO} \quad subject\ to\ P[PR_{CDO}(Q,D) \le v_0] \le \eta_{CDO};\ \pi_{CDO}(Q) \ge v_1$$

6. Numerical Examples with Uniformly Distributed Demand

For exploring the potential of the CDO approach in general risk management, in this section, VaR-NBP defined in Section 4 would be compared with VaR-NBP-CDO introduced in Section 5 through numerical examples. In order to conduct such numerical experiments systematically, we assume that the demand D is uniformly distributed, i.e. the p.d.f. $f_D(x)$ of D is defined by

$$f_D(x) = \begin{cases} \frac{1}{b} & if\ a \le x \le a+b \\ 0 & otherwise \end{cases} . \tag{44}$$

The distribution function and the survival function of D can be written as

$$F_D(x) = \begin{cases} 0 & if\ x \le a \\ \frac{x-a}{b} & if\ a \le x \le a+b \\ 1 & if\ a+b \le x \end{cases} \qquad \bar{F}_D(x) = \begin{cases} 1 & if\ x \le a \\ 1-\frac{x-a}{b} & if\ a \le x \le a+b \\ 0 & if\ a+b \le x \end{cases} . \tag{45}$$

The basic set of the parameter values to be employed in this section is provided in Table 6.1, which we assume unless specified otherwise.

Table 6.1 Basic Set of Parameter Values

p	the unit sales price	3
c	the unit procurement cost	1
r	the unit residual value	0.1
s	the unit opportunity cost	0.5
μ_D	the mean of the demand	5000
r_f	the risk free rate	0.0001
K_a	the attachment point	500
K_d	the detachment point	1000, 2000, 3000

We note that two parameters a and b for the distribution of the demand D are related to each other, when μ_D is fixed, as $a = \mu_D - b/2$.

From (26), one finds that the optimal order quantity Q^*_{NBP} maximizing the expected profit is given by

$$Q^*_{NBP} = a + \frac{c_U}{c_O + c_U} b \quad . \tag{46}$$

The corresponding maximum expected profit is then obtained from (27) as

$$\pi_{NBP}(Q^*_{NBP}) = ac_U + \frac{bc_U^2}{2(c_O + c_U)} - \frac{s(2a+b)}{2} \; . \tag{47}$$

From (27) together with (46) and (47), it follows that

$$\pi_{NBP}(Q) \;\; = \;\; \pi_{NBP}(Q^*_{NBP}) - \frac{c_O + c_U}{2b}\left(Q - Q^*_{NBP}\right)^2 \quad . \tag{48}$$

The expected loss can then be given from (27) as

$$\mu_{l:NBP}(Q) = (p-c)\mu_D - \pi_{NBP}(Q) \quad . \tag{49}$$

Figures 6.1 and 6.2 depict π(Q) and $\mu_{l:NBP}(Q)$ for b=2000, 2500 and 3000.

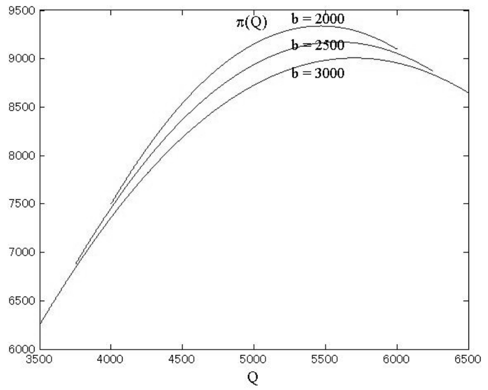


Figure 6.1 Expected Profit π(Q)

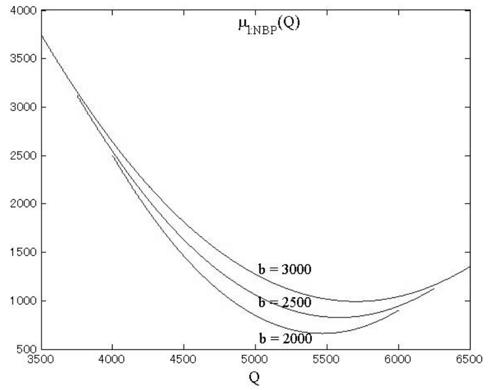


Figure 6.2 Expected Loss $\mu_{l:NBP}(Q)$

We next turn our attention to VaR-NBP introduced in Section 4. The feasible region is denoted by

$$\mathcal{F}_I(v_1) \stackrel{\text{def}}{=} \{Q : \pi_{NBP}(Q) \geq v_1\} \quad . \tag{50}$$

