

Department of Social Systems and Management

Discussion Paper Series

No. 1226

**Enhancement of m-Commerce via Mobile Access to the
Internet**

by

Ushio Sumita and Jun Yoshii

February 2009

UNIVERSITY OF TSUKUBA
Tsukuba, Ibaraki 305-8573
JAPAN

Enhancement of m-Commerce via Mobile Access to the Internet

Ushio Sumita , Jun Yoshii

*Graduate School of Systems and Information Engineering, University of Tsukuba,
1-1-1, Tenoudai, Tsukuba, Ibaraki, 305-8573, Japan
(E-mail address: sumita@sk.tsukuba.ac.jp;yoshii40@sk.tsukuba.ac.jp)*

Abstract

The potential of the Internet has been expanded substantially by a new generation of mobile devices, opening the door for rapid growth of m-commerce. While the traditional PC access to the Internet continues to be vital for exploiting the advantages of the Internet, the mobile access appears to attract more people because of flexible accesses to the Internet in a ubiquitous manner. Accordingly, e-commerce is now in the process of being converted into m-commerce. The purpose of this paper is to develop and analyze a mathematical model for comparing e-commerce via the traditional PC access only with m-commerce which accommodates both the traditional PC access and the mobile access. The distribution of the number of products purchased by time t and the distribution of the time required for selling K products are derived explicitly, enabling one to assess the impact of mobile devices on e-businesses. Numerical examples are given for illustrating behavioral differences between m-commerce consumers and traditional e-commerce consumers.

Key words: M-commerce, E-commerce, Consumer behavior, Semi-Markov process, Sales completion time

1. Introduction

The potential of the Internet has been expanded substantially by a new generation of mobile devices, opening the door for rapid growth of m-commerce. While the traditional PC access to the Internet continues to be vital for exploiting the advantages of the Internet, the mobile access appears to attract more people because of flexible accesses to the Internet in a ubiquitous manner. Accordingly, e-commerce is now in the process of being converted into m-commerce.

Because of the fact that the mobile technology is still young, the study of the impacts of mobile devices on e-businesses is also rather new in the literature. Roto[1] and Kim[2] provide the current state of mobile devices and m-businesses. Chae and Kim[3] discuss the business implications of m-commerce, and Barwise[4] and Hammond[5] predict the evolutionary trend of m-commerce in the foreseeable future. Wu and Hisa[6] propose the hypercube innovation model for analyzing the characteristics of m-commerce with focus on three axes: changes in business models, changes in core components and stake holders. Siau, Sheng and Nah[7], and Park and Fader[8] investigate the benefits of m-commerce to consumers and how e-commerce has changed the consumer behavior. Büyüközkan[9] develops an analytical approach for determining the mobile commerce user requirements. All of these papers are either empirical, qualitative or static in their analytical nature and, to the best knowledge of the authors, no study exists in the literature for capturing behavioral differences between e-commerce and m-commerce based on a mathematical stochastic model.

The purpose of this paper is to develop and analyze a mathematical model for comparing e-commerce via the traditional PC access only with m-commerce which accommodates both the traditional PC access and the mobile access. More specifically, a consumer behavior for m-commerce is formulated as a semi-Markov process having three transient states and two absorbing states. The three transient states describe the first period of a day in which the mobile access is available from time to time for the private use of the Internet (e.g. working hours), the second period of a day in which the PC access is available from time to time (e.g. evening hours at home), and the third period of a day in which the consumer is inactive in using the Internet (e.g. sleeping hours). Given a specific product under consideration, the two absorbing states represent the decision of purchasing the product and that of not purchasing the product. For the case of e-commerce, the access

to the Internet by a user is not allowed in both the first period and the third period. In modeling m-commerce, the access to the Internet is allowed in the first two periods but not in the third period. By considering a group of such consumers, the two stochastic performance measures of interest can be evaluated: the distribution of the number of products sold by time t and the distribution of the time required for selling K products. This analysis, in turn, enables one to assess the impact of mobile devices on e-businesses by comparing such stochastic performance measures for m-commerce against those for traditional e-commerce.

The structure of this paper is as follows. In Section 2, a mathematical model is developed for capturing the consumer behavior in m-commerce based on a semi-Markov process approach. Section 3 is devoted to dynamic analysis of the semi-Markov model. The two stochastic performance measures are introduced in Section 4 and the associated distributions are derived explicitly, which can be computed based on the results in Section 3. Numerical examples are given in Section 5, for illustrating behavioral differences between m-commerce consumers and traditional e-commerce consumers.

Throughout the paper, vectors and matrices are indicated by underbar and doubleunderbar respectively, e.g. $\underline{\xi}$, $\underline{\underline{P}}(t)$, etc. The vector with all components equal to 0 is denoted by $\underline{0}$. The identity matrix is denoted by \underline{I} .

2. Development of Mathematical Model for m-Commerce Consumer Behavior: Semi-Markov Process Approach

We consider an m-commerce consumer who intends to decide whether or not he/she should buy a product by exploring the Internet for information. In order to capture the behavioral pattern of the m-commerce consumer, each day is decomposed into three periods. During the first period of a day, only the mobile access is available from time to time for the private use of the Internet (e.g. working hours). Throughout the second period of a day, the PC access is available from time to time, overriding the mobile access to the Internet (e.g. evening hours at home). The third period of a day represents to take a rest in which the m-commerce consumer is inactive in using the Internet (e.g. sleeping hours). Each time the Internet is accessed for information, he/she makes one of the three decisions: to purchase the product, not to purchase the product, or to remain undecided. Of interest is then to find out what the status of the m-commerce consumer would be at time t .

For capturing the m-commerce consumer behavior described above more formally, we consider a semi-Markov process $\{J(t) : t \geq 0\}$ defined on $\mathcal{N} = \{0, 1, 2, 3, 4\}$. Here, the i -th period of a day is represented by state i , $i = 1, 2, 3$. The two states 0 and 4 are absorbing, where the former corresponds to the decision of purchasing the product while the latter represents the decision of not purchasing the product. Given that neither the decision of purchasing nor that of not purchasing is made, we assume, for the time being, that the dwell time of the semi-Markov process in state i is absolutely continuous with probability density function (p.d.f.) $a_i(x)$, $i = 1, 2, 3$. The corresponding distribution function, the survival function and the hazard rate function are denoted by

$$A_i(x) = \int_0^x a_i(x)dx; \quad \bar{A}_i(x) = 1 - A_i(x); \quad \eta_i(x) = \frac{a_i(x)}{\bar{A}_i(x)}. \quad (2.1)$$

In state i for $i = 1, 2$, the Internet accesses occur according to a Poisson process with intensity λ_i . For each access, the decision of purchasing (not purchasing) the product is made with probability α_i (β_i) where $0 < \alpha_i + \beta_i < 1$. With probability $1 - (\alpha_i + \beta_i) > 0$, the m-commerce consumer remains undecided. The transition structure of the semi-Markov process is depicted in Figure 1.1, where $\xi_i = \alpha_i \lambda_i$ and $\theta_i = \beta_i \lambda_i$ represent the hazard rates from state i to state 0 and 4 respectively for $i = 1, 2$. In order to deal with

the case in which the three periods are constant, we subsequently choose, for each $i \in \{1, 2, 3\}$, a sequence of distribution functions $(A_i(k, x))_{k=1}^{\infty}$ such that $A_i(k, x) \rightarrow U(x)$ as $k \rightarrow \infty$, where $U(x)$ is a step function defined by $U(x) = 1$ for $x \geq 0$ and $U(x) = 0$ for $x < 0$.

