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The Intertemporal Elasticity of Substitution: An Analysis Based on Japanese Data

by

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Abstract

This paper investigates whether the intertemporal elasticity of substitution (IES) for Japan possesses similar properties to that found for the United States in the literature. As the existing empirical findings are mixed, there is no clear consensus. This paper uses a model with both production and nonseparability between nondurables and durables in order to control for possible factors that cause a bias in the IES. We present the empirical results demonstrating that the IES is quite similar between the two countries.

The intertemporal elasticity of substitution (IES) is a key parameter that characterizes household consumption and saving behavior, and it plays a crucial role in determining policy and welfare assessments, where the positive value of the IES is the basic premise of theoretical or quantitative analysis. As is well known, contrary to this premise, empirical research encountered a finding that the IES is sometimes negative and not significantly different from zero (Hall 1988). In response to this finding, many researchers attempted explanations of why the IES is biased downward from various perspectives, including nonseparable preferences across nondurables and durables (e.g., Ogaki and Reinhart 1998, and Pakoš 2007), econometric issues such as time aggregation and weak identification (e.g., Hansen and Singleton 1996, Stock and Wright 2000, Neely et al. 2001, and Yogo 2004), and heterogeneity among consumers (e.g., Attanasio and Weber 1993, 1995, Beaudry and van Wincoop 1996, Vissing-Jørgensen 2002, and Guvenen 2006). However, because these recent studies use U.S. data with few exceptions, despite much work little is known about whether the U.S. findings at the heart of the debate, such as the small magnitude and the downward bias, universally serve as a starting point for the debate on the IES.

Given that consumption and saving decisions are inextricably linked, one way to answer this question may be to examine the differences in household saving across countries. In this comparison, a particularly noticeable and well-known fact is that the U.S. saving rate is low and stable, while Japan's saving rate is high compared with most other countries,[1] which has

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[1] At least until the mid 1990s, this fact can be confirmed for member countries of the Organisation for Economic Co-operation and Development (OECD). According to the OECD (2006), in 1990, household net saving rates (as a percentage of household disposable income) were 13.0% for Canada, 9.3% for France, 13.9% for Germany, 13.9% for Japan, and 7.0% for the United States. In 1999, they were 4.0% for Canada, 11.5% for France, 9.5% for Germany, 10.7% for Japan, and 2.4% for the United States. However, after 2000, Japan's saving rate is not the highest (in 2004 they were 1.4% for Canada, 11.8% for France, 10.5% for Germany, 6.9% for Japan, and 1.8% for the United States.). Because of the scarcity of data, which is related to the extension of our model presented in this paper, our sample period was inevitably limited up to 1999.

attracted the interest of many researchers and policymakers (see, e.g., Chen et al. 2006a,b for recent work). If this fact is true and it reflects a difference of households' attitude with respect to intertemporal substitution of consumption, the high saving rate implies that Japan's intertemporal substitution is stronger than that of the United States, and of most other countries. In other words, the applicability of the U.S. data-based debate may be limited. However, as Hayashi (1986, 1997) and Horioka (1995) pointed out, Japan's high saving rate can be a statistical illusion in that appropriate adjustments of various concepts can lead to a saving rate that is not as high as commonly thought. As their finding suggests, if there is no large disparity in the saving behavior of households in the two countries from a macroeconomic perspective, the IES for Japan is probably similar to that of the United States.[2] Thus, understanding the Japanese IES can be a challenge for evaluating the debate (therefore theory) tailored for U.S. data, as is understanding Japanese saving behavior.[3]

In this paper, we focus on the national accounts data of Japan and examine whether the estimated IES is significantly greater than zero, and if so, whether the IES for Japan possesses similar properties to that of the United States. We introduce an extended framework in order to respond to debates on the empirical study of intertemporal substitution using aggregate data. First, we use a constant elasticity of substitution (CES) utility function to allow for a downward bias that will arise by ignoring the intratemporal substitution effect between nondurables and durables. Ogaki and Reinhart (1998) focus on this misspecification bias, and estimate the IES

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[2] More recently, using a neoclassical growth model, Chen et al. (2006a) argue that differences in consumer preferences are not needed to explain the differences in the U.S. and Japanese saving rates. Atkeson and Ogaki (1996) demonstrate that the IES can depend on household's income level, in which the difference in the IES is as great as 0.13 between the United States and India. In this respect, given that there is no large difference in per capita GDP between the United States and Japan, their findings may be interpreted as supporting this conjecture.

[3] In the literature regarding Japan, most previous studies have tended to place a major focus on tests for overidentifying restrictions or model diagnoses using the methods of Hansen and Jagannathan (1991, 1997), rather than on the estimation of the IES itself. See, e.g., Saito (1999) for a comprehensive survey. Given a serious lack of empirical investigation, it seems fair to state that there is no consensus among researchers regarding the value of the IES for Japanese consumers.

to be around 0.4, using U.S. postwar data. Second, we incorporate the production side into the two-good model in order to respond to another debate that bond returns may not be a good indicator of the interest rate (Kitamura and Fujiki 1997 and Mulligan 2002). The point is that if there are any distortions in financial markets, they could cause a deviation between returns on bonds and capital (Mulligan 2002), and this problem will be particularly indispensable for countries without an equivalent of the U.S. treasury bill rate (Kitamura and Fujiki 1997). The model in this paper therefore allows both the intratemporal substitutability and the use of the marginal product of capital.

In recent work, Pakoš (2004) proposes a representative agent model with nonhomothetic utility in nondurables and durables consumption. Okubo (2007a) estimates this nonhomothetic utility model using Ogaki and Reinhart's U.S. postwar data and demonstrates that incorporating the nonseparability between nondurable and durable goods rather than nonhomotheticity is the dominant factor for obtaining reasonable estimates of the IES. Hence, the model in this paper concentrates on the homothetic CES utility function.

Following convention (see, e.g., Hall 1988 and Vissing-Jørgensen 2002), we interpret the IES as the elasticity of the consumption ratio to the real interest rate. Therefore, while empirical findings provided in this paper can be viewed as another guide to estimates of the IES, they should not be taken as evidence for reconciling the debate over the equity premium puzzle (Mehra and Prescott 1985). To address this puzzle, for example, the model may need to incorporate the recursive function of Epstein and Zin (1989, 1991) for households' intertemporal utility. As Yogo (2006) demonstrates, even in this case (i.e., the class of nonexpected utility), it is necessary to incorporate the role of durables by using the nonseparable utility. In addition, his findings suggest that as long as the nonseparable utility is used, the effect of imposing expected utility on the IES is extremely small relative to the large variation in the coefficient of relative risk

aversion. Hence, in this paper, we consciously limit our model to the class of expected utility with homothetic CES preferences, while we examine the robustness of the estimation results to other factors such as a shift in the utility weight and weak identification.

This paper is organized as follows. In Section I, we describe the two-good model with both production and nonseparable preferences in nondurables and durables, and provide an outline of a two-step approach that combines a cointegrating regression with the generalized method of moments (GMM). Section II describes the consumption and return data used in the empirical work with graphs and descriptive statistics. In Section III, we present the empirical results demonstrating that the IES in the two countries is comparable in terms of both its magnitude and the importance of incorporating the intratemporal substitution effect. We also discuss how the results based on aggregate data should be taken as a whole. Section IV contains concluding remarks. A separate appendix (Okubo 2007b) provides detailed descriptions of the data and additional tests for ensuring our results.

I. Framework

The Model

Suppose that a representative consumer's lifetime utility is specified by:

$$U = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1 - 1/\sigma} \right) \{ u(t)^{1-1/\sigma} - 1 \} \right], \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, $E_t[\cdot]$ is the expectations operator conditional on the information available at time t, $\sigma > 0$ is the intertemporal elasticity of substitution (IES), and $u(t)$ is the period utility function. To allow for the intratemporal substitution effect, we assume that the consumer derives the utility in each period according to the following constant

elasticity of substitution (CES) function:

$$u(t) = \left[C_t^{1-1/\epsilon} + a S_t^{1-1/\epsilon} \right]^{1/(1-1/\epsilon)}, \tag{2}$$

where $a > 0$, C_t is the consumption of a nondurable good in period t, S_t is the stock of a durable good in period t, and $\epsilon > 0$ is the elasticity of substitution between C_t and S_t. In principle, utility is derived from a flow of services from the good. Specification (2) therefore assumes that the service flow from the durable good is proportional to the stock of the durable good (see, e.g., Mankiw 1985, Fauvel and Samson 1991, and Ogaki and Reinhart 1998 for this convention of durable consumption models).[4] The durables stock is related to the purchase by:

$$S_t = (1 - \delta)S_{t-1} + D_t, \tag{3}$$

where D_t is expenditure on the durable good in period t, and $\delta \in (0,1)$ is the depreciation rate.

The consumer holds bonds B_t at the beginning of period t, realizing the gross rate of return R_t at the end of period t. Output is given by the production function, $Y_t = F(K_t, L_t)$, where K_t is the stock of capital at the beginning of period t and L_t is the labor input given exogenously in each period. As usual, the function $F(\cdot)$ exhibits positive and diminishing marginal products with respect to each input: $F'(\cdot) > 0$ and $F''(\cdot) < 0$. The capital stock depreciates at a rate $\delta_K \in (0,1)$, and it is related to investment I_t by:

$$K_{t+1} = (1 - \delta_K)K_t + I_t. \tag{4}$$

The durable good starts to yield services at the time of its purchase, while the capital contributes to the production of output from a period after its purchase. Letting P_t denote the price of the

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[4] In this specification, whether the preference weight a is attached to the nondurable good or to the service flow from the durable good is only a problem of its definition, and it is not essential. A formal explanation of why the preference weight is attached to one good can be found in Pakoš (2007). The point is that we do not make an a priori assumption that the factor of proportionality is normalized to one.

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durable good in terms of the nondurable good, the consumer's budget constraint in this economy

is given by:

$$B_{t+1} = R_t B_t + Y_t - C_t - P_t[S_t - (1-\delta)S_{t-1}] - [K_{t+1} - (1-\delta_K)K_t]. \tag{5}$$

The consumer's problem is to choose paths for C, S, B, and K so as to maximize utility (1)

subject to the constraint (5).