From (48), we see that $\mathcal{F}_I(v_1)$ can be rewritten as

$$\mathcal{F}_I(v_1) \;\; = \;\; \{Q : Q_{v_{1:L}} \leq Q \leq Q_{v_{1:R}}\} \quad , \tag{51}$$

where

$$Q_{v_{1:L}} = max\left\{Q^*_{NBP} - \sqrt{\frac{2b}{c_O + c_U}\{\pi_{NBP}(Q^*_{NBP}) - v_1\}}, a\right\} \; ;$$

$$Q_{v_{1:R}} = min\left\{Q^*_{NBP} + \sqrt{\frac{2b}{c_O + c_U}\{\pi_{NBP}(Q^*_{NBP}) - v_1\}}, a+b\right\} \; . \tag{52}$$

In order to evaluate $W_{NBP}(Q,x)$, we recall from Theorem 4.1 (b) that

$$W_{NBP}(Q,x) \;=\; \begin{cases} F_D\left(\rho_F(Q,x)\right) + \overline{F}_D\left(\rho_{\overline{F}}(Q,x)\right) & if\ x \le (p-c)Q \\ 1 & otherwise \end{cases} \; , \tag{53}$$

where $$\rho_F(Q,x) = \frac{c_O Q + x}{p-r} \; ; \; \rho_{\overline{F}}(Q,x) = \frac{c_U Q - x}{s} \; . \tag{54}$$

From (45), it can be seen that

$$F_D\left(\rho_F(Q,v_0)\right) \;=\; \begin{cases} 0 & if\ Q \le q_{F:a}(v_0) \\ \frac{\rho_F(Q,v_0)-a}{b} & if\ q_{F:a}(v_0) \le Q \le q_{F:b}(v_0) \\ 1 & if\ q_{F:b}(v_0) \le Q \end{cases} \tag{55}$$

and

$$\overline{F}_D\left(\rho_{\overline{F}}(Q,v_0)\right) \;=\; \begin{cases} 1 & if\ Q \le q_{\overline{F}:a}(v_0) \\ 1 - \frac{\rho_{\overline{F}}(Q,v_0)-a}{b} & if\ q_{\overline{F}:a}(v_0) \le Q \le q_{\overline{F}:b}(v_0) \\ 0 & if\ q_{\overline{F}:b}(v_0) \le Q \end{cases} \; , \tag{56}$$

where

$$q_{F:a}(v_0) \overset{\text{def}}{=} \frac{a(p-r)-v_0}{c_O}, \qquad q_{F:b}(v_0) \overset{\text{def}}{=} \frac{(a+b)(p-r)-v_0}{c_O} \; ,$$

$$q_{\overline{F}:a}(v_0) \overset{\text{def}}{=} \frac{sa+v_0}{c_U} \; , \qquad q_{\overline{F}:b}(v_0) \overset{\text{def}}{=} \frac{s(a+b)+v_0}{c_U} \; .$$

The next lemma then immediately follows.

Lemma 6.1

(a) $v_0 \ge a(p-c) - \frac{c_O}{c_O + c_U} sb \Leftrightarrow q_{F:a}(v_0) \le q_{\overline{F}:b}(v_0)$

(b) $v_0 \ge a(p-c) \Leftrightarrow q_{F:a}(v_0) \le q_{\overline{F}:a}(v_0)$

(c) $v_0 \le (a+b)(p-c) \Leftrightarrow q_{\overline{F}:b}(v_0) \le q_{F:b}(v_0)$

Let

$$\mathcal{F}_{II}(v_0) \;=\; \{Q : v_0 \le (p-c)Q, a \le Q \le a+b\} \tag{57}$$

and assume that

$$a(p-c) \leq v_0 \leq (a+b)(p-c) \quad . \tag{58}$$

Lemma 6.1 then yields the next lemma.

Lemma 6.2

Under the assumption of (58), the following statements hold true.