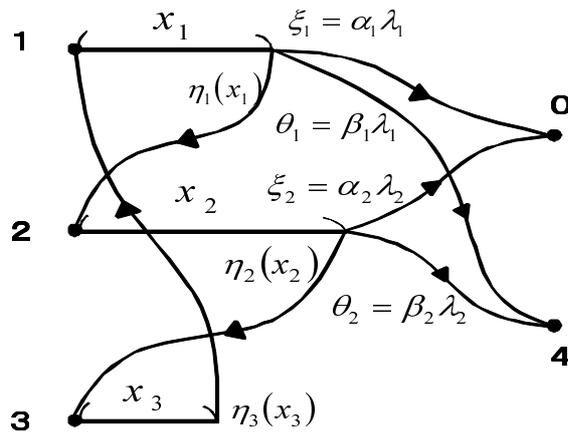


Figure 1.1 : Transition Structure of the Semi-Markov Process

3. Dynamic Analysis of the Semi-Markov Process

In this section, we derive explicitly the transition probability matrix $\underline{\underline{P}}(t)$ of the semi-Markov process $J(t)$, where $\underline{\underline{P}}(t)$ is defined by

$$\underline{\underline{P}}(t) = [P_{ij}(t)]; \quad P_{ij}(t) \stackrel{\text{def}}{=} \text{P}[J(t) = j | J(0) = i], \quad i, j \in \mathcal{N}. \quad (3.1)$$

For this purpose, the age process $X(t)$ associated with the semi-Markov process $J(t)$ is introduced as the elapsed time since the last transition of $J(t)$ into the current state at time t . Clearly the bivariate process $[J(t), X(t)]$ becomes Markov and the first step of our analysis is to evaluate the joint distribution function defined by

$$F_{ij}(x, t) = \text{P}[X(t) \leq x, J(t) = j | J(0) = i], \quad (3.2)$$

and the corresponding joint p.d.f.

$$\frac{d}{dx} F_{ij}(x, t) = f_{ij}(x, t), \quad (3.3)$$

where the delta function $\delta(t)$ is employed for describing the boundary conditions with respect to x . More specifically, one sees that

$$f_{i1}(0+, t) = \delta_{\{i=1\}}\delta(t) + \int_0^\infty f_{i3}(x, t)\eta_3(x)dx, \quad (3.4)$$

$$f_{i2}(0+, t) = \delta_{\{i=2\}}\delta(t) + \int_0^\infty f_{i1}(x, t)\eta_1(x)dx, \quad (3.5)$$

$$f_{i3}(0+, t) = \delta_{\{i=3\}}\delta(t) + \int_0^\infty f_{i2}(x, t)\eta_2(x)dx. \quad (3.6)$$

Here, $\delta_{\{STATEMENT\}} = 1$ if STATEMENT is true, $\delta_{\{STATEMENT\}} = 0$ otherwise, and the delta function $\delta(t)$ is the unit operator associated with convolution, i.e. $g(t) = \int_0^\infty g(x)\delta(t-x)dx$ for any integrable function $g(t)$ on $[0, \infty)$.

In order to evaluate the joint p.d.f. given in (3.3), we introduce the following Laplace transforms.

$$\alpha_i(s) = \int_0^\infty e^{-sx} a_i(x) dx \quad \text{for } i = 1, 2, 3 \quad (3.7)$$

$$\beta_i(s) = \int_0^\infty e^{-sx} \bar{A}_i(x) dx = \frac{1 - \alpha_i(s)}{s} \quad \text{for } i = 1, 2, 3 \quad (3.8)$$

$$\underline{\hat{\zeta}}(0+, s) = [\hat{\zeta}_{ij}(0+, s)]; \quad \hat{\zeta}_{ij}(0+, s) \stackrel{\text{def}}{=} \int_0^\infty e^{-st} f_{ij}(0+, t) dt \quad \text{for } i, j \in \mathcal{N} \quad (3.9)$$

$$\underline{\hat{\varphi}}(x, s) = [\hat{\varphi}_{ij}(x, s)]; \quad \hat{\varphi}_{ij}(x, s) \stackrel{\text{def}}{=} \int_0^\infty e^{-st} f_{ij}(x, t) dt \quad \text{for } i, j \in \mathcal{N} \quad (3.10)$$

$$\underline{\hat{\varphi}}(v, s) = [\hat{\varphi}_{ij}(v, s)]; \quad \hat{\varphi}_{ij}(v, s) \stackrel{\text{def}}{=} \int_0^\infty e^{-vx} \hat{\varphi}_{ij}(x, s) dx \quad \text{for } i, j \in \mathcal{N} \quad (3.11)$$

For notational convenience, we also define $C_i = \xi_i + \theta_i$ for $i = 1, 2$ and

$$d(\underline{\xi}, \underline{\theta}, s) = 1 - \alpha_1(s + C_1)\alpha_2(s + C_2)\alpha_3(s) \quad (3.12)$$

$$\underline{\psi}_0(\underline{\xi}, \underline{\theta}, s) = \begin{bmatrix} \xi_1\beta_1(s + C_1) + \xi_2\beta_2(s + C_2)\alpha_1(s + C_1) \\ \xi_1\beta_1(s + C_1)\alpha_2(s + C_2)\alpha_3(s) + \xi_2\beta_2(s + C_2) \\ \xi_1\beta_1(s + C_1)\alpha_3(s) + \xi_2\beta_2(s + C_2)\alpha_1(s + C_1)\alpha_3(s) \end{bmatrix} \quad (3.13)$$

$$\underline{\psi}_4(\underline{\xi}, \underline{\theta}, s) = \begin{bmatrix} \theta_1\beta_1(s + C_1) + \theta_2\beta_2(s + C_2)\alpha_1(s + C_1) \\ \theta_1\beta_1(s + C_1)\alpha_2(s + C_2)\alpha_3(s) + \theta_2\beta_2(s + C_2) \\ \theta_1\beta_1(s + C_1)\alpha_3(s) + \theta_2\beta_2(s + C_2)\alpha_1(s + C_1)\alpha_3(s) \end{bmatrix} \quad (3.14)$$

$$\underline{B}(\underline{\xi}, \underline{\theta}, s) = \begin{bmatrix} 1 & \alpha_1(s + C_1) & \alpha_1(s + C_1)\alpha_2(s + C_2) \\ \alpha_2(s + C_2)\alpha_3(s) & 1 & \alpha_2(s + C_2) \\ \alpha_3(s) & \alpha_1(s + C_1)\alpha_3(s) & 1 \end{bmatrix} \quad (3.15)$$

Then the following theorem holds.