First-Order Conditions

Optimal consumption of the nondurable and durable good is determined so as to satisfy an

intratemporal first-order condition that the user cost equals the marginal rate of substitution

between the service flow from the durable good and the nondurable good:

$$Q_t = a\left(\frac{S_t}{C_t}\right)^{-1/\epsilon}. \tag{6}$$

As usual, the user cost, Q_t, represents the net expense of purchasing the durable good in the

current period and selling it in the next period, given by:

$$Q_t = P_t - (1-\delta)E_t\left[\frac{\beta P_{t+1} mu_{t+1}}{mu_t}\right], \tag{7}$$

where

$$mu_t = C_t^{-1/\epsilon}\left[C_t^{1-1/\epsilon} + aS_t^{1-1/\epsilon}\right]^{(\sigma-\epsilon)/\sigma(\epsilon-1)}. \tag{8}$$

Equation (6) implies that an increase in the user cost leads to a shift in demand from the service

flow of the durable good to the nondurable good. The necessary conditions also include an Euler

equation:

$$E_t\left[\beta\left(\frac{mu_{t+1}}{mu_t}\right)R_{t+1}\right] = 1, \tag{9}$$

and a condition that equates the marginal product of capital to the rate of return:

$$R_{t+1} = 1 + \frac{\partial Y_{t+1}}{\partial K_{t+1}} - \delta_K. \tag{10}$$

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Implicit in the condition (10) is the assumption that there are no adjustment costs to invest-ment. As Cochrane (1991, 1996) shows, adjustment costs models predict that stock returns and investment returns (inferred from investment data through a production function) should be equal. As discussed later, we use this condition to define a proxy of real interest rates calculated from risk-free assets such as U.S. treasury bills, rather than a proxy of stock returns.

Estimation Procedures

Taking natural logarithms of both sides of equation (6) and then applying the add-and-subtract strategy about $(1/\epsilon) \ln D_t$ and $\ln P_t$ yields:

$$\ln(a) + \frac{1}{\epsilon} \ln \left(\frac{C_t}{D_t} \right) - \ln P_t = \ln \left(\frac{Q_t}{P_t} \right) + \frac{1}{\epsilon} \ln \left(\frac{S_t}{D_t} \right). \tag{11}$$

Suppose that $\ln C_t$, $\ln D_t$, and $\ln P_t$ are difference stationary. Following Ogaki and Reinhart (1998), we assume that the ratio of marginal utility, mu_{t+1}/mu_t, is stationary.[5] Then $\ln(Q_t/P_t)$ and $\ln(S_t/D_t)$ are stationary,[6] so that the model implies a cointegration relationship between $\ln(C_t/D_t)$ and $\ln P_t$. This cointegration relationship provides a way to estimate the elasticity of substitution, ϵ. Specifically, we consider the following cointegrating regression:

$$\ln(C_t/D_t) = \eta + \epsilon \ln P_t + \zeta_t, \tag{12}$$

where η is a constant term and ζ_t is a stationary error term.

The advantages of using equation (12) are that (i) it does not require observations on the user cost for the durable good, and (ii) its appropriate estimation yields a superconsistent estimate of ϵ. As a consequence of the latter property, we can estimate the remaining parameters, σ, β,

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[5] Test results for this assumption are provided in Section C of the separate appendix.
[6] From equation (7), Q_t/P_t is a function of stationary variables, P_{t+1}/P_t and mu_{t+1}/mu_t. On the other hand, as equation (3) can be rewritten as $S_t = D_t + (1-\delta)D_{t-1} + (1-\delta)^2 D_{t-2} + \cdots$ by repeated substitution, S_t/D_t is a function of D_{t-i}/D_t $(i = 1, 2, \ldots)$.

and a, by applying Hansen's (1982) GMM to the moment conditions with the estimate of ϵ from regression (12). Letting z_t denote a vector of variables that are in the information set at time t, the moment conditions are given by:

$$E\left[\left(\beta\frac{\hat{mu}_{t+1}}{\hat{mu}_t}R_{t+1}-1\right)z_t\right]=0, \tag{13}$$

$$E\left[\left((1-\delta)\beta\frac{P_{t+1}}{P_t}\frac{\hat{mu}_{t+1}}{\hat{mu}_t}+\frac{a}{P_t}\left(\frac{S_t}{C_t}\right)^{-1/\hat{\epsilon}}-1\right)z_t\right]=0, \tag{14}$$

where '^' denotes that the value of ϵ is fixed at the estimate from regression (12). Equation (13) represents the moment condition implied by the Euler equation. Equation (14) represents the moment condition implied by the intratemporal first-order condition (6) together with equation (7). The condition (14) is imposed mainly for the purpose of identifying the preference weight a, as in Yogo (2006) and Pakoš (2007). An alternative method for obtaining the weight parameter a, which was adopted by Ogaki and Reinhart (1998), is to calculate its sample average from the intratemporal first-order condition (6) by using the estimate of ϵ and an approximated value of the user cost, Q_t, based on a vector autoregression (VAR) estimation. Although the two approaches both rely on the same intratemporal first-order condition, the present paper adopts the former approach to avoid an additional bias that may arise from the VAR approximation. These moment conditions are exploited for testing the model as well as estimating the remaining parameters.

II. Data

Consumption

Quarterly consumption data for this study are taken from the *2005 Annual Report on National Accounts* compiled by the Economic and Social Research Institute (ESRI), Cabinet Office, Government of Japan. The *2005 Annual Report* releases quarterly data for the period from 1980:1

to 2004:1.[7] For the variables C_t and D_t in the model, we use real expenditures on nondurables and durables that are divided by the total population (averaged over each quarter). The relative price of durables, P_t, is calculated as the ratio of the price for durables to the price for non-durables, where the price is defined as the ratio of the nominal value of consumption expenditure to its real value.

Nondurables expenditure consists of items such as food (including alcoholic beverages and cigarettes), lighting and heating, household nondurables, drugs, books and other printed matter, and other miscellaneous goods. This category is similar to the U.S. National Income and Product Accounts (NIPA). However, the Japanese National Accounts do not classify items with durability such as clothing and shoes as nondurable goods.[8]

Major items of durables expenditure are furniture and floor coverings, household appliances, personal transport equipment, information transmission equipment, and other durable goods. The stock part of the Japanese National Accounts has year-end values for the stocks of these five items of consumer durables. We use a total of the five stock series as the initial value and construct quarterly series of the durables stock, S_t, by equation (3).[9] From the data available in the stock part of the *2005 Annual Report*, the implicit depreciation rate for consumer durables (the ratio of replacement-cost depreciation to the stock of consumer durables) is about 4% per quarter.[10] Therefore, in equation (3), we set $1 - \delta = 0.96$. See Section A of the separate

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[7] The Japanese National Accounts were estimated on the international standard (System of National Accounts 1968 (SNA68)) until 2000 and on the new standard recommended by the United Nations in 1993 (System of National Accounts 1993 (SNA93)) thereafter. The implementation of SNA93 by the ESRI goes back to 1980 at the time of this writing. Therefore, Japan's macro time-series data are now available for the period since 1980 under SNA93.

[8] See ESRI (2000, p.67) for more details on classification of consumption by type.

[9] Unfortunately, the total stock of the five consumer durables does not provide the stock of all consumer durables because the data on the stock of consumer durables are a subsidiary to the major consumer durables rather than all consumer durables. Following Horioka (1995), we estimated the stock of all consumer durables by assuming that the ratio of expenditures on major consumer durables to expenditures on all consumer durables is equal to the ratio of the stock of major consumer durables to the stock of all consumer durables. See Section A of the separate appendix for more details.

[10] This depreciation rate is low compared with the U.S. rate. In Ogaki and Reinhart (1998) and a more recent

appendix for a detailed description of the calculation.

Figure 1 displays in log scale the ratio of nondurables expenditure to durables expenditure and the price of durables relative to nondurables. These variables are included as the dependent variable and the independent variable in equation (12). The figure reveals two features. First, the log ratio $\ln(C_t/D_t)$ as a whole has a downward trend, which is consistent with the decline in the relative price of durables to nondurables. This suggests that the elasticity of substitution between nondurables and durables is positive. Second, it also demonstrates regarding durables consumption: (i) a rise and a gradual decline over the period 1987–1997, and (ii) a sharp decline around 1997:2 and a declining trend after that time. This feature is consistent with (i) the period of Japan's economic bubble and its burst, and (ii) reaction against the last-minute push of durables purchases before the consumption tax hike (from 3% to 5%) in April 1997 and the subsequent sluggishness in durables consumption.[11] The time path of the log ratio $\ln(C_t/D_t)$ appears as though it is reverting to the trend before Japan's economic bubble. A simple interpretation for this is probably that there was some shift in consumers' preferences (including the utility weight parameter in our model) between nondurables and durables. This point is discussed in Section III.

Figure 2 shows the constructed quarterly series of the stock of durables S_t and its growth rate (measured as the log difference, $\ln S_t - \ln S_{t-1}$). Although nondurables and durables consumption

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study, Yogo (2006), it is set to 6% per quarter. To evaluate this low depreciation rate for consumer durables, we examined (i) whether it is also obtained from the data under the previous standard (called SNA68) adopted in Japan up to 2000, (ii) whether it is consistent with the depreciation rate implicit in the perpetual inventory method, and (iii) whether the depreciation rate for the United States of about 6% per quarter is replicable using our calculation method (i.e., the ratio of replacement-cost depreciation to the stock of consumer durables). From these investigations, a depreciation rate for Japan of about 4% per quarter appears to be supported. These results are available from the author upon request. In Section III, however, we will also attempt a depreciation rate of 6% to check the sensitivity of the empirical results.

[11] A similar change in durables consumption did not appear when the consumption tax was first introduced in April 1989, because the commodity tax was abolished at the same time. See Cabinet Office (1997, 1998) for an official opinion. Also, this sluggishness in durables consumption has been recognized as one of the features that was not present in past economic recovery periods (Cabinet Office 2001).

data start in 1980, the stock of consumer durables is available only since the end of 1980 (the beginning of 1981). The sample period therefore starts in 1981:1. The growth rate of S_t possesses the time-series properties similar to durables consumption illustrated in Figure 1: a rise and a decline for the period 1987–1997, a sharp fall around 1997:2, and the subsequent sluggishness.

Using this constructed series, Figure 3 graphs the ratio of the stock of durables to nondurables expenditure, S_t/C_t, along with the relative price of durables. For the upward trend in S_t/C_t, there are two explanations in the literature. The first reason is the substitution effect related to the downward trend in the relative price. The second reason is the income effect, that is, a change in the relative demand caused by the difference in the income elasticity between nondurable and durable goods. Using nonhomothetic preferences, Pakoš (2004) points out that the upward trend in S_t/C_t for the United States can be explained by the income effect. As demonstrated by Figure 1, however, the substitution effect is expected to be significant for the present data. His main claim is that ϵ has an upward bias under homothetic preferences. Within the present framework, how a smaller value of ϵ affects the IES is discussed in Section III.