(a) $Q \in \mathcal{F}_{II}(v_0) \Rightarrow q_{F:a}(v_0) \leq q_{\overline{F}:a}(v_0) \leq q_{\overline{F}:b}(v_0) \leq q_{F:b}(v_0)$

(b) $Q \in \mathcal{F}_{II}(v_0) \Rightarrow \rho_F(Q,x) \leq \rho_{\overline{F}}(Q,x)$

We are now in a position to prove the next theorem.

Theorem 6.3

Under the condition of (58), the optimal solution of VaR-NBP can be obtained as

$$\eta_{NBP}^{**} = \begin{cases} \frac{\rho_F(Q_{v_1:L},v_0)-a}{b} & if\ q_{\overline{F}:b}(v_0) \leq Q_{v_1:L} \\ \frac{\rho_F(q_{\overline{F}:b}(v_0),v_0)-a}{b} & if\ Q_{v_1:L} \leq q_{\overline{F}:b}(v_0) \leq Q_{v_1:R} \\ \frac{b+\rho_F(Q_{v_1:R},v_0)-\rho_{\overline{F}}(Q_{v_1:R},v_0)}{b} & if\ Q_{v_1:R} \leq q_{\overline{F}:b}(v_0) \end{cases} .$$

Proof

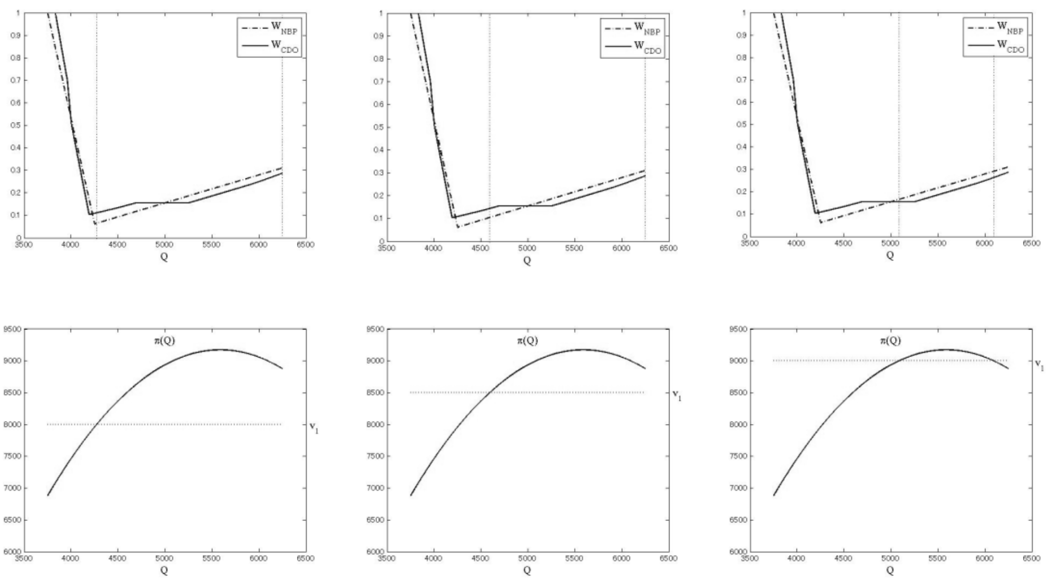
From (53) through (56) combined with Lemma 6.2, one finds that

$$W_{NBP}(Q,v_0) = \begin{cases} 1 & if\ Q \leq q_{\overline{F}:a}(v_0) \\ \frac{b+\rho_F(Q,v_0)-\rho_{\overline{F}}(Q,v_0)}{b} & if\ q_{\overline{F}:a}(v_0) \leq Q \leq q_{\overline{F}:b}(v_0) \\ \frac{\rho_F(Q,v_0)-a}{b} & if\ q_{\overline{F}:b}(v_0) \leq Q \leq q_{F:b}(v_0) \\ 1 & if\ q_{F:b}(v_0) \leq Q \end{cases} . \tag{59}$$

It can be seen that, as a function of Q, $W_{NBP}(Q,v_0)$ is linearly decreasing given by $\frac{b+\rho_F(Q,v_0)-\rho_{\overline{F}}(Q,v_0)}{b}$ for $q_{\overline{F}:a}(v_0) \leq Q \leq q_{\overline{F}:b}(v_0)$ and linearly increasing as $\frac{\rho_F(Q,v_0)-a}{b}$ for $q_{\overline{F}:b}(v_0) \leq Q \leq q_{F:b}(v_0)$. The theorem then follows from (50) and (51).