Theorem 3.1. *Let $\underline{\hat{\zeta}}(0+, s)$ and $\underline{\hat{\varphi}}(v, s)$ be as in (3.9) and (3.11) respectively. One then has:*

$$a) \quad \underline{\hat{\zeta}}(0+, s) = \frac{1}{d(\underline{\xi}, \underline{\theta}, s)} \begin{bmatrix} 0 & \underline{0}^T & 0 \\ \underline{\psi}_0(\underline{\xi}, \underline{\theta}, s) & \underline{B}(\underline{\xi}, \underline{\theta}, s) & \underline{\psi}_4(\underline{\xi}, \underline{\theta}, s) \\ 0 & \underline{0}^T & 0 \end{bmatrix},$$

where $d(\underline{\xi}, \underline{\theta}, s)$, $\underline{\psi}_0(\underline{\xi}, \underline{\theta}, s)$, $\underline{\psi}_4(\underline{\xi}, \underline{\theta}, s)$ and $\underline{B}(\underline{\xi}, \underline{\theta}, s)$ are as given in (3.12), (3.13), (3.14) and (3.15) respectively.

$$b) \quad \underline{\hat{\varphi}}(v, s) = \underline{\hat{\zeta}}(0+, s) \times \text{diag} \left\{ \frac{1}{s+v}, \beta_1(s+v+C_1), \beta_2(s+v+C_2), \beta_3(s+v), \frac{1}{s+v} \right\},$$

where $\text{diag}\{a_1, \dots, a_n\}$ denotes an $n \times n$ diagonal matrix with diagonal elements a_1, \dots, a_n .

Proof. In addition to the boundary conditions in (3.4), (3.5) and (3.6) for states 1, 2 and 3 respectively, one sees that, for state 0 and 4,

$$f_{i0}(0+, t) = \xi_1 \int_0^\infty f_{i1}(x, t) dx + \xi_2 \int_0^\infty f_{i2}(x, t) dx \quad (3.16)$$

and

$$f_{i4}(0+, t) = \theta_1 \int_0^\infty f_{i1}(x, t) dx + \theta_2 \int_0^\infty f_{i2}(x, t) dx. \quad (3.17)$$

By taking the Laplace transform of (3.4) through (3.6) and the above two equations with respect to t , one finds that

$$\hat{\zeta}_{i0}(0+, s) = \sum_{j=1}^2 \xi_j \hat{\zeta}_{ij}(0+, s) \beta_j(s + C_j), \quad (3.18)$$

$$\hat{\zeta}_{i1}(0+, s) = \delta_{\{i=1\}} + \hat{\zeta}_{i3}(0+, s) \alpha_3(s), \quad (3.19)$$

$$\hat{\zeta}_{i2}(0+, s) = \delta_{\{i=2\}} + \hat{\zeta}_{i1}(0+, s) \alpha_1(s + C_1), \quad (3.20)$$

$$\hat{\zeta}_{i3}(0+, s) = \delta_{\{i=3\}} + \hat{\zeta}_{i2}(0+, s) \alpha_2(s + C_2), \quad (3.21)$$

$$\hat{\zeta}_{i4}(0+, s) = \sum_{j=1}^2 \theta_j \hat{\zeta}_{ij}(0+, s) \beta_j(s + C_j). \quad (3.22)$$

By describing (3.18) through (3.22) in matrix form, it follows that

$$\underline{\hat{\zeta}}^T(0+, s) = \begin{bmatrix} \underline{0}^T \\ \underline{u}_1^T \\ \underline{u}_2^T \\ \underline{u}_3^T \\ \underline{0}^T \end{bmatrix} + \underline{\hat{\zeta}}^T(0+, s) \underline{\gamma}(\underline{\xi}, \underline{\theta}, s),$$

where \underline{u}_i is the i -th unit vector in R^5 and

$$\underline{\gamma}(\underline{\xi}, \underline{\theta}, s) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \xi_1 \beta_1(s + C_1) & 0 & \alpha_1(s + C_1) & 0 & \theta_1 \beta_1(s + C_1) \\ \xi_2 \beta_2(s + C_2) & 0 & 0 & \alpha_2(s + C_2) & \theta_2 \beta_2(s + C_2) \\ 0 & \alpha_3(s) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (3.23)$$

It then leads to

$$\underline{\hat{\zeta}}^T(0+, s) = \begin{bmatrix} \underline{0}^T \\ \underline{u}_1^T \\ \underline{u}_2^T \\ \underline{u}_3^T \\ \underline{0}^T \end{bmatrix} \left[\underline{I} - \underline{\gamma}(\underline{\xi}, \underline{\theta}, s) \right]^{-1}. \quad (3.24)$$

It can be shown from (3.23), after a little algebra, that

$$\left[\underline{I} - \underline{\gamma}(\underline{\xi}, \underline{\theta}, s) \right]^{-1} = \frac{1}{d(\underline{\xi}, \underline{\theta}, s)} \begin{bmatrix} d(\underline{\xi}, \underline{\theta}, s) & \underline{0}^T & 0 \\ \underline{\psi}_0(\underline{\xi}, \underline{\theta}, s) & \underline{B}(\underline{\xi}, \underline{\theta}, s) & \underline{\psi}_4(\underline{\xi}, \underline{\theta}, s) \\ 0 & \underline{0}^T & d(\underline{\xi}, \underline{\theta}, s) \end{bmatrix},$$

and part a) follows from (3.24).