Marginal Product of Capital and Real Interest Rates

The marginal product of capital (excluding depreciation) is calculated as:[12]

$$SK_t \left(\frac{Y_t}{K_t} \right) (1 - \tau) - \delta_K, \tag{15}$$

where SK_t is the capital share in quarter t, Y_t is real GDP in quarter t, K_t is the real stock of capital at the beginning of quarter t, and τ is the tax rate on capital income. The existing literature suggests two points associated with this calculation: (i) the stock of infrastructure

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[12] The specification of (15) is equivalent to assuming a Cobb-Douglas constant returns to scale (CRS) production function. Miyagawa et al. (2006) find some evidence that the CRS technology may be valid for the Japanese production function. Also recent quantitative research that calibrates a model to the Japanese economy commonly assumes the CRS technology. See, e.g., Hayashi and Prescott (2002), Chen et al. (2006a,b), and Nakajima (2006).

(such as highways, other transportation systems, water and sewer lines, and communication systems, hereafter called public capital) is an important input in the production of output (see e.g., Gramlich 1994 for a review), and (ii) estimation of the consumption-based capital asset pricing model (CCAPM) using the marginal product of capital that excludes public capital leads to imprecise estimates of the preference parameters (Kitamura and Fujiki 1997). Therefore, the real stock of capital is measured as the sum of private capital and public capital. If the capital stock in equation (15) excludes the public capital stocks, then the output–capital ratio, Y_t/K_t, namely the marginal product of capital, is overestimated. The private capital stock is taken from the *Gross Capital Stock of Private Enterprises* (GCSPE) compiled by the ESRI. The public capital stock is from the *Social Capital of Japan* (SCJ) edited by the Cabinet Office Director-General for Economic Research. Because the SCJ's data is annual, a quarterly series is constructed from SCJ's original data, which is available up to 1999:1. See Section A of the separate appendix for more details. Because of the availability of the data thus far described, the sample period is finally limited to 1981:1–1999:1.

There are some possible definitions of public capital. In this paper, the ESRI's (2000) definition of public capital, which was adopted in the *2005 Annual Report*, is used. The ESRI definition adopts a relatively narrow definition of the capital stock owned by the general government (central and local governments), compared with that of the SCJ. See Section A of the separate appendix for details. Although the *2005 Annual Report* does not provide the real value of this public capital stock, it is possible to construct it by using the SCJ public capital series.

Figure 4 plots the output–capital ratio with and without public capital for the period 1981:1–1999:1. The figure reveals that there is a significant difference in the output–capital ratio depending on whether public capital is included or not. The output–capital ratio with public capital, which is labeled as the ESRI Definition, decreases from 0.551 in 1981:1 to 0.327 in 1999:1.

If the capital stock excludes public capital, the output–capital ratio is 0.793 in 1981:1 and 0.489 in 1999:1. The figure also demonstrates the effect of using the relatively narrow definition of the ESRI. Looking at the line labeled the SCJ Definition, we notice that there is no significant difference.[13] In what follows, the results based on the ESRI definition of public capital are mainly reported, unless otherwise noted.

Figure 5 plots the marginal product of capital for the same sample period, together with real interest rates (calculated as the log return of the short-term money market rate minus the inflation rate using the nondurables price). On the basis of data available from the Japanese National Accounts, the tax rate on capital income and the depreciation rate for capital are set to the average rates in the 1981–1999 period: $\tau =0.525$ and $\delta_K =0.020$ (0.030 if public capital is excluded). See Section A of the separate appendix for more details. The marginal product of capital has a downward trend, which is consistent with the downward trend in the output–capital ratio.[14] Figure 5 further reveals that the real interest rate frequently takes negative values, especially in the late 1990s.[15]

Some Summary Statistics and Pretests

Table 1 reports descriptive statistics for the data underlying the above figures, where the growth rate is calculated as the log difference of variables. The following can be revealed from the table. First, the consumption data possess a well-known property that durables consumption is more volatile than nondurables consumption. In this sample, the standard deviations for nondurables and durables consumption growth are 1.21% and 4.52%, respectively. Second, the marginal

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[13] In this case, the output–capital ratio is 0.520 in 1981:1 and 0.311 in 1999:1.

[14] The capital share is 0.342 in 1981:1 and 0.270 in 1999:1. Its sample average over 1981–1999 is 0.313. Therefore, the effect of the change in the capital share on the marginal product of capital is relatively small.

[15] This tendency is due to Japan's nominal interest rate being close to 0%, and it is also ongoing after this period.

product of capital has a mean of 4.46% and a standard deviation of 1.28% (mean 6.50% and standard deviation 1.85% if the capital stock excludes public capital), whereas the real interest rate has a mean of 0.81% and a standard deviation of 0.85%. That is, in terms of the standard deviation, the volatility of the two measures is quite similar, but there is a substantial difference in the rates of return. Third, regarding the correlation coefficients, the real interest rate has a negative correlation with nondurables consumption growth.[16]   Together with the result from Figure 5, this finding indicates that the real interest rate calculated from the money-market rate is not a good indicator variable for the rates of return. In contrast, the correlation coefficient between the marginal product of capital and nondurables consumption growth is 0.135 (0.130 if public capital is excluded). Therefore, we can evade the negative correlation issue by using the marginal product of capital.

As previously discussed with Figure 1, the time-series plot of $\ln(C_t/D_t)$ and $\ln P_t$ suggests that the elasticity of substitution between the two consumption goods is positive. If the elasticity of substitution is sufficiently high (i.e., $\sigma < \epsilon$), because a rise in the real return leads to an increase in the user cost for the durable good, the consumer may substitute from durables to nondurables in the same period. If that is the case, the IES will be underestimated unless this effect is appropriately controlled (Ogaki and Reinhart 1998). A way to examine this possibility is to look at the correlation between the relative demand S_t/C_t and the real return R_{t+1}.[17] Equation (6) implies that the rise in Q_t that follows the increase in R_{t+1} causes the decrease in S_t/C_t. In fact, as expected from Figures 3 and 5, the correlation coefficient between the relative demand and the marginal product of capital is -0.953 (-0.951 if public capital is excluded); its

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[16] The timing convention used for consumption is the "beginning-of-quarter" (i.e., a given quarter's consumption is interpreted as measuring consumption at the beginning of the quarter), following Campbell (2003, pp. 813–814).

[17] As Ogaki and Reinhart (1998) note, there is a difficulty that the calculation of the user cost requires an approximation of the expected values of the relative price and marginal utility, in conjunction with some additional assumptions.

correlation coefficient with the real interest rate is -0.469. Therefore, the relative demand is negatively correlated with the real return.

To sum up, Figures 1–5 and Table 1 suggest that without (i) an alternative to the real interest rate and (ii) the consideration of *intra*temporal substitution between nondurables and durables, it is difficult to retrieve accurate estimates of the IES from the Japanese National Accounts data.

We end this section by examining the possibility of nonstationarity of the two variables, $\ln(C_t/D_t)$ and $\ln P_t$. In this paper we use both the Augmented Dickey–Fuller (ADF) test developed by Said and Dickey (1984) and the J test proposed by Park (1990). These tests are designed to take trend stationarity as the alternative hypothesis, as both $\ln(C_t/D_t)$ and $\ln P_t$ exhibit downward trends over the sample period. The test results are reported in Table 2. For both variables, we find that the null hypothesis of difference stationarity with drift cannot be rejected even at the 10% significance level.[18]

III. Estimation Results

Substitution between Nondurables and Durables

Given that both $\ln(C_t/D_t)$ and $\ln P_t$ follow difference stationary processes with drift, the model implies both the deterministic cointegration restriction and the restriction that the two variables are stochastically cointegrated with the cointegrating vector $(1, -\epsilon)'$, in the terminology used by Ogaki and Park (1998) and Campbell and Perron (1991). To test these restrictions explicitly, we estimate equation (12) using Park's (1992) canonical cointegrating regression (CCR)

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[18] We also attempted other unit-root tests: the ADF test with automatic lag selection, the DF–GLS test of Elliott, Rothenberg, and Stock (1996), and the modified tests of Ng and Perron (2001). As the results were robust, we did not provide them in the table.

procedure,[19] and apply Park's (1990) H(p,q) tests for the null hypotheses of stochastic cointegration and the deterministic cointegration restriction.

Table 3 reports estimation results of equation (12) and results of the H(0,1) and H(1,q) tests.[20] We consider the regression with and without a dummy variable for the period 1987:1–1999:1, as explained with Figure 1. For the case with the dummy variable, we obtain an estimate of $\epsilon = 1.409$ with a standard error of 0.245. Ogaki and Reinhart (1998) estimate equation (12) using quarterly U.S. data and report an estimate of $\epsilon = 1.167$ with a standard error of 0.099. Yogo (2006) obtains an estimate of $\epsilon = 0.790$ with a standard error of 0.082 for a longer sample period. Our estimate of ϵ is therefore similar to that of Ogaki and Reinhart, as long as any shift in consumer preferences is taken into account. For the case with the dummy variable, the H(0,1) and H(1,q) test statistics are not large enough to reject the null hypotheses of the deterministic cointegration restriction and stochastic cointegration, respectively. The dummy variable is significant at the 1% level, implying a shift of the constant term from 1.585 to 1.069. Comparing the two rows of Table 3, the H(p,q) test statistics are more favorable for the case with the dummy variable in terms of p-values. Hence, we use the value of ϵ obtained from the regression with the dummy variable in the following analysis.

The Intertemporal Elasticity of Substitution

Panel A of Table 4 presents the GMM results for the two-good model based on the marginal product of capital. Estimation is by continuous updating GMM of Hansen, Heaton, and Yaron

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[19] Other asymptotically efficient estimators are proposed by Phillips and Hansen (1990), Saikkonen (1991), and Stock and Watson (1993). As these all have the same limiting properties as the CCR estimator, the following estimation results are unlikely to be influenced by the choice of the estimation methods. For comparison with Ogaki and Reinhart (1998), we report results from the CCR procedure.

[20] We used the VAR(1) prewhitening technique of Andrews and Monahan (1992) to estimate the long-run variance matrix of the disturbances in the system. We report the third-stage CCR estimates and the fourth-stage H(p,q) statistics. For details on these points, see Park and Ogaki (1991) and Han (1996).