Unfortunately, the counterpart of Theorem 6.3 for VaR-NBP-CDO is not available and η_{CDO}^{**} cannot be evaluated explicitly. One has to resort to numerical solutions based on the bi-section method using Theorems A.1 through A.6 given in Appendix.

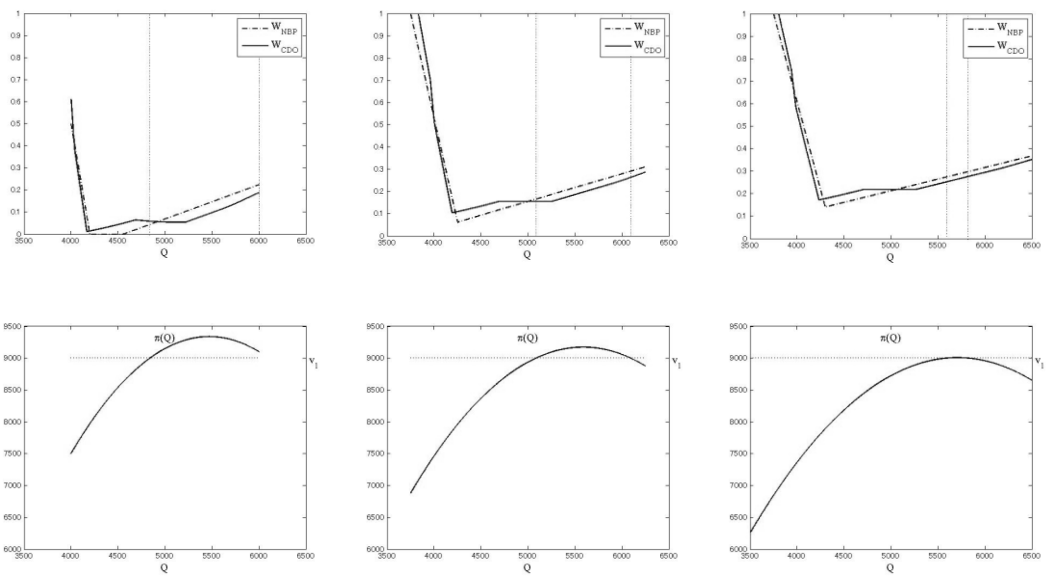
In Figure 6.3, $W_{NBP}(Q,7500)$ and $W_{CDO}(Q,7500)$ are plotted along with $\pi(Q)$, where v_1 is varied for 8000, 8500 and 9000, while K_d=1000 and b=2500 are fixed. One sees that the feasible region $\mathcal{F}_I(v_1)$ becomes narrower as the threshold, v_1, of the expected profit increases. Accordingly, both η^{**}_{NBP} and η^{**}_{CDO}, the optimal solutions for VaR-NBP and VaR-NBP-CDO respectively, become worse and increase as v_1 increases. It is worth noting that the CDO approach is effective only when v_1 becomes sufficiently large.



v_1=8000 v_1=8500 v_1=9000

Figure 6.3 π(Q) and W_{NBP}(Q,7500) v.s. W_{CDO}(Q,7500) [K_d=1000, b=2500]

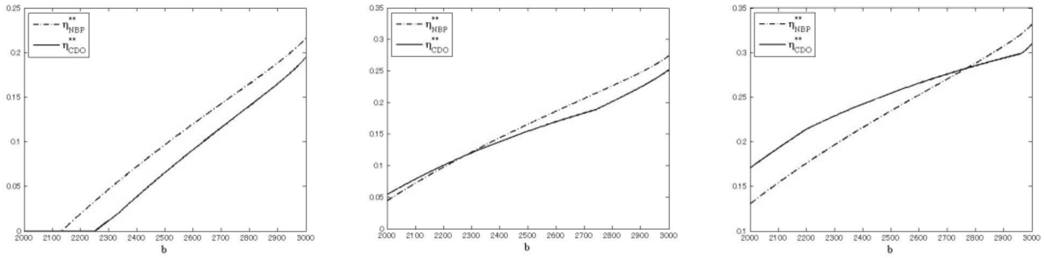
Similarly to Figure 6.3, $W_{NBP}(Q,7500)$, $W_{CDO}(Q,7500)$ and π(Q) are depicted in Figure 6.4 where b is varied for 2000, 2500 and 3000, while v_1 is fixed at 9000 with K_d=1000. It should be noted that π(Q) decreases while both $W_{NBP}(Q,7500)$ and $W_{CDO}(Q,7500)$ increase as b increases. This means that it becomes more difficult to control the profit as the variability of the stochastic demand becomes larger. While $\eta^{**}_{NBP} < \eta^{**}_{CDO}$ for b = 2000, this inequality is reversed and the CDO approach becomes effective for b = 2500 or 3000.