For part b), we note that

$$f_{i0}(x, t) = f_{i0}(0+, t - x), \quad (3.25)$$

$$f_{ij}(x, t) = f_{ij}(0+, t - x) \bar{A}_j(x) e^{-C_j x} \quad j = 1, 2, \quad (3.26)$$

$$f_{i3}(x, t) = f_{i3}(0+, t - x) \bar{A}_3(x), \quad (3.27)$$

$$f_{i4}(x, t) = f_{i4}(0+, t - x). \quad (3.28)$$

Since states 0 and 4 are absorbing, for the process to be in one of the two states at time t with age x , it should have entered the state at time $t - x$, explaining (3.25) and (3.28). For the process to be in state j at time t for $j = 1, 2$, as shown in (3.26), it should have entered the state at time $t - x$, having no transition to any other state until time t . The case for state 3 in (3.27) is similar except that transitions from state 3 to state 0 or state 4 are not possible.

By taking the Laplace transform of (3.25) through (3.28) with respect to t , it can be seen that

$$\hat{\varphi}_{i0}(x, s) = \hat{\zeta}_{i0}(0+, s) e^{-sx}, \quad (3.29)$$

$$\hat{\varphi}_{ij}(x, s) = \hat{\zeta}_{ij}(0+, s) e^{-(s+C_j)x} \bar{A}_j(x) \quad j = 1, 2, \quad (3.30)$$

$$\hat{\varphi}_{i3}(x, s) = \hat{\zeta}_{i3}(0+, s) e^{-sx} \bar{A}_3(x), \quad (3.31)$$

$$\hat{\varphi}_{i4}(x, s) = \hat{\zeta}_{i4}(0+, s) e^{-sx}. \quad (3.32)$$

Again by taking the Laplace transform of (3.29) through (3.32) with respect to x and putting the results into matrix form, the theorem follows.

□

Let the Laplace transform of $\underline{\underline{P}}(t)$ with respect to t be denoted by $\underline{\underline{\pi}}(s)$, i.e.

$$\underline{\underline{\pi}}(s) = \int_0^\infty e^{-st} \underline{\underline{P}}(t) dt . \quad (3.33)$$

From the definition of $\underline{\underline{P}}(t)$ in (3.1), one easily sees that $\underline{\underline{\pi}}(s) = \underline{\underline{\hat{\varphi}}}(0, s)$. The next theorem is then immediate from Theorem 3.1.

Theorem 3.2.

$$\underline{\underline{\pi}}(s) = \underline{\underline{\hat{\zeta}}}(0+, s) \text{diag} \left\{ \frac{1}{s}, \beta_1(s + C_1), \beta_2(s + C_2), \beta_3(s), \frac{1}{s} \right\}$$

So far, we have assumed that the dwell time of the semi-Markov process in state i is absolutely continuous with p.d.f. $a_i(x)$, $i = 1, 2, 3$, given that neither the decision of purchasing nor that of not purchasing is made. In reality, however, the three periods of a day should be treated as constants τ_1, τ_2 and τ_3 . This case can be dealt with by considering a sequence of Laplace transforms of p.d.f.'s $(\alpha_i(k, s))_{k=1}^\infty$ where $\alpha_i(k, s) \rightarrow e^{-s\tau_i}$ as $k \rightarrow \infty$, $i = 1, 2, 3$. We emphasize the limit by using the symbol \sim , i.e. $\tilde{\alpha}_i(s) = e^{-s\tau_i}$. At the limit, the corresponding Laplace transform $\tilde{\underline{\underline{\pi}}}(s)$ of the transition probability matrix $\tilde{\underline{\underline{P}}}(t)$ can be obtained by substituting $\tilde{\alpha}_i(s) = e^{-s\tau_i}$ into Theorems 3.1 and 3.2. Assuming that a day starts with period 1, of particular interest to our analysis are $\tilde{\pi}_{10}(s)$ and $\tilde{\pi}_{14}(s)$ as summarized in the next theorem.

Theorem 3.3. *Suppose that the three periods of a day are represented by constants τ_1, τ_2 and τ_3 . Let $\tau = \tau_1 + \tau_2 + \tau_3$. One then has:*

$$a) \quad \tilde{\pi}_{10}(s) = \frac{1}{s} \cdot \frac{1}{1 - e^{-(s\tau + C_1\tau_1 + C_2\tau_2)}} \\ \times \left\{ \xi_1 \frac{1 - e^{-(s+C_1)\tau_1}}{s + C_1} + \xi_2 e^{-(s+C_1)\tau_1} \frac{1 - e^{-(s+C_2)\tau_2}}{s + C_2} \right\} ,$$

$$b) \quad \tilde{\pi}_{14}(s) = \frac{1}{s} \cdot \frac{1}{1 - e^{-(s\tau + C_1\tau_1 + C_2\tau_2)}} \\ \times \left\{ \theta_1 \frac{1 - e^{-(s+C_1)\tau_1}}{s + C_1} + \theta_2 e^{-(s+C_1)\tau_1} \frac{1 - e^{-(s+C_2)\tau_2}}{s + C_2} \right\} .$$

We are now in a position to prove the following main theorem by inverting the results in Theorem 3.3 a) and b) into the real domain. For notational convenience, the following intervals are introduced for $k = 0, 1, 2, \dots$.

$$\text{Int}[k, 1] = \{t : k\tau \leq t < k\tau + \tau_1\} \quad (3.34)$$

$$\text{Int}[k, 2] = \{t : k\tau + \tau_1 \leq t < k\tau + \tau_1 + \tau_2\} \quad (3.35)$$

$$\text{Int}[k, 3] = \{t : k\tau + \tau_1 + \tau_2 \leq t < (k+1)\tau\} \quad (3.36)$$

We also write $[x]$ to mean the integer part of a real number x .

Theorem 3.4. *Let $\text{Int}[k, 1]$, $\text{Int}[k, 2]$ and $\text{Int}[k, 3]$ be as in (3.34), (3.35) and (3.36) respectively. Let τ and C_i be as in Theorem 3.3 and define $m(t) = [\frac{t}{\tau}]$. Then the probability $\tilde{P}_{10}(t)$ can be obtained as follows.*

i) *If $t \in \text{Int}[m(t), 1]$, then*

$$\begin{aligned} \tilde{P}_{10}(t) = & \frac{\xi_1}{C_1} (1 - e^{-C_1\tau_1}) \frac{1 - e^{-(C_1\tau_1 + C_2\tau_2)m(t)}}{1 - e^{-(C_1\tau_1 + C_2\tau_2)}} \\ & + \frac{\xi_1}{C_1} e^{-(C_1\tau_1 + C_2\tau_2 - C_1\tau)m(t)} (e^{-C_1m(t)\tau} - e^{-C_1t}) \\ & + \frac{\xi_2}{C_2} e^{-C_1\tau_1} (1 - e^{-C_2\tau_2}) \frac{1 - e^{-(C_1\tau_1 + C_2\tau_2)m(t)}}{1 - e^{-(C_1\tau_1 + C_2\tau_2)}}. \end{aligned}$$