(1996), using the heteroskedasticity-robust version of the weighting matrix (see Stock and Wright 2000). This estimator has desirable properties in terms of higher-order asymptotic approximation because of the absence of any bias from the preliminary estimator (Newey and Smith 2004), implying less bias in finite samples than the two-step or iterated estimators. The numerical optimization of the continuous updating GMM objective function requires initial values for the parameters. We set the initial values equal to the two-step GMM estimates, using the identity matrix in the first stage.[21]   The instrumental variables used are a constant, the realized real interest rate, the growth rates of C_t and D_t, the growth rate of C_t/D_t, and the growth rate of P_t. To control for the time aggregation problem caused by the use of quarterly data, all instruments are lagged two periods.

The first row of Table 4 reports the result when the parameters β, a, and σ are estimated simultaneously, and the second to fifth rows of Table 4 report the results when the discount factor β is fixed. It is known that β and σ are negatively correlated. For this reason, unless β is fixed, it is difficult to discuss the direction of change in the estimates of σ as in the analysis below. To make the results from the two-good and the one-good models comparable, the discount factor is fixed at $\beta = 0.999$, 0.995, 0.990, and 0.985, encompassing values that the literature previously used or obtained for quarterly data.

As shown in Panel A of Table 4, for all cases, the estimated IES is positive and significantly different from zero. Hansen's J-test of overidentifying restrictions does not reject the model at conventional significance levels. In the first row, the estimate of β is slightly above one, but in the light of interval estimation it does not eliminate the possibility of taking a value that is less than one.

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[21] The numerical minimization of the continuous updating GMM objective function is implemented by the Nelder–Mead method. This choice follows Stock and Wright (2000), who used this method and did Monte Carlo experiments and empirical exercises in their paper.

Panel B of Table 4 reports the results of sensitivity analysis with respect to possible values of the parameters, ϵ, δ, and τ, given exogenously in the second step of GMM. The sixth row of Table 4 reports the result when the value of ϵ is reduced by two standard errors. Pakoš (2004) points out that ϵ becomes smaller than one when nonhomothetic preferences over nondurable and durable goods are used. A decrease in the value of ϵ corresponds to making the substitution effect less important in our framework, as Ogaki and Reinhart (1998) discuss. Therefore, if the substitution effect is important for obtaining higher estimates of σ, such changes in ϵ will yield a smaller estimate of the IES. The result indicates that this prediction is true for our estimate of σ, while the estimated IES remains positive and significant.[22] Again, the model is not rejected by the J-test.

The asymptotic distribution of the GMM estimator in the second step is not affected by the cointegrating regression estimator in the first step because of its superconsistency; however, in finite samples, a bias in the cointegrating regression estimator can affect the distribution of the GMM estimator. Therefore, it is preferable that the estimate of σ is not sensitive to changes in the value of ϵ. Looking at the result again with this point in mind, although the estimate of σ in the sixth row of Table 4 decreases as expected, its change is very small relative to the size of the change in ϵ. On the other hand, the estimation in the sixth row involves an increase in the estimate of the preference weight a, implying an increase in weight of the durable good in the consumer's preferences. Because this change in the preference weight is interpreted as indicating that the role of durable goods becomes more important, we conversely expect it to yield a larger estimate of the IES. However, the result suggests that this effect through the preference weight

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[22] Because the decrease in σ may be due to the increase in β, we also tried an alternative estimation in which β was fixed at the same values as those in the base runs. The results supported this prediction. For example, the estimate of the IES is $\sigma=0.267$ with a standard error of 0.014 when $\beta=0.999$, and it is $\sigma=0.285$ with a standard error of 0.016 when $\beta=0.995$. These values of σ are lower than those reported in the second and third rows of Table 4, respectively.

does not dominate that of ϵ and β mentioned above.

The seventh row of Table 4 reports the result for a different value of the depreciation rate where $1 - \delta$ is set to 0.94, used commonly for the U.S. data as discussed in Section II. As before, the estimated IES is positive and significant. The J-statistic is also not sensitive to the change in the value of the deprecation rate. In this case, the estimate of σ rises to 0.425, but this can be explained by the decrease in the estimate of β from 1.007 to 0.980 rather than by the change in the depreciation rate.[23]

In comparison to recent quantitative research on the Japanese economy, our capital income tax rate $\tau = 0.525$ appears to be somewhat large. The eighth row of Table 4 presents the result when the capital income tax rate is set to $\tau = 0.435$, which is the sample average over the period 1981–1999 calculated from Table A-1 of Chen et al. (2006a).[24] Both the estimate of σ and the J-statistic are not very sensitive to this change in τ.

In Panel B of Table 4, we also tried estimation under other combinations of instrumental variables. The first one excludes the growth rate of D_t from the instruments set, which is close to the choice of Ogaki and Reinhart (1998), and the second one instead excludes the growth rate of C_t/D_t. The corresponding results are presented in the ninth and tenth rows of Table 4. The estimates of σ are essentially the same as that of the first row; the J-test does not reject the model at the conventional significance levels.

Thus far, our model evaluation has been based on the conventional GMM inference. However, when there is weak identification (commonly called weak instruments in linear models), such conventional inference based on point estimates, standard errors, and the J-test can be invalid.

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[23] Indeed, the results when β is fixed were similar to those reported in Panel A of Table 4. For example, σ is estimated to be 0.263 with a standard error of 0.020 when β=0.999, and σ is estimated to be 0.288 with a standard error of 0.022 when β=0.995.

[24] To calibrate a model to the Japanese economy, Hayashi and Prescott (2002) set it to $\tau = 0.480$; on the other hand, Nakajima (2006) uses $\tau = 0.442$. Therefore, these alternatives fall into the range of 0.435–0.525 tried in this paper.

To see the robustness of our results to weak identification, we compute a confidence set that is constructed from the S-statistic proposed by Stock and Wright (2000) (see the Appendix for a more complete description). The confidence set, which we refer to as the S-set, is the set of parameter values for which the joint null hypothesis that the parameters are the true values and the moment conditions hold is not rejected. If the weak identification problem is acute, then the S-set and conventional GMM confidence set have substantially different areas. When reexamining our results by this criterion, we consider two types of the S-set for two reasons. First, the discount factor β is known to be estimated tightly in the literature, so that it is frequently regarded as being strongly identified. We therefore treat σ and a as weakly identified parameters and evaluate the effect of weak identification through looking at the two-dimensional S-set for (σ, a). Second, σ is the main parameter that is suspected of being weakly identified in the literature. For example, Yogo (2004) examines this question by using a counterpart of the S-set for σ in the context of linear models. We consider the S-set for σ associated with our nonlinear model.

Panel (a) of Figure 6 plots the S-set for (σ, a), denoted by the shaded area, and the conventional GMM confidence ellipse, which is comparable to the first row of Panel A in Table 4.[25] The last two columns of Table 4 report the S-sets for σ and the conventional confidence intervals calculated using the point estimates and standard errors. The results indicate that the S-sets and conventional GMM confidence sets closely agree for all cases in Panels A and B, implying that the conventional GMM inference is reliable.

Panels A and B of Table 4 overall reveal that the IES under the two-good model is significantly different from zero and is within a relatively narrow range. At first glance, our estimates

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[25] Although it is possible to calculate the S-sets for (σ, a) with respect to the other cases from the second to the tenth rows, we omit them to save space.

of the IES around 0.2 under the two-good model appears small compared with the U.S. case, when Ogaki and Reinhart's (1998) IES estimates in the range of 0.32–0.45 are taken as a reference point. However, the upper end of the S-sets for σ in Panels A and B includes values of the IES around 0.45. In a more recent study, Yogo (2004) concludes that the IES for the United States is around 0.2 for a longer sample period 1947:3–1998:4. In this respect, the magnitudes of the IES for both the United States and Japan are quite similar.

Next, we examine how ignoring the substitution effect affects the estimates of the IES. Panel A of Table 5 reports the GMM results for the one-good model that assumes $\sigma = \epsilon$ and $a = 0$, in which the marginal product of capital is used as the return data. For the reason already mentioned, we report the results when β is fixed at the four values. Because the one-good model does not involve the intratemporal first-order condition (6), it is estimated and tested through only the moment condition implied by the Euler equation. For the case of $\beta =0.999$, 0.995, and 0.990, we find that the separability assumption ($\sigma = \epsilon$) yields smaller point estimates of σ. The S-sets for σ, which contain negative values of the IES, also lead to the same conclusion. This finding can be viewed as a strong confirmation of the downward bias in the IES. That is, the IES for Japan possesses the same property as that previously found by Ogaki and Reinhart (1998) for the U.S. data. For the case of $\beta =0.985$, we encountered a computational problem, so that it was impossible to obtain the estimate of σ. However, given the strong evidence in favor of the two-good model, this is a possibility under the misspecified model.

We now turn to GMM estimation of σ based on the real interest rate. The results are given in Panel C of Table 4 for the two-good model and Panel B of Table 5 for the one-good model. As before, Panel B of Table 5 reports the results when β is fixed at the four values. Because we have already confirmed how a change in the values of ϵ, δ, and τ biases the estimate of σ, here we concentrate on the base-run cases. The corresponding S-set for (σ, a) and conventional

GMM confidence ellipse are shown in Panel (b) of Figure 6. The following can be seen from the two panels and the figure.

First, for the case of simultaneous estimation in the eleventh row of Table 4, the J-test does not reject the model. However, as shown in Panel (b) of Figure 6 and the last two columns in the eleventh row of Table 4, the S-sets and conventional GMM confidence sets do not agree in this case, leading to substantially different conclusions with respect to possible values of σ. Second, when β is fixed at 0.999 and 0.995, both the two-good and one-good models are not rejected by the J-test. For the two cases, the separability assumption ($\sigma = \epsilon$) appears to yield a smaller point estimate of σ. However, for the case of β=0.995, σ is estimated to be 1.230 under the two-good model; that is, evidence against the separability assumption is weak for this choice of β. In addition, σ is estimated to be 0.445 under the one-good model, but the S-set for σ is [-1.524, 2.052] as in the sixth row of Table 5. That is, it is difficult to interpret the value of σ=0.445 under the one-good model as the evidence of the downward bias caused by ignoring the intratemporal substitution effect. This evidence rather can be viewed as an example that the conventional GMM inference leads to an erroneous conclusion because it does not allow for weak identification. Third, for the case of β=0.990 and 0.985, σ is significantly estimated under the two-good model, but the estimate of σ is not available under the one-good model because it took exceptionally high values. This can also be explained by weak identification. In summary, the results for the real interest rate overall bear the weak identification problem. If we restrict our concern to the case of β =0.999, we find that the value of σ is much larger than that of the marginal product of capital (i.e., compared with σ =0.299 in the second row) and, only in that case, we can again confirm the downward bias in the IES.