b=2000 b=2500 b=3000

Figure 6.4 π(Q) and W_{NBP}(Q,7500) v.s. W_{CDO}(Q,7500) [K_d=1000 , v_1=9000]

In order to observe the impact of the variability of the stochastic demand on the optimal solutions more closely, Figure 6.5 depicts η^{**}_{NBP} and η^{**}_{CDO} as functions of b, where v_0 is varied for 7000, 7500 and 8000, while v_1 = 9000 and K_d=1000 are fixed. One sees that the CDO approach dominates the performance without CDO for v_0 = 7000. For v_0 = 7500 and 8000, the CDO approach becomes effective when b becomes sufficiently large, and this breaking point becomes larger as v_0 becomes larger.



$v_0 = 7000$ $v_0 = 7500$ $v_0 = 8000$

Figure 6.5 η^{**}_{NBP} **v.s.** η^{**}_{CDO} **[K_d=1000 , v_1=9000]**

Finally, Figure 6.6 illustrates how η^{**}_{NBP} and η^{**}_{CDO} are impacted when (v_0, v_1) and (b, K_d) are changed, where the white areas represent the regions in which the CDO approach is effective. It can be observed that the CDO approach can be effective only when v_1 is sufficiently large. The area in which the CDO approach performs better shifts toward the lower side of v_0 and becomes larger as b increases or K_d decreases.

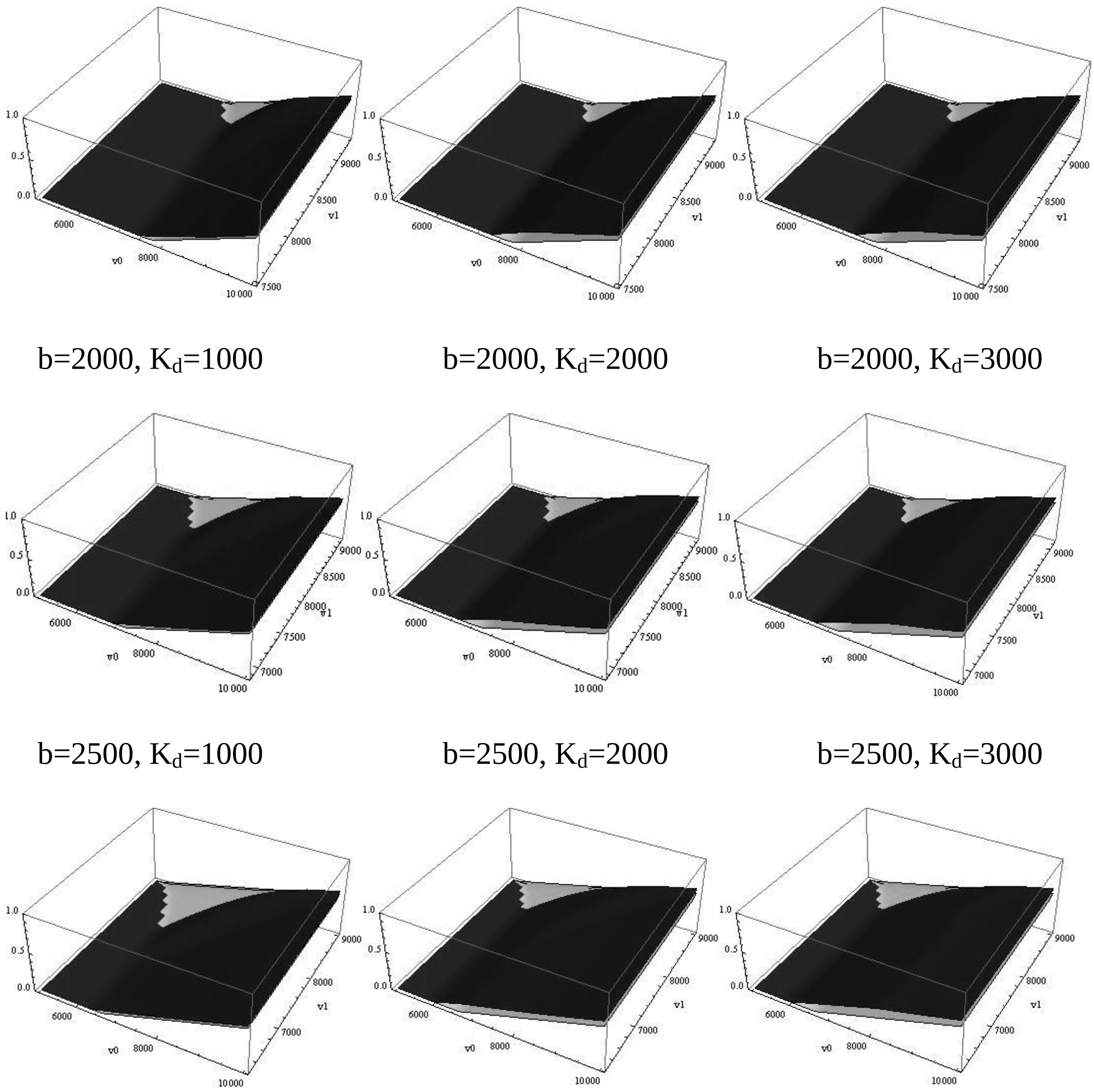
In summary, assuming that the stochastic demand D is uniformly distributed, the CDO approach could become effective if