ii) *If $t \in \text{Int}[m(t), 2]$, then*

$$\begin{aligned} \tilde{P}_{10}(t) = & \frac{\xi_1}{C_1} (1 - e^{-C_1\tau_1}) \frac{1 - e^{-(C_1\tau_1 + C_2\tau_2)(m(t)+1)}}{1 - e^{-(C_1\tau_1 + C_2\tau_2)}} \\ & + \frac{\xi_2}{C_2} e^{-C_1\tau_1} (1 - e^{-C_2\tau_2}) \frac{1 - e^{-(C_1\tau_1 + C_2\tau_2)m(t)}}{1 - e^{-(C_1\tau_1 + C_2\tau_2)}} \\ & + \frac{\xi_2}{C_2} e^{-(C_1 - C_2)\tau_1} e^{-(C_1\tau_1 + C_2\tau_2 - C_2\tau)m(t)} (e^{-C_2(m(t)\tau + \tau_1)} - e^{-C_2t}). \end{aligned}$$

iii) *If $t \in \text{Int}[m(t), 3]$, then*

$$\begin{aligned} \tilde{P}_{10}(t) = & \frac{\xi_1}{C_1} (1 - e^{-C_1\tau_1}) \frac{1 - e^{-(C_1\tau_1 + C_2\tau_2)(m(t)+1)}}{1 - e^{-(C_1\tau_1 + C_2\tau_2)}} \\ & + \frac{\xi_2}{C_2} e^{-C_1\tau_1} (1 - e^{-C_2\tau_2}) \frac{1 - e^{-(C_1\tau_1 + C_2\tau_2)(m(t)+1)}}{1 - e^{-(C_1\tau_1 + C_2\tau_2)}}. \end{aligned}$$

Proof. From Theorem 3.3, one sees that

$$s\tilde{\pi}_{10}(s) = \frac{1}{1 - e^{-(s\tau + C_1\tau_1 + C_2\tau_2)}} \left\{ \xi_1 \frac{1 - e^{-(s+C_1)\tau_1}}{s + C_1} + \xi_2 e^{-(s+C_1)\tau_1} \frac{1 - e^{-(s+C_2)\tau_2}}{s + C_2} \right\}.$$

The geometric expansion of the first factor on the right hand side of the above equation then leads to

$$s\tilde{\pi}_{10}(s) = \sum_{k=0}^{\infty} e^{-(s\tau + C_1\tau_1 + C_2\tau_2)k} \left\{ \xi_1 \frac{1 - e^{-(s+C_1)\tau_1}}{s + C_1} + \xi_2 e^{-(s+C_1)\tau_1} \frac{1 - e^{-(s+C_2)\tau_2}}{s + C_2} \right\}.$$

By arranging terms involving k or s appropriately, it follows that

$$\begin{aligned} s\tilde{\pi}_{10}(s) &= \frac{\xi_1}{s + C_1} \sum_{k=0}^{\infty} e^{-(C_1\tau_1 + C_2\tau_2)k} e^{-sk\tau} \\ &\quad - \frac{\xi_1 e^{-C_1\tau_1}}{s + C_1} \sum_{k=0}^{\infty} e^{-(C_1\tau_1 + C_2\tau_2)k} e^{-s(k\tau + \tau_1)} \\ &\quad + \frac{\xi_2 e^{-C_1\tau_1}}{s + C_2} \sum_{k=0}^{\infty} e^{-(C_1\tau_1 + C_2\tau_2)k} e^{-s(k\tau + \tau_1)} \\ &\quad - \frac{\xi_2 e^{-(C_1\tau_1 + C_2\tau_2)}}{s + C_2} \sum_{k=0}^{\infty} e^{-(C_1\tau_1 + C_2\tau_2)k} e^{-s(k\tau + \tau_1 + \tau_2)}. \quad (3.37) \end{aligned}$$

We recall that the Laplace transform $\frac{1}{s+\alpha}e^{-s\beta}$ can be inverted into the real domain as $\mathcal{L}^{-1}\left[\frac{1}{s+\alpha}e^{-s\beta}\right] = e^{-\alpha t} * \delta(t - \beta) = \int_0^t e^{-\alpha(t-y)}\delta(y - \beta) dy = \delta_{\{0 \leq \beta \leq t\}} e^{-\alpha(t-\beta)}$. Applying this inversion formula to (3.37) and noticing $\tilde{P}_{10}(0) = 0$, one finds, after a little algebra, that

$$\begin{aligned} \frac{d}{dt}\tilde{P}_{10}(t) &= \xi_1 e^{-C_1 t} \sum_{k=0}^{\infty} \delta_{\{k\tau \leq t\}} e^{-(C_1\tau_1 + C_2\tau_2 - C_1\tau)k} \\ &\quad - \xi_1 e^{-C_1 t} \sum_{k=0}^{\infty} \delta_{\{k\tau + \tau_1 \leq t\}} e^{-(C_1\tau_1 + C_2\tau_2 - C_1\tau)k} \\ &\quad + \xi_2 e^{-(C_1 - C_2)\tau_1} e^{-C_2 t} \sum_{k=0}^{\infty} \delta_{\{k\tau + \tau_1 \leq t\}} e^{-(C_1\tau_1 + C_2\tau_2 - C_2\tau)k} \\ &\quad - \xi_2 e^{-(C_1 - C_2)\tau_1} e^{-C_2 t} \sum_{k=0}^{\infty} \delta_{\{k\tau + \tau_1 + \tau_2 \leq t\}} e^{-(C_1\tau_1 + C_2\tau_2 - C_2\tau)k}. \end{aligned}$$

Since $\delta_{\{k\tau \leq t\}} - \delta_{\{k\tau + \tau_1 \leq t\}} = \delta_{\{t \in \text{Int}[k, 1]\}}$ and $\delta_{\{k\tau + \tau_1 \leq t\}} - \delta_{\{k\tau + \tau_1 + \tau_2 \leq t\}} = \delta_{\{t \in \text{Int}[k, 2]\}}$, this then leads to

$$\begin{aligned} \frac{d}{dt} \tilde{P}_{10}(t) &= \xi_1 e^{-C_1 t} \sum_{k=0}^{\infty} \delta_{\{t \in \text{Int}[k, 1]\}} e^{-(C_1 \tau_1 + C_2 \tau_2 - C_1 \tau)k} \\ &\quad + \xi_2 e^{-(C_1 - C_2) \tau_1} e^{-C_2 t} \sum_{k=0}^{\infty} \delta_{\{t \in \text{Int}[k, 2]\}} e^{-(C_1 \tau_1 + C_2 \tau_2 - C_2 \tau)k}. \end{aligned} \quad (3.38)$$