Given the results above, the question is which is the preferred IES estimate, the marginal product of capital or the real interest rate? Because of our model extension that allows for the

production side, there is no reason to restrict our view to the literature on the IES and the CCAPM, in order to ask this question. As compared to recent quantitative research on the Japanese economy, our choice of the variables necessary for calculating the marginal product of capital, that is, the capital income tax rate, the output-capital ratio, and the capital share is reasonable. On the other hand, we do not have a basis for judging whether the short-term money market rate, a typical measure in the literature, is a good proxy for the rate of return. Thinking of the above-mentioned econometric problems in the case of the real interest rate and the data properties discussed in Section II, we conclude that the IES estimates based on the marginal product of capital are preferable; in that case, the IES agrees with the U.S. case in terms of both its magnitude and the importance of incorporating the intratemporal substitution effect.

Further Discussion

The results presented in this paper suggest that there is no significant disparity in the magnitude of the IES between the United States and Japan. We have reached this conclusion by tackling some of issues identified in the literature, i.e., by theoretically allowing for durables and a different rate of return measure and econometrically allowing for the weak identification problem. However, the other issues remain intact, many of which were recognized in empirical microeconomic studies including those from labor economics. They include at least the following: (i) nonhomotheticity (i.e., inconsistency with the fact that budget shares for necessary goods and luxury goods change over time); (ii) nonseparability between different components of consumption, other than between nondurables and durables; (iii) nonseparability of consumption from male and/or female labor supply; (iv) differences resulting from factors such as household size and composition, age, and cohort; (v) differences between asset holders and non-asset holders or

households with high wealth and low wealth. See, e.g., Attanasio and Weber (1993, 1995), Blundell, Browning, and Meghir (1994), Attanasio and Browning (1995), Meghir and Weber (1996), Vissing-Jørgensen (2002), Vissing-Jørgensen and Attanasio (2003), and Guvenen (2006).[26]

The broad surveys suggest that each of these factors is equally important for obtaining sharp IES estimates, while the current literature estimates the IES by concentrating on one issue or only a few issues. For example, in the context of macroeconomics, it is frequently argued that the IES estimates from aggregate time-series data may be unreliable because they do not control for differences in stock or bond holdings among households, i.e., factor (v) (see e.g., Vissing-Jørgensen 2002 and Guvenen 2006). Although the aggregation problem across households can be avoided with panel data, the current literature along this line does not address both the nonseparability of consumption from male and/or female labor supply and the nonseparability between different components of consumption including durables (and, consequently, nonhomotheticity of preferences as well). In this respect, an impartial view would be that the current literature based on micro data also has controlled for only one of the issues listed above to obtain the IES, so that it still suffers from the other shortcomings. Here, we have considered (v) as an example (simply because it was not discussed in Browning, Hansen, and Heckman (1999)), but a similar discussion is also possible for the other factors. Indeed, looking at Tables 3.1 and 3.2 of Browning, Hansen, and Heckman (1999), which summarized the main results in the early literature including those from labor economics, we see that the empirical microeconomic studies also have produced a wide range of values for the IES, depending on which factors of (i) to (iv) are allowed for.

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[26] Because the purpose here is to give a balanced view of what has been demonstrated in this paper, we do not intend to mention individually the main results of this large literature. A comprehensive evaluation of the IES estimates from empirical microeconomic studies on consumption up to the 1990s, which focuses on (i)–(iv), can be found in Browning, Hansen, and Heckman (1999, Section 3), who in particular emphasize a serious lack of consideration for (iii) in the existing literature. A brief, but helpful survey including (v) can be found in Guvenen (2006, Section 1.1 and Section 3.1.2).

Thus, our estimates of the IES based on aggregate data are far from complete, but it seems fair to state that the same criticism is also true for the existing estimates of the IES based on micro data. In this situation, a way to look at the estimates of the IES under the framework of this paper (including U.S. studies using a similar one) will be a role as a reference point when evaluating estimates of the IES from micro data or aggregate data under a different framework. For example, suppose that a researcher obtained an estimate of the IES around 0.8 for a group of households. A natural question that arises here is: is it high or low? The current literature using the U.S. data typically asks this question by comparing it with Hall's (1988) estimates based on aggregate data and then provides an explanation that Hall's estimates were biased by aggregation over groups with different IES. This approach is typical in the literature demonstrating heterogeneity in the IES and the bias resulting from the aggregation. In more recent research, such as Guvenen (2006), Ogaki and Reinhart's (1998) estimates play the same role as Hall's. What this means is that one needs a good reference point when looking at estimates of the IES from micro data (e.g., when considering which factors are more crucial for the IES and which directions in bias are introduced by ignoring some factor).

Given these perspectives, probably the only implication that we can draw from the findings in this paper is that the U.S. debate on the IES such as the small magnitude and the downward bias can be a good starting point even for Japan. On the face of things, this implication may sound inconsiderable but, from a macroeconomic point of view for example, it would be helpful in that researchers who care about Japan become less uncertain than ever about the value of the IES to be used in their quantitative work; or, from a microeconomic point of view, it can be useful in evaluating future empirical work on the IES for Japanese households, as the U.S. literature did. However, we should hesitate to draw further implications (e.g., about risk aversion) from our estimates of the IES.

IV. Concluding Remarks

In this paper, we have estimated the IES using Japanese aggregate data, and examined whether the IES for Japan possesses similar properties to those identified for the United States in the literature. To control for possible factors that cause a bias in the IES, we have used a model with both production and nonseparability between nondurables and durables. This attempt is important because it has been recognized that Japan is a country that differs from the United States from the macroeconomic perspective of saving behavior, given Japan's historically high saving rate.

We have found strong evidence that supports the claim of Ogaki and Reinhart (1998) that allowing for the intratemporal substitution between nondurables and durables is important in estimating the IES. Our empirical results indicate that the IES is significantly different from zero, and the point estimates of the IES are around 0.2–0.4 when the marginal product of capital is used as the return data and around 0.9–1.0 when the real interest rate is used. We have argued that the IES estimates based on the marginal product of capital are preferable.

We also have mentioned that the finding that the IES for Japan is similar to that for the United States may be bound up with the claim that Japan's high saving rate is a statistical illusion. Unfortunately, our analysis has been limited to the sample period of 1981:1–1999:1 because of a scarcity of data. For this reason, it is impossible to associate our empirical results with the recent decline in the Japanese saving rate. However, if the saving rate is a good indicator, the reduction in the gap between the United States and Japan may ensure our result that the IES is quite similar between the two countries.

In this paper, we have assumed expected utility with homothetic preferences over nondurable and durable goods. The recent literature has found that imposing homotheticity leads to an

upward bias in the elasticity of substitution (e.g., Pakoš 2004, 2007). However, our sensitivity analysis with respect to ϵ suggests that it would not substantially affect the IES estimates. This finding, on the other hand, does not mean that the model in this paper outperforms the nonhomothetic utility model proposed by Pakoš (2004). Rather, it should be regarded as evidence showing that nonhomotheticity does not seem to cause practical problems for our data set.[27] Thus, it is still of interest to reinvestigate models with such nonhomothetic preferences at the time when the data necessary for constructing a longer sample period became available. A more substantive issue of our model is that the expected utility does not allow the separation of the IES from risk aversion. In other words, the model assumes that the consumer is indifferent with respect to the timing of the resolution of uncertainty (Epstein and Zin 1989 and Weil 1990). One way to allow for the timing of the resolution is to use nonexpected utility, as previously mentioned in the Introduction. Because Yogo (2006) found that the nonexpected utility model with nonseparability in nondurable and durable goods yields a smaller and significant IES, we may expect that the IES for Japan also would take a value below 0.2 in this case. Furthermore, to better understand why the Japanese IES is low at the aggregate level, empirical studies need to proceed to micro data. These further investigations and extensions remain as future research along the lines of this paper.

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[27] We tried estimation of Pakoš's (2004) nonhomothetic utility model. Unfortunately, the cointegrating regression in the first step did not yield estimates of curvature parameters with a theoretically expected sign, and so we were unable to judge whether nonhomotheticity is significant for our data set. Probably one reason for this is related to the shortness of the time period used in this study. That is, compared with previous studies that pointed out the importance of nonhomotheticity or nonlinearity of Engel curves, the sample period used in this study is not long enough to identify the effect of nonhomotheticity. For example, Pakoš's (2007) sample period is 1956:Q1–2001:Q4, which is about twice as long as the sample period used in this study. Ogaki's (1992) sample period is 1929–1988, which is about three times as long as the sample period used in this study.

Let θ be an n-dimensional parameter vector with the true value θ_0. The true parameter value is assumed to satisfy G conditional moment conditions:

$$E_t[h(y_t, \theta_0)] = 0,$$

where h is a G-dimensional vector-valued function and y_t is a vector of model variables. Let z_t be a vector of K instrumental variables known at time t and define the moment function $g_t(\theta) = h(y_t, \theta_0) \otimes z_t$. Then the parameter vector is identified by the following unconditional moment conditions:

$$E[g_t(\theta_0)] = 0.$$

Weak identification occurs when $E[g_t(\theta)]$ is close to zero for $\theta \neq \theta_0$. In this situation, conventional GMM tests such as Hansen's J test may be invalid because consistency and asymptotic normality of GMM estimators are not ensured. See Stock, Wright, and Yogo (2002) for a survey of weak identification in nonlinear GMM.

Stock and Wright (2000) propose a test that is valid even when there is weak identification, based on the following continuous updating GMM objective function (Hansen, Heaton, and Yaron 1996):

$$S_T(\theta) = T g(\theta)' \Omega(\theta)^{-1} g(\theta)$$

where

$$g(\theta) = \frac{1}{T} \sum_{t=1}^{T} g_t(\theta)$$

$$\Omega(\theta) = \frac{1}{T} \sum_{t=1}^{T} g_t(\theta) g_t(\theta)'.$$

Despite the presence of the weak identification, under the null hypothesis $\theta = \theta_0$, the statistic $S_T(\theta)$ follows the asymptotic χ^2 distribution with GK degrees of freedom (Stock and Wright

2000, Theorem 2). In principle, a confidence set for θ, called an S-set in the context of weak identification, can be constructed by inverting the test based on $S_T(\theta)$ numerically, but the inversion is only feasible for a small dimension of the parameter vector θ. If it is known that some parameters in θ are strongly identified, the following approach is also valid.