(i) the underlying risk is high in that the variability of the stochastic demand D is substantially large;

(ii) the expected profit should be held above a high level;

(iii) the probability of having a huge loss should be contained; and

(iv) the detachment point K_d should be held relatively low.



b=2000, K_d=1000 b=2000, K_d=2000 b=2000, K_d=3000

b=2500, K_d=1000 b=2500, K_d=2000 b=2500, K_d=3000

b=3000, K_d=1000 b=3000, K_d=2000 b=3000, K_d=3000

Figure 6.6 η^{**}_{NBP} v.s. η^{**}_{CDO} as (v_0, v_1) and (b, K_d) Change

7. Concluding Remarks

It is widely believed that the Collateralized Debt Obligations (CDOs) played a major role in the ongoing worldwide financial crisis. Naturally, the CDO became notorious for its role in destructing the world economy. However, the fact that the misuse of the CDO resulted in collapse of the world economy does not necessarily imply that the CDO itself would be useless or even hazardous. It is still worth asking whether or not the CDO would be a genuine financial tool for managing risks. The purpose of this paper is to answer this question by exploring the potential of the CDO for controlling general risks. In order to examine the essential structure of the CDO in a neutral manner, we stay away from the problem of controlling financial risks and apply the CDO approach to the classical NBP, where the optimal probability, the optimal order quantity and the resulting expected profit for a value at risk problem without CDO would be compared against those with CDO.

A general multi-term CDO model is formally described first and the risk-neutral unit premium is derived for assuring no-arbitrage. By incorporating the revenue and cost structure within the framework of the CDO, it is also shown that the CDO would not affect the expected profit. The CDO idea is then applied to the classical NBP. The fundamental structure of the classical NBP is first described and the associated value at risk problem is formulated. In order to solve the value at risk problem numerically, the distribution function of the profit is derived explicitly. The CDO approach for the classical NBP is developed and the value at risk problem is rewritten with CDO. The distribution function of the profit with CDO is then obtained in a closed form. Numerical examples are given for illustrating the merits of the CDO approach under certain conditions.

Extensive numerical experiments reveal that the overall effect of CDO is rather limited. It could be effective, however, if (i) the underlying risk is high in that the variability of the stochastic demand D is substantially large; (ii) the expected profit should be held above a high level; (iii) the probability of having a huge loss should be contained; and (iv) the detachment point K_d should be held relatively low.