By integrating both sides of the above equation from 0 to t , it can be seen that

$$\begin{aligned} \tilde{P}_{10}(t) &= \xi_1 \sum_{k=0}^{\infty} e^{-(C_1 \tau_1 + C_2 \tau_2 - C_1 \tau)k} \int_0^t \delta_{\{t' \in \text{Int}[k, 1]\}} e^{-C_1 t'} dt' \\ &\quad + \xi_2 e^{-(C_1 - C_2) \tau_1} \sum_{k=0}^{\infty} e^{-(C_1 \tau_1 + C_2 \tau_2 - C_2 \tau)k} \int_0^t \delta_{\{t' \in \text{Int}[k, 2]\}} e^{-C_2 t'} dt'. \end{aligned} \quad (3.39)$$

We now consider the following three cases separately.

i) $t \in \text{Int}[m(t), 1]$

In this case, $m(t)$ days have passed and it is currently in the first period of the $m(t)$ -th day. Accordingly, one has

$$\begin{aligned} \tilde{P}_{10}(t) &= \xi_1 \sum_{k=0}^{m(t)-1} \left\{ e^{-(C_1 \tau_1 + C_2 \tau_2 - C_1 \tau)k} \int_{k\tau}^{k\tau + \tau_1} e^{-C_1 t'} dt' \right\} \\ &\quad + \xi_1 e^{-(C_1 \tau_1 + C_2 \tau_2 - C_1 \tau)m(t)} \int_{m(t)\tau}^t e^{-C_1 t'} dt' \\ &\quad + \xi_2 e^{-(C_1 - C_2) \tau_1} \sum_{k=0}^{m(t)-1} e^{-(C_1 \tau_1 + C_2 \tau_2 - C_2 \tau)k} \int_{k\tau + \tau_1}^{k\tau + \tau_1 + \tau_2} e^{-C_2 t'} dt', \end{aligned}$$

where $\sum_a^b = 0$ if $a > b$ according to the mathematical convention. Calculating the exponential integrals and summing the resulting geometric sequences, part i) follows.

ii) $t \in \text{Int}[m(t), 2]$

This case implies that $m(t)$ days have passed and it is currently in the second period of the $m(t)$ -th day. Hence one sees that

$$\begin{aligned}\tilde{P}_{10}(t) &= \xi_1 \sum_{k=0}^{m(t)} e^{-(C_1\tau_1+C_2\tau_2-C_1\tau)k} \int_{k\tau}^{k\tau+\tau_1} e^{-C_1t'} dt' \\ &\quad + \xi_2 e^{-(C_1-C_2)\tau_1} \sum_{k=0}^{m(t)-1} e^{-(C_1\tau_1+C_2\tau_2-C_2\tau)k} \int_{k\tau+\tau_1}^{k\tau+\tau_1+\tau_2} e^{-C_2t'} dt' \\ &\quad + \xi_2 e^{-(C_1-C_2)\tau_1} e^{-(C_1\tau_1+C_2\tau_2-C_2\tau)m(t)} \int_{m(t)\tau+\tau_1}^t e^{-C_2t'} dt' .\end{aligned}$$

As for part i), a little algebra then would prove part ii).

iii) $t \in \text{Int}[m(t), 3]$

In the third case, $m(t)$ days have passed and it is currently in the third period of the $m(t)$ -th day. It can be seen that

$$\begin{aligned}\tilde{P}_{10}(t) &= \xi_1 \sum_{k=0}^{m(t)} e^{-(C_1\tau_1+C_2\tau_2-C_1\tau)k} \int_{k\tau}^{k\tau+\tau_1} e^{-C_1t'} dt' \\ &\quad + \xi_2 e^{-(C_1-C_2)\tau_1} \sum_{k=0}^{m(t)} e^{-(C_1\tau_1+C_2\tau_2-C_2\tau)k} \int_{k\tau+\tau_1}^{k\tau+\tau_1+\tau_2} e^{-C_2t'} dt' .\end{aligned}$$

By conducting integrations and summations as for part i) and part ii), the theorem follows. □

The counterpart of Theorem 3.4 for $\tilde{P}_{14}(t)$ can be obtained via similar arguments, as shown in Theorem 3.5 below. Proof is omitted.

Theorem 3.5. *Let $\text{Int}[k, 1]$, $\text{Int}[k, 2]$ and $\text{Int}[k, 3]$ be as in (3.34), (3.35) and (3.36) respectively. Let τ and C_i be as in Theorem 3.3 and define $m(t) = \lceil \frac{t}{\tau} \rceil$. Then the probability $\tilde{P}_{14}(t)$ can be obtained as follows.*

i) If $t \in \text{Int}[m(t), 1]$, then

$$\begin{aligned}\tilde{P}_{14}(t) &= \frac{\theta_1}{C_1} (1 - e^{-C_1\tau_1}) \frac{1 - e^{-(C_1\tau_1+C_2\tau_2)m(t)}}{1 - e^{-(C_1\tau_1+C_2\tau_2)}} \\ &\quad + \frac{\theta_1}{C_1} e^{-(C_1\tau_1+C_2\tau_2-C_1\tau)m(t)} (e^{-C_1m(t)\tau} - e^{-C_1t}) \\ &\quad + \frac{\theta_2}{C_2} e^{-C_1\tau_1} (1 - e^{-C_2\tau_2}) \frac{1 - e^{-(C_1\tau_1+C_2\tau_2)m(t)}}{1 - e^{-(C_1\tau_1+C_2\tau_2)}}.\end{aligned}$$

ii) If $t \in \text{Int}[m(t), 2]$, then

$$\begin{aligned}\tilde{P}_{14}(t) &= \frac{\theta_1}{C_1} (1 - e^{-C_1\tau_1}) \frac{1 - e^{-(C_1\tau_1+C_2\tau_2)(m(t)+1)}}{1 - e^{-(C_1\tau_1+C_2\tau_2)}} \\ &\quad + \frac{\theta_2}{C_2} e^{-C_1\tau_1} (1 - e^{-C_2\tau_2}) \frac{1 - e^{-(C_1\tau_1+C_2\tau_2)m(t)}}{1 - e^{-(C_1\tau_1+C_2\tau_2)}} \\ &\quad + \frac{\theta_2}{C_2} e^{-(C_1-C_2)\tau_1} e^{-(C_1\tau_1+C_2\tau_2-C_2\tau)m(t)} (e^{-C_2(m(t)\tau+\tau_1)} - e^{-C_2t}).\end{aligned}$$

iii) If $t \in \text{Int}[m(t), 3]$, then

$$\begin{aligned}\tilde{P}_{14}(t) &= \frac{\theta_1}{C_1} (1 - e^{-C_1\tau_1}) \frac{1 - e^{-(C_1\tau_1+C_2\tau_2)(m(t)+1)}}{1 - e^{-(C_1\tau_1+C_2\tau_2)}} \\ &\quad + \frac{\theta_2}{C_2} e^{-C_1\tau_1} (1 - e^{-C_2\tau_2}) \frac{1 - e^{-(C_1\tau_1+C_2\tau_2)(m(t)+1)}}{1 - e^{-(C_1\tau_1+C_2\tau_2)}}.\end{aligned}$$

From Theorems 3.4 and 3.5, the two absorbing probabilities e_0 and e_4 can be obtained by letting $t \rightarrow \infty$.