Let us partition the parameter vector as $\theta = (\theta'_w, \theta'_s)'$, where θ_w is an n_1-dimensional vector of weakly identified parameters, and θ_s is an n_2-dimensional vector of strongly identified parameters. Replace θ_s with the GMM estimator of θ_s conditional on a given value of θ_w, say $\bar{\theta}_w$:

$$\widehat{\theta}_s(\bar{\theta}_w) = \arg \min_{\theta_s \in \Theta_s} S_T(\theta)|_{\theta_w = \bar{\theta}_w}.$$

Stock and Wright (2000, Theorem 3) show that the objective function at the true value, $S_T(\theta_{w0}, \widehat{\theta}_s(\theta_{w0}))$, has an asymptotic χ^2 distribution with $GK - n_2$ degrees of freedom. The S-set for θ_w can be defined by inverting the test of $\theta_w = \theta_{w0}$ based on this objective function. When θ_w is a scalar (i.e., $n_1 = 1$), the S-set forms a confidence interval. The S-set for (σ, a) and the S-set for σ in the text are constructed using this result.

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Table 1
Descriptive Statistics

Variable	Mean (%)	Std.Dev (%)	Correlation				
			Nondurables	Durables	Service Flow	Interest Rate	MPK1
(1) Nondurables	0.241	1.210					
(2) Durables	1.693	4.522	0.317				
(3) Service Flow	1.751	0.732	0.024	-0.007			
(4) Interest Rate	0.810	0.849	-0.084	-0.045	0.053		
(5) MPK1	6.502	1.851	0.130	0.237	0.075	0.388	
(6) MPK2	4.461	1.278	0.135	0.231	0.065	0.395	0.999

Note: Nondurables, durables, and the service flow from durables are growth rates defined as the logarithmic difference. The service flow from durables is calculated using $1 - \delta = 0.96$. MPK1 and MPK 2 denote the marginal product of capital without and with public capital, respectively.

Table 2
Unit Root Test Results

Variable	ADF(1)	ADF(4)	ADF(7)	J(1,5)
$\ln(C_t/D_t)$	-0.234	-0.921	-1.276	3.246
	[0.991]	[0.947]	[0.885]	
$\ln P_t$	-1.581	-1.647	-1.943	2.496
	[0.791]	[0.764]	[0.621]	

Note: ADF(r) denotes the Augmented Dickey–Fuller test with r lags. The numbers in square brackets are p-values calculated from MacKinnon's (1996) numerical distribution functions. The 1%, 5%, and 10% critical values for J(1,5) are 0.123, 0.295, and 0.452, respectively. When the J(1,5) statistic is smaller than these values, the null hypothesis of difference stationarity is rejected.

Table 3
Cointegrating Regression Results

η	ϵ	d	H(0,1)	H(1,2)	H(1,3)	H(1,4)
1.177	1.821		3.460	6.775	6.949	7.562
(0.102)	(0.540)		[0.063]	[0.009]	[0.031]	[0.056]
1.585	1.409	-0.516	0.050	3.680	5.126	5.130
(0.083)	(0.245)	(0.076)	[0.823]	[0.055]	[0.077]	[0.162]

Note: The third column presents the estimated coefficient of the intercept dummy variable for the period 1987:1–1999:1. The numbers in parentheses are standard errors. H(0,1) denotes a χ^2 test statistic with one degree of freedom for the deterministic cointegration restriction. H(1,q) denotes a χ^2 test statistic with $q-1$ degrees of freedom for stochastic cointegration. The numbers in square brackets are asymptotic p-values.

Table 4
GMM Estimates of the Intertemporal Elasticity of Substitution
(The Two-Good Model)

	$1-\delta$	ϵ	β	a	σ	J_T	90%S-set for σ	90%CI for σ
Panel A: Base-Run Results								
(1)	0.96	1.409	1.007	3.043	0.262	13.406	[0.140, 0.457]	[0.137, 0.387]
			(0.017)	(0.037)	(0.076)	[0.145]		
(2)	0.96	1.409	0.999	3.036	0.299	13.621	[0.256, 0.343]	[0.274, 0.324]
			(0.037)	(0.015)		[0.191]		
(3)	0.96	1.409	0.995	3.031	0.321	13.912	[0.275, 0.372]	[0.295, 0.347]
			(0.037)	(0.016)		[0.177]		
(4)	0.96	1.409	0.990	3.022	0.355	14.459	[0.303, 0.415]	[0.324, 0.386]
			(0.037)	(0.019)		[0.153]		
(5)	0.96	1.409	0.985	3.010	0.396	15.190	[0.341, 0.467]	[0.360, 0.432]
			(0.038)	(0.022)		[0.125]		
Panel B: Results of Sensitivity Analysis								
(6)	0.96	0.919	1.021	5.086	0.199	14.711	[0.134, 0.291]	[0.102, 0.296]
			(0.023)	(0.130)	(0.059)	[0.099]		
(7)	0.94	1.409	0.980	2.536	0.425	13.443	[0.170, 0.808]	[0.245, 0.605]
			(0.008)	(0.024)	(0.110)	[0.144]		
(8)	0.96	1.409	0.999	3.046	0.246	13.436	[0.134, 0.423]	[0.131, 0.361]
			(0.018)	(0.037)	(0.070)	[0.144]		
(9)	0.96	1.409	1.022	3.044	0.210	12.673	[0.103, 0.307]	[0.066, 0.354]
			(0.030)	(0.038)	(0.088)	[0.080]		
(10)	0.96	1.409	1.019	3.045	0.218	12.644	[0.121, 0.310]	[0.079, 0.357]
			(0.028)	(0.038)	(0.085)	[0.081]		
Panel C: Results Based on the Real Interest Rate								
(11)	0.96	1.409	0.987	2.982	2.546	13.539	[0.952, ∞]	[-1.090, 6.182]
			(0.005)	(0.036)	(2.217)	[0.140]		
(12)	0.96	1.409	0.999	3.052	0.964	15.949	[0.883, 1.043]	[0.885, 1.043]
			(0.037)	(0.048)		[0.101]		
(13)	0.96	1.409	0.995	3.028	1.230	14.710	[1.074, 1.413]	[1.107, 1.353]
			(0.036)	(0.075)		[0.143]		
(14)	0.96	1.409	0.990	2.995	1.857	13.719	[1.506, 2.480]	[1.578, 2.136]
			(0.036)	(0.170)		[0.186]		
(15)	0.96	1.409	0.985	2.973	3.902	13.623	[2.655, 9.400]	[2.651, 5.153]
			(0.035)	(0.763)		[0.191]		

Note: The instrumental variables used are a constant, the realized real interest rate, the growth rates of C_t and D_t, the growth rate of C_t/D_t, and the growth rate of P_t, except for rows (9) and (10). In row (9), the growth rate of D_t is excluded; in row (10), the growth rate of C_t/D_t is instead excluded. All instrumental variables are lagged by two periods. In row (8), the tax rate on capital income is changed from $\tau = 0.525$ to $\tau = 0.435$. Estimation is by continuous updating GMM. Standard errors are in parentheses. J_T denotes the J-statistic of the overidentifying restrictions, and the p-values for the J-test are in square brackets. CI denotes the confidence interval calculated using the point estimate and standard error.

Table 5

GMM Estimates of the Intertemporal Elasticity of Substitution

(The One-Good Model)

	β	σ	J_T	90%S-set for σ	90%CI for σ
Panel A: Base-Run Results					
(1)	0.999	0.029	4.015	[-0.055, 0.194]	[0.011, 0.047]
		(0.011)	[0.404]		
(2)	0.995	0.028	3.997	[-0.064, 0.214]	[0.010, 0.046]
		(0.011)	[0.406]		
(3)	0.990	0.027	3.986	[-0.078, 0.245]	[0.011, 0.043]
		(0.010)	[0.408]		
(4)	0.985	NA	—	—	—
Panel B: Results Based on the Real Interest Rate					
(5)	0.999	0.292	3.811	[-0.476, 0.975]	[0.130, 0.454]
		(0.099)	[0.432]		
(6)	0.995	0.445	4.347	[-1.524, 2.052]	[0.217, 0.673]
		(0.139)	[0.361]		
(7)	0.990	NA	—	—	—
(8)	0.985	NA	—	—	—

Note: The instrumental variables used are a constant, the realized real interest rate, the growth rates of C_t and D_t, and the growth rate of C_t/D_t. All instrumental variables are lagged by two periods. Estimation is by continuous updating GMM. Standard errors are in parentheses. J_T denotes the J-statistic of the overidentifying restrictions, and the p-values for the J-test are in square brackets. CI denotes the confidence interval calculated using the point estimate and standard error. NA means that the point estimate is not available because it took an exceptionally high value or because of a computational problem. The horizontal line in the NA case denotes that it is impossible to calculate the J_T, the S-set, or the CI.