While the positive effect of the CDO approach could be demonstrated through numerical examples, it is difficult to establish a necessary and sufficient condition under which the CDO approach would be worth doing for the protection buyer. Furthermore, the CDO model discussed in this paper may be expanded so as to accommodate multiple terms and multiple markets, and also to incorporate the motivation analysis of the protection seller which is totally ignored in the current model. These theoretical challenges would be pursued further in the future.

APPENDIX

Theorem A.1.

Let $G_1(Q,x)$ be as in Theorem 5.2 and define

$$\zeta_1(x) = \frac{1}{p-r}\left\{x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) + c_O Q\right\} .$$

Let $x_{1:Q}$ and $x_{1:a}$ be defined by

$$\zeta_1(x_{1:Q}) = Q \quad and \quad \zeta_1(x_{1:a}) = [Q - \frac{K_a}{\tilde{c}_O}]^+$$

respectively. Then the following statements hold.

$$(a)\ x_{1:Q} = (p-c)Q - e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q)$$

$$(b)\ x_{1:a} = \begin{cases} x_{1:Q} - (p-r)\frac{K_a}{\tilde{c}_O} & if\ \frac{K_a}{\tilde{c}_O} \le Q \\ x_{1:Q} - (p-r)Q & if\ Q < \frac{K_a}{\tilde{c}_O} \end{cases}$$

$$(c)\ x_{1:a} \le x_{1:Q}$$

$$(d)\ G_1(Q,x) = \begin{cases} 0 & if\ x \in (-\infty, x_{1:a}) \\ F_D(\zeta_1(x)) - F_D(Q - \frac{K_a}{\tilde{c}_O}) & if\ x \in [x_{1:a}, x_{1:Q}] \\ F_D(Q) - F_D(Q - \frac{K_a}{\tilde{c}_O}) & if\ x \in (x_{1:Q}, \infty) \end{cases} .$$

Theorem A.2

Let $G_2(Q,x)$ be as in Theorem 5.2 and define

$$\zeta_2(x) = -\frac{1}{s}\left\{x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) - c_U Q\right\} .$$

Let $x_{2:Q}$ and $x_{2:a}$ be defined by

$$\zeta_2(x_{2:Q}) = Q \quad and \quad \zeta_2(x_{2:a}) = Q + \frac{K_a}{\tilde{c}_U}$$

respectively. Then the following statements hold.

$$(a)\ x_{2:Q} = x_{1:Q}$$

$$(b)\ x_{2:a} = x_{1:Q} - \frac{sK_a}{\tilde{c}_U}$$

$$(c)\ x_{2:a} \le x_{2:Q}$$

$$(d)\ G_2(Q,x) = \begin{cases} 0 & if\ x \in (-\infty, x_{2:a}) \\ F_D(Q + \frac{K_a}{\tilde{c}_U}) - F_D(\zeta_2(x)) & if\ x \in [x_{2:a}, x_{2:Q}] \\ F_D(Q + \frac{K_a}{\tilde{c}_U}) - F_D(Q) & if\ x \in (x_{2:Q}, \infty) \end{cases} .$$

Theorem A.3.

Let $G_3(Q,x)$ be as in Theorem 5.2 and define

$$\zeta_3(x) = \frac{1}{p-c}\left\{x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) + e^{-r_f\tau}K_a\right\} .$$

Let $x_{3:d}$ and $x_{3:a}$ be defined by

$$\zeta_3(x_{3:d}) = [Q - \frac{K_d}{\tilde{c}_O}]^+ \quad and \quad \zeta_3(x_{3:a}) = [Q - \frac{K_a}{\tilde{c}_O}]^+$$

respectively. Then the following statements hold.

$$(a)\ x_{3:d} = \begin{cases} x_{1:Q} - e^{-r_f\tau}K_a - (p-c)\frac{K_d}{\tilde{c}_O} & if\ \frac{K_d}{\tilde{c}_O} \le Q \\ x_{1:Q} - (p-c)Q - e^{-r_f\tau}K_a & if\ Q < \frac{K_a}{\tilde{c}_O} \end{cases}$$

$$(b)\ x_{3:a} = \begin{cases} x_{1:Q} - e^{-r_f\tau}K_a - (p-c)\frac{K_a}{\tilde{c}_O} & if\ \frac{K_a}{\tilde{c}_O} \le Q \\ x_{1:Q} - (p-c)Q - e^{-r_f\tau}K_a & if\ Q < \frac{K_a}{\tilde{c}_O} \end{cases}$$