Theorem 3.6. *Let e_0 and e_4 be the absorbing probabilities in state 0 and state 4 respectively. One then has*

$$\begin{aligned}e_0 = \tilde{P}_{10}(\infty) &= \left\{ \frac{\xi_1}{C_1} (1 - e^{-C_1\tau_1}) + \frac{\xi_2}{C_2} e^{-C_1\tau_1} (1 - e^{-C_2\tau_2}) \right\} \frac{1}{1 - e^{-(C_1\tau_1+C_2\tau_2)}}, \\ e_4 = \tilde{P}_{14}(\infty) &= \left\{ \frac{\theta_1}{C_1} (1 - e^{-C_1\tau_1}) + \frac{\theta_2}{C_2} e^{-C_1\tau_1} (1 - e^{-C_2\tau_2}) \right\} \frac{1}{1 - e^{-(C_1\tau_1+C_2\tau_2)}}.\end{aligned}$$

For those users who have only the PC access to the Internet, the counterparts of Theorems 3.4 through 3.6 can be obtained, in principle, by setting $\xi_1 = \theta_1 = 0$. Since some terms take an indefinite form in that both

the denominator and the numerator vanish as ξ_1 and θ_1 go to 0, however, L'Hopital's rule should be employed wherever appropriate. The resulting transition probability matrix at the limit is denoted by $\underline{\underline{\tilde{Q}}}(t) = [\underline{\underline{\tilde{Q}}}_{ij}(t)]$. The next two theorems then hold true.

Theorem 3.7. *Let $Int[k, 1], Int[k, 2]$ and $Int[k, 3]$ be as in (3.34), (3.35) and (3.36) respectively. Let τ and C_2 be as in Theorem 3.3 and define $m(t) = \lfloor \frac{t}{\tau} \rfloor$. Then the probability $\tilde{Q}_{10}(t)$ can be obtained as follows.*

i) If $t \in Int[m(t), 1]$, then

$$\tilde{Q}_{10}(t) = \frac{\xi_2}{C_2} (1 - e^{-C_2 \tau_2 m(t)}) .$$

ii) If $t \in Int[m(t), 2]$, then

$$\tilde{Q}_{10}(t) = \frac{\xi_2}{C_2} (1 - e^{-C_2 [t - (\tau - \tau_2)m(t) - \tau_1]}) .$$

iii) If $t \in Int[m(t), 3]$, then

$$\tilde{Q}_{10}(t) = \frac{\xi_2}{C_2} (1 - e^{-C_2 \tau_2 [m(t)+1]}) .$$

Theorem 3.8. *Let $Int[k, 1], Int[k, 2]$ and $Int[k, 3]$ be as in (3.34), (3.35) and (3.36) respectively. Let τ and C_2 be as in Theorem 3.3 and define $m(t) = \lfloor \frac{t}{\tau} \rfloor$. Then the probability $\tilde{Q}_{14}(t)$ can be obtained as follows.*

i) If $t \in Int[m(t), 1]$, then

$$\tilde{Q}_{14}(t) = \frac{\theta_2}{C_2} (1 - e^{-C_2 \tau_2 m(t)}) .$$

ii) If $t \in Int[m(t), 2]$, then

$$\tilde{Q}_{14}(t) = \frac{\theta_2}{C_2} (1 - e^{-C_2 [t - (\tau - \tau_2)m(t) - \tau_1]}) .$$

iii) If $t \in Int[m(t), 3]$, then

$$\tilde{Q}_{14}(t) = \frac{\theta_2}{C_2} (1 - e^{-C_2 \tau_2 [m(t)+1]}) .$$

Corresponding to Theorem 3.6, one has the following theorem by letting $t \rightarrow \infty$ in Theorems 3.7 and 3.8.

Theorem 3.9. *Let q_0 and q_4 be the absorbing probabilities in state 0 and state 4 respectively with $\xi_1 = \theta_1 = 0$. One then has*

$$q_0 = \tilde{Q}_{10}(\infty) = \frac{\xi_2}{C_2} ; \quad q_4 = \tilde{Q}_{14}(\infty) = \frac{\theta_2}{C_2} .$$

4. Analysis of Dynamic Sales Volume and Sales Completion Time

Using the results of the semi-Markov model discussed in Section 3, we are now in a position to assess the impact of the mobile access to the Internet on enhancement of e-businesses. In this regard, it should be noted that the semi-Markov model with $\xi_1 = \theta_1 = 0$ describes a consumer who does not have the capability of the mobile access to the Internet and utilizes the Internet only through the PC access. Let N_{BOTH} be the number of consumers having both the PC access and the mobile access to the Internet. Let M_{PC} be the number of consumers who have only the PC access to the Internet. We recall that the stochastic behavior of those counted for N_{BOTH} is characterized by $\underline{\tilde{P}}(t) = [\tilde{P}_{ij}(t)]$, while that of those counted for M_{PC} is described by $\underline{\tilde{Q}}(t) = [\tilde{Q}_{ij}(t)]$. Given N_{BOTH} and M_{PC} , of interest then is the distribution of the sales volume at time t . Also, of equal importance would be the distribution of the sales completion time for K products. In this section, we derive these two distributions explicitly.