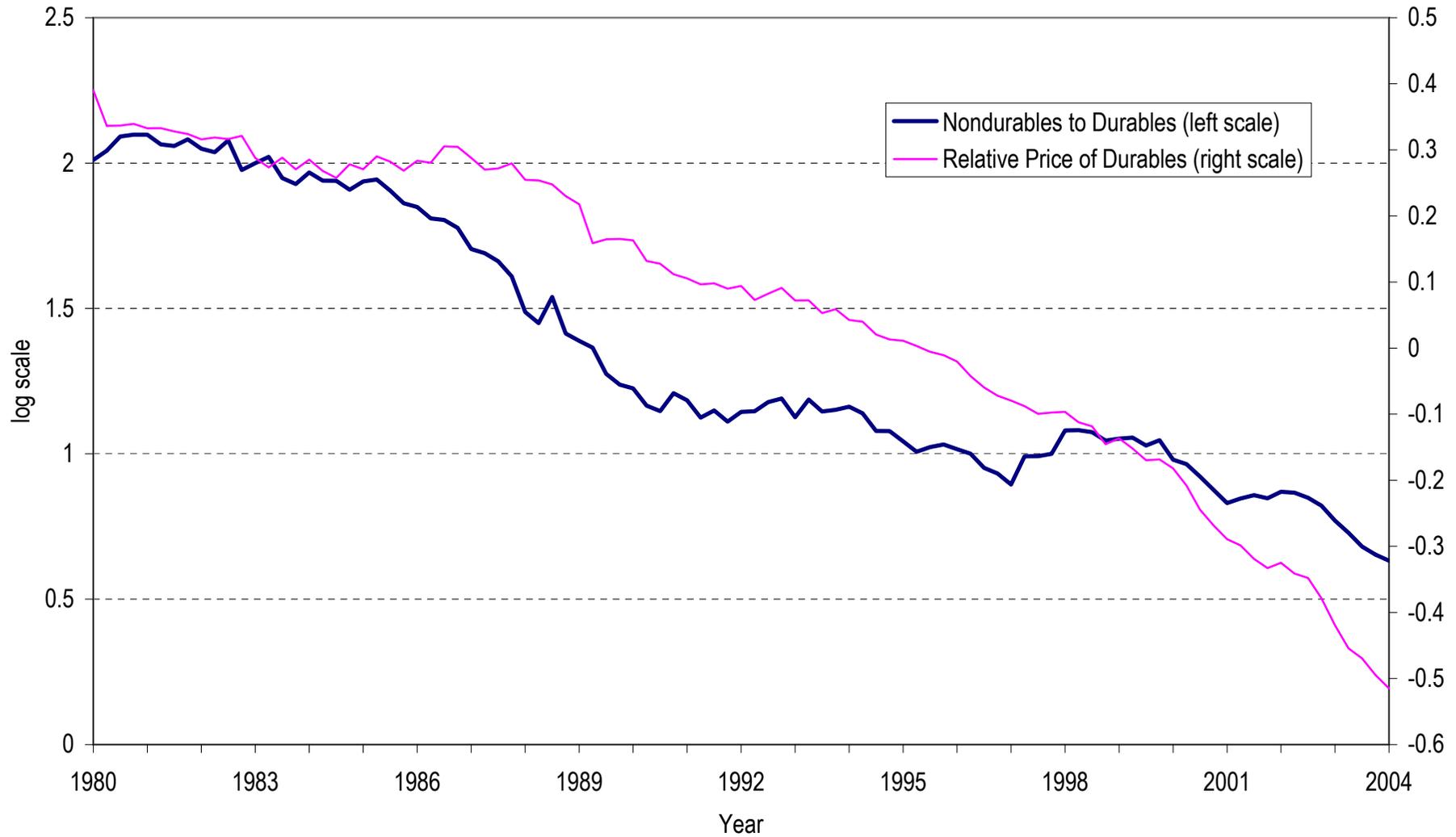# Figure 1: Price and Ratio of Nondurables to Durables Expenditure

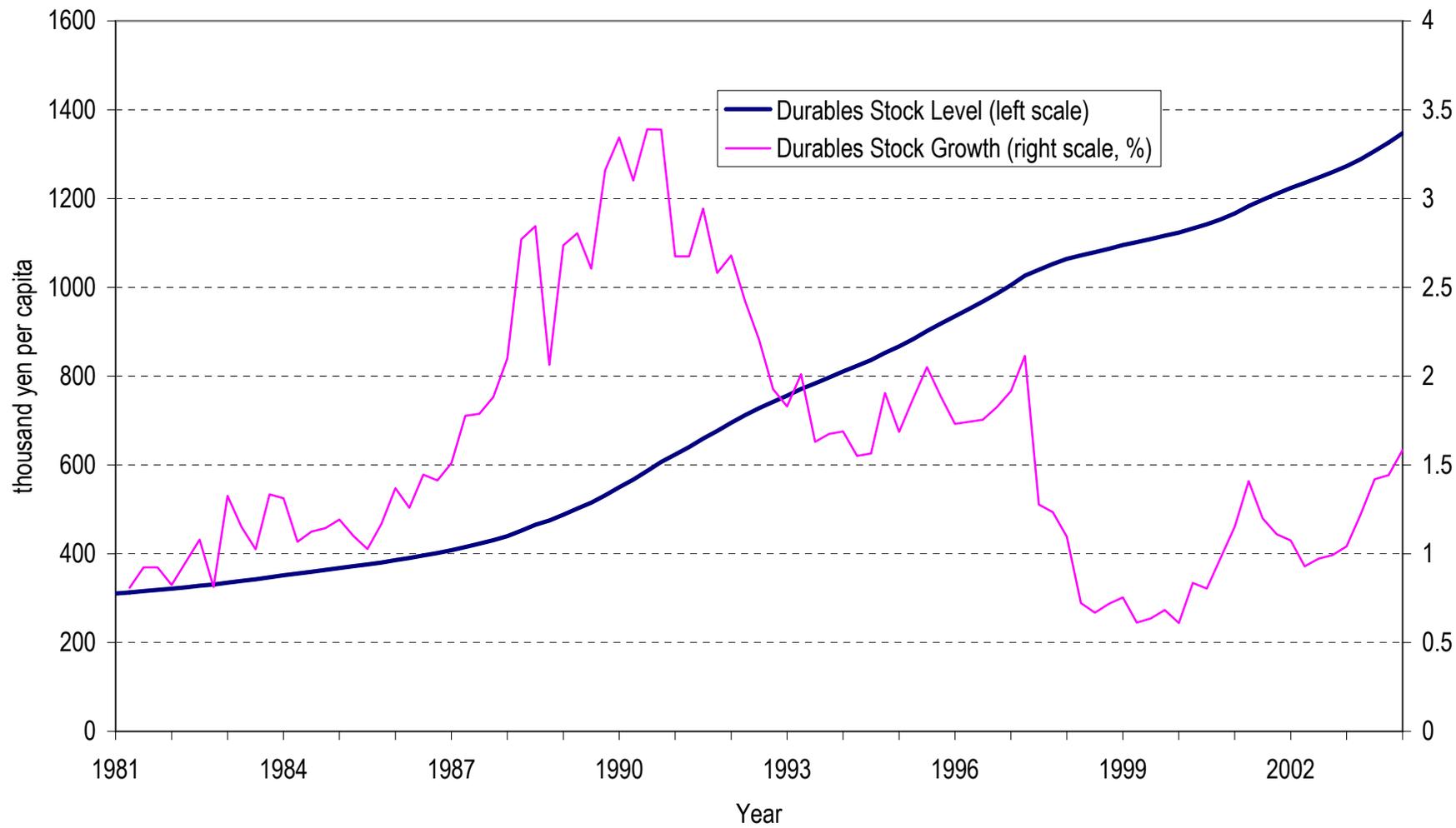**Figure 2: Durables Stock Level and Growth Rate**

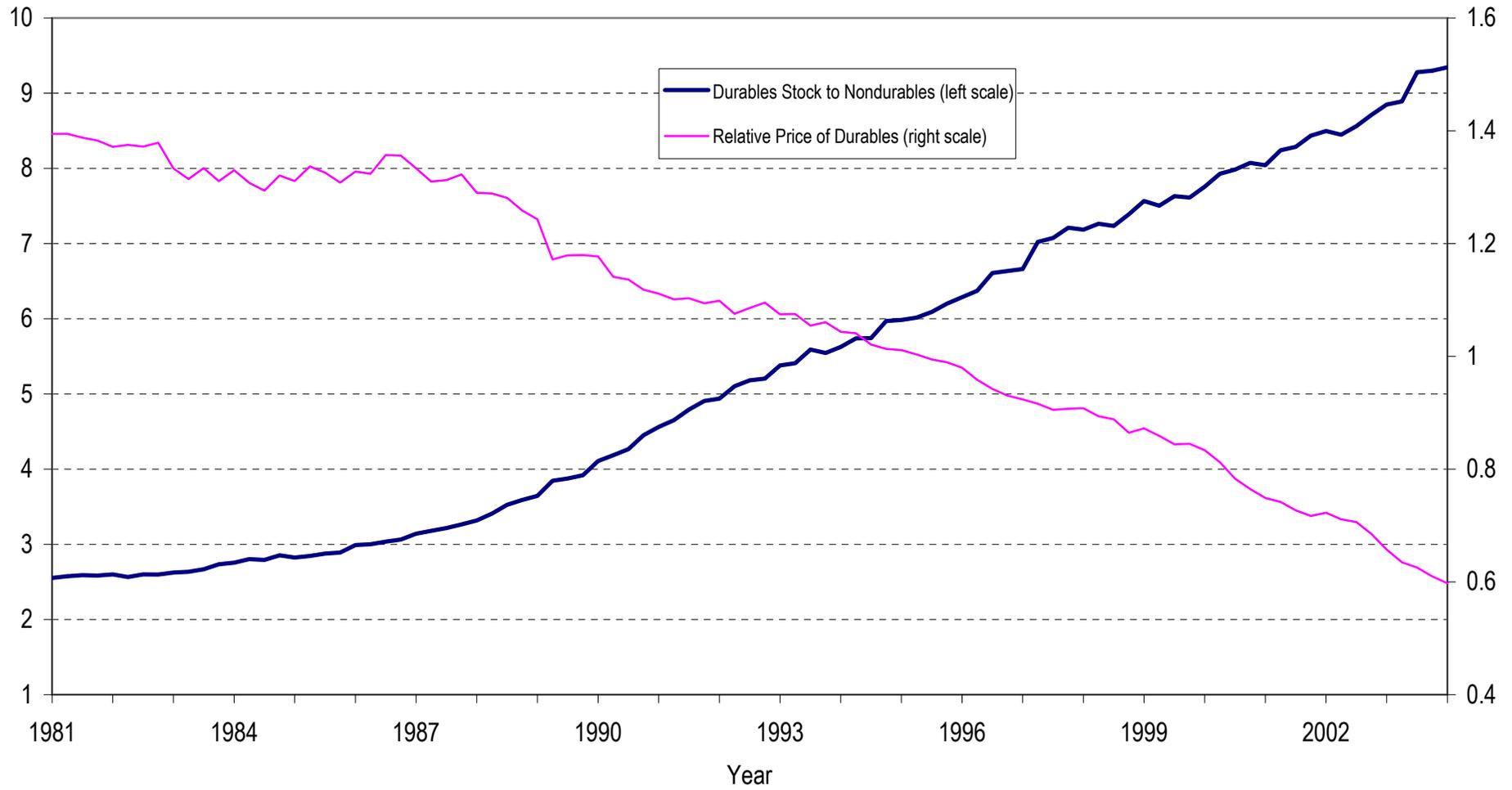**Figure 3: Price and Ratio of Durables Stock to Nondurables Expenditure**