$$(c)\ x_{3:d} \le x_{3:a}$$

$$(d)\ G_3(Q,x) = \begin{cases} 0 & if\ x \in (-\infty, x_{3:d}) \\ F_D(\zeta_3(x)) - F_D(Q - \frac{K_d}{\tilde{c}_O}) & if\ x \in [x_{3:d}, x_{3:a}] \\ F_D(Q - \frac{K_a}{\tilde{c}_O}) - F_D(Q - \frac{K_d}{\tilde{c}_O}) & if\ x \in (x_{3:a}, \infty) \end{cases} .$$

Theorem A.4.

Let $G_4(Q,x)$ be as in Theorem 5.2 and define

$$\zeta_4(x) = \frac{1}{p-c}\left\{x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) + e^{-r_f\tau}K_a\right\} .$$

Let $x_{4:a}$ and $x_{4:d}$ be defined by

$$\zeta_4(x_{4:a}) = Q + \frac{K_a}{\tilde{c}_U}\quad and\quad \zeta_4(x_{4:d}) = Q + \frac{K_d}{\tilde{c}_U}$$

respectively. Then the following statements hold.

$$(a)\ x_{4:a} = x_{1:Q} - \frac{sK_a}{\tilde{c}_U}$$

$$(b)\ x_{4:d} = x_{1:Q} - e^{-r_f\tau}K_a + (p-c)\frac{K_d}{\tilde{c}_U}$$

$$(c)\ x_{4:a} \le x_{4:d}$$

$$(d)\ G_4(Q,x) = \begin{cases} 0 & if\ x \in (-\infty, x_{4:a}) \\ F_D(\zeta_4(x)) - F_D(Q + \frac{K_a}{\tilde{c}_U}) & if\ x \in [x_{4:a}, x_{4:d}] \\ F_D(Q + \frac{K_d}{\tilde{c}_U}) - F_D(Q + \frac{K_a}{\tilde{c}_U}) & if\ x \in (x_{4:d}, \infty) \end{cases} .$$

Theorem A.5.

Let $G_5(Q,x)$ be as in Theorem 5.2 and define

$$\zeta_5(x) = \frac{1}{p-r}\left\{x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) - e^{-r_f\tau}(K_d - K_a) + c_O Q\right\} .$$

Let $x_{5:d}$ be defined by

$$\zeta_5(x_{5:d}) = Q - \frac{K_d}{\tilde{c}_O} .$$

Then the following statements hold.

$$(a)\ x_{5:d} = \begin{cases} x_{1:Q} + \frac{(c-p)K_d}{\tilde{c}_O} - e^{-r_f\tau}K_a & if\ \frac{K_d}{\tilde{c}_O} \le Q \\ x_{1:Q} - (p-r)Q + e^{-r_f\tau}(K_d - K_a) & if\ Q < \frac{K_d}{\tilde{c}_O} \end{cases}$$

$$(b)\ G_5(Q,x) = \begin{cases} F_D(\zeta_5(x)) & if\ x \in (-\infty, x_{5:d}] \\ F_D(Q - \frac{K_d}{\tilde{c}_O}) & if\ x \in (x_{5:d}, \infty) \end{cases} .$$

Theorem A.6.

Let $G_6(Q,x)$ be as in Theorem 5.2 and define

$$\zeta_6(x) = -\frac{1}{s}\left\{x + e^{-r_f\tau}\mu_{\tilde{L}[K_a,K_d]}(Q) - e^{-r_f\tau}(K_d - K_a) - c_U Q\right\} .$$

Let $x_{6:d}$ be defined by

$$\zeta_6(x_{6:d}) = Q + \frac{K_d}{\tilde{c}_U} .$$

Then the following statements hold.

$$(a)\ x_{6:d} = x_{1:Q} - e^{-r_f\tau}K_a + (p-c)\frac{K_d}{\tilde{c}_U}$$

$$(b)\ G_6(Q,x) = \begin{cases} \bar{F}_D(\zeta_6(x)) & if\ x \in (-\infty, x_{6:d}] \\ \bar{F}_D(Q + \frac{K_d}{\tilde{c}_U}) & if\ x \in (x_{6:d}, \infty) \end{cases} .$$

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