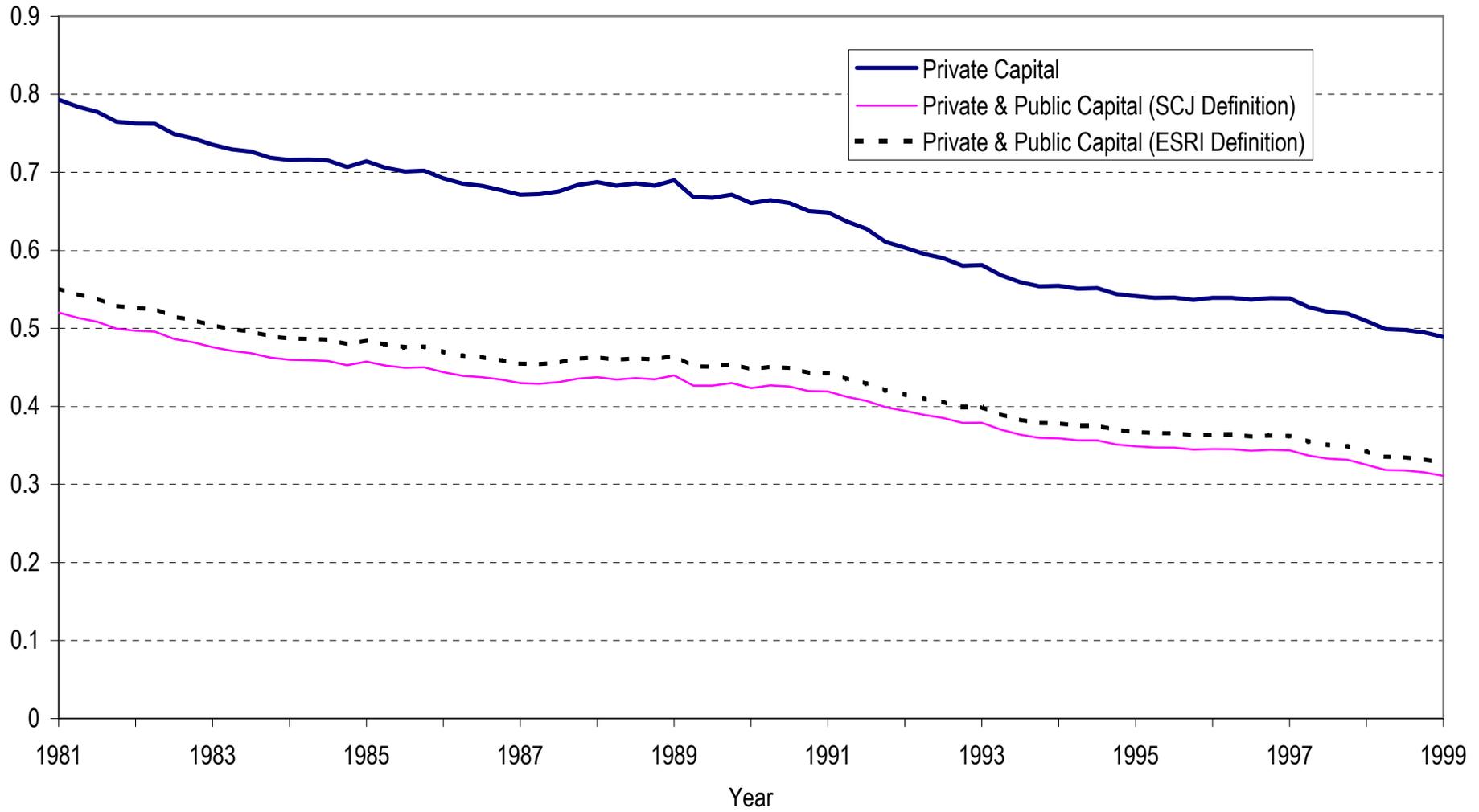# Figure 4: Ratio of Real GDP to Capital Stock

Legend:
- Private Capital
- Private & Public Capital (SCJ Definition)
- Private & Public Capital (ESRI Definition)

Year

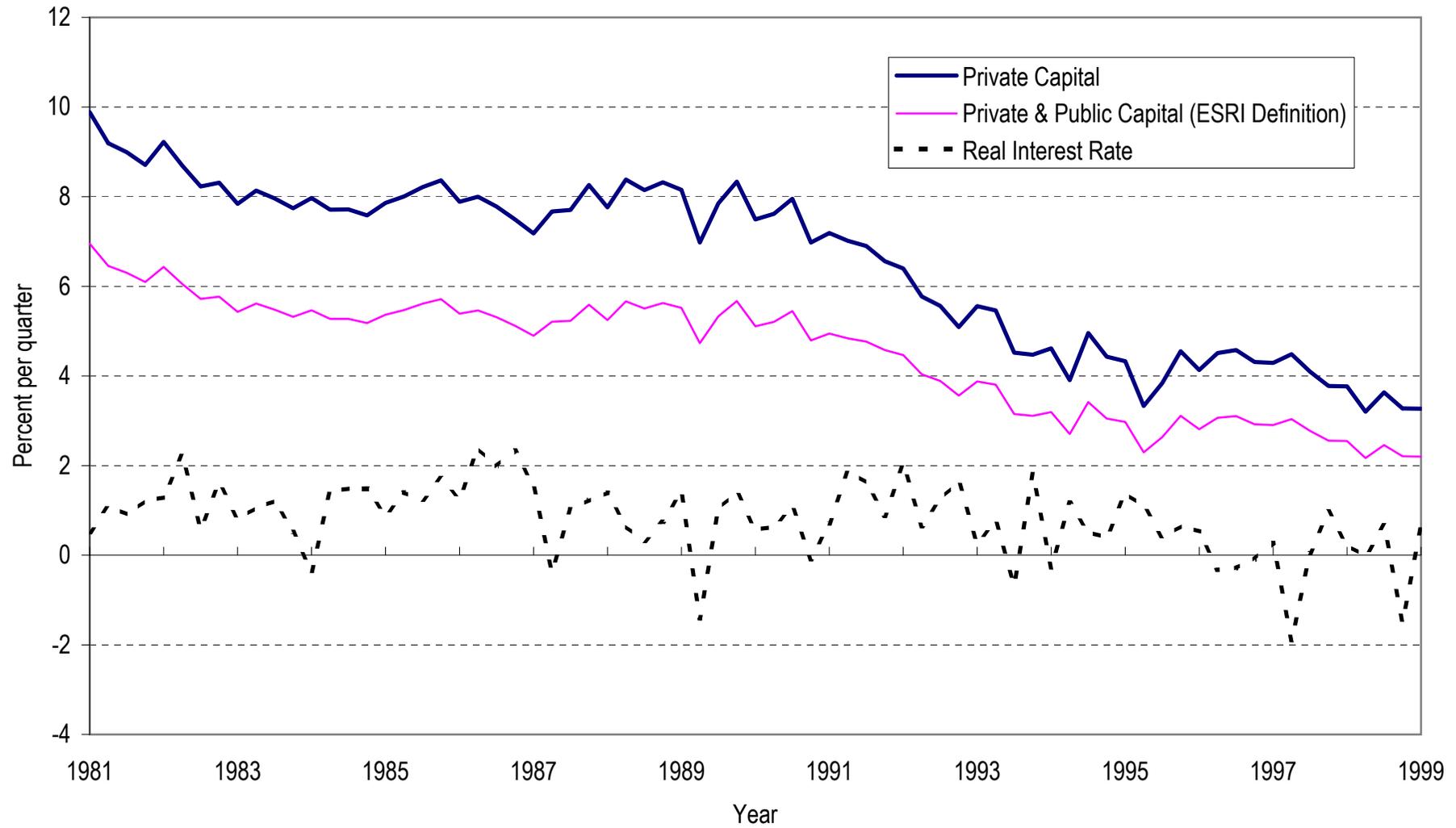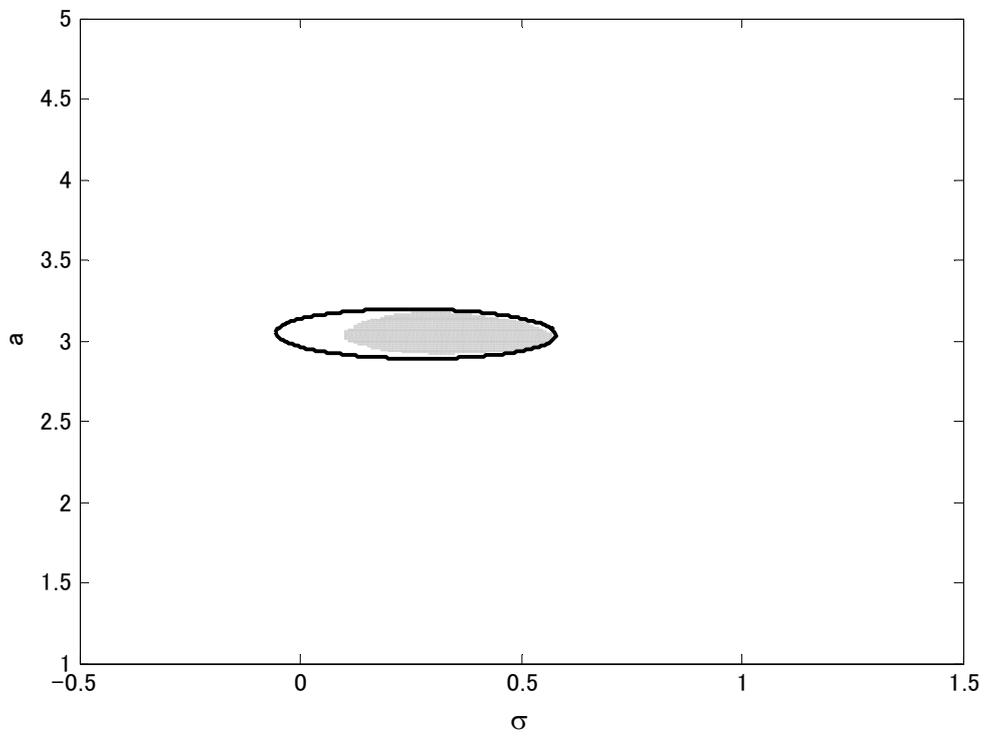Figure 5: Marginal Product of Capital and Real Interest Rate

Figure 6: The 90% *S*-set and GMM Confidence Ellipse

(a) Result based on the marginal product of capital

(b) Result based on the real interest rate