In order to capture individual consumer behaviors better from an application point of view, we redefine the state space of the semi-Markov model $\mathcal{N} = \{0, 1, 2, 3, 4\}$ as $\mathcal{S} = \{Buy, UD, \neg Buy\}$, where *Buy* corresponds to state 0, *UD* (*UnDecided*) aggregates the three states $\{1, 2, 3\}$, and $\neg Buy$ means state 4. Accordingly, we define

$$P_{Buy}(t) = \tilde{P}_{10}(t); P_{UD}(t) = \tilde{P}_{11}(t) + \tilde{P}_{12}(t) + \tilde{P}_{13}(t); P_{\neg Buy}(t) = \tilde{P}_{14}(t), \quad (4.1)$$

and

$$Q_{Buy}(t) = \tilde{Q}_{10}(t); Q_{UD}(t) = \tilde{Q}_{11}(t) + \tilde{Q}_{12}(t) + \tilde{Q}_{13}(t); Q_{\neg Buy}(t) = \tilde{Q}_{14}(t), \quad (4.2)$$

which can be readily computed from Theorems 3.4, 3.5, 3.7 and 3.8. We now introduce the following trivariate generating functions.

$$\chi_{IND:P}(U, V, W, t) = P_{Buy}(t)U + P_{UD}(t)V + P_{\neg Buy}(t)W \quad (4.3)$$

$$\chi_{IND:Q}(U, V, W, t) = Q_{Buy}(t)U + Q_{UD}(t)V + Q_{\neg Buy}(t)W \quad (4.4)$$

Given N_{BOTH} , let $N_{Buy}(t)$, $N_{UD}(t)$ and $N_{\neg Buy}(t)$ be the number of consumers who have bought the product by time t , the number of consumers who have not decided in either way by time t and the number of consumers who have decided not to buy the product by time t , respectively, where

$N_{BOTH} = N_{Buy}(t) + N_{UD}(t) + N_{\neg Buy}(t)$ for any $t \geq 0$. Assuming that individual consumers behave independently of each other, the collective consumer behavior can then be described by

$$\begin{aligned}\chi_{N_{BOTH}:P}(U, V, W, t) &= E[U^{N_{Buy}(t)} V^{N_{UD}(t)} W^{N_{\neg Buy}(t)}] \\ &= \{ \chi_{IND:P}(U, V, W, t) \}^{N_{BOTH}} .\end{aligned}\quad (4.5)$$

Accordingly, the joint probability of $N_{Buy}(t)$, $N_{UD}(t)$ and $N_{\neg Buy}(t)$ is given by

$$\begin{aligned}P[N_{Buy}(t) = N_1, N_{UD}(t) = N_2, N_{\neg Buy}(t) = N_3] \\ = \binom{N_{BOTH}}{N_1, N_2, N_3} P_{Buy}(t)^{N_1} P_{UD}(t)^{N_2} P_{\neg Buy}(t)^{N_3} .\end{aligned}$$

Based on these observations, the next theorem can be shown.

Theorem 4.1. *Given N_{BOTH} , let $K_{BOTH}(t)$ be the number of products sold by time t . Then $K_{BOTH}(t)$ has the binomial distribution with mean $N_{BOTH} \cdot P_{Buy}(t)$, i.e.*

$$P[K_{BOTH}(t) = k] = \binom{N_{BOTH}}{k} P_{Buy}(t)^k \{1 - P_{Buy}(t)\}^{N_{BOTH}-k} .$$

Proof. Since $E[U^{N_{Buy}(t)}] = \chi_{N_{BOTH}:P}(U, 1, 1, t)$, one sees from (4.3) and (4.5) that

$$E[U^{N_{Buy}(t)}] = \{ P_{Buy}(t)U + (1 - P_{Buy}(t)) \}^{N_{BOTH}} ,$$

proving the theorem. □

We next turn our attention to the sales completion time for K products given N_{BOTH} . More formally, let $T_{BOTH}(K)$ be the time until K products have been sold among N_{BOTH} , i.e.

$$T_{BOTH}(K) = \inf\{t : K_{BOTH}(t) = K\} .\quad (4.6)$$

Let $\bar{H}_{BOTH(K)}(t)$ be the survival function of $T_{BOTH}(K)$, i.e.

$$\bar{H}_{BOTH(K)}(t) = P[T_{BOTH}(K) > t] .\quad (4.7)$$

The next theorem then holds true.

Theorem 4.2. Let $\bar{H}_{BOTH(K)}(t)$ be as in (4.7). Then one has

$$\bar{H}_{BOTH(K)}(t) = \sum_{k=0}^{K-1} \binom{N_{BOTH}}{k} P_{Buy}(t)^k \{1 - P_{Buy}(t)\}^{N_{BOTH}-k}.$$

Proof. From (4.6), one easily sees that $T_{BOTH}(K) > t$ if and only if $K_{BOTH}(t) < K$. This dual relationship between $T_{BOTH}(K)$ and $K_{BOTH}(t)$ then implies that

$$\bar{H}_{BOTH(K)}(t) = \mathbb{P}[T_{BOTH}(K) > t] = \mathbb{P}[K_{BOTH}(t) < K],$$

proving the theorem. □

Clearly, Theorems 4.1 and 4.2 hold true for sales among M_{PC} except that $P_{Buy}(t), P_{UD}(t), P_{-Buy}(t)$ should be replaced by $Q_{Buy}(t), Q_{UD}(t), Q_{-Buy}(t)$. These results are summarized below.

Theorem 4.3. Given M_{PC} , let $K_{PC}(t)$ be the number of products sold by time t . Then $K_{PC}(t)$ has the binomial distribution with mean $M_{PC} \cdot Q_{Buy}(t)$, i.e.

$$\mathbb{P}[K_{PC}(t) = k] = \binom{M_{PC}}{k} Q_{Buy}(t)^k \{1 - Q_{Buy}(t)\}^{M_{PC}-k}.$$

Let $T_{PC}(K)$ be the time until K products have been sold among M_{PC} , i.e.

$$T_{PC}(K) = \inf\{t : K_{PC}(t) = K\}. \quad (4.8)$$

Let $\bar{H}_{PC(K)}(t)$ be the survival function of $T_{PC}(K)$, i.e.

$$\bar{H}_{PC(K)}(t) = \mathbb{P}[T_{PC}(K) > t]. \quad (4.9)$$

Theorem 4.4. Let $\bar{H}_{PC(K)}(t)$ be as in (4.9). Then one has

$$\bar{H}_{PC(K)}(t) = \sum_{k=0}^{K-1} \binom{M_{PC}}{k} Q_{Buy}(t)^k \{1 - Q_{Buy}(t)\}^{M_{PC}-k}.$$

In general, the market has both types of users and the two distributions for the whole market should be derived by convolving the corresponding distributions for N_{BOTH} and M_{PC} . More formally, let L be the number of consumers in the whole market given by $L = N_{BOTH} + M_{PC}$. Among L consumers, let $K(t)$ be the number of products sold by time t , and let $T(K)$ be the time required for selling K products. One then has the following theorem.

Theorem 4.5. Let L , $K(t)$ and $T(K)$ be as described above, and define the survival function of $T(K)$ by $\bar{H}_{(K)}(t) = P[T(K) > t]$.

$$a) P[K(t) = k] = \sum_{i=0}^k P[K_{BOTH}(t) = i] \cdot P[K_{PC}(t) = k - i],$$

where $P[K_{BOTH}(t) = k]$ and $P[K_{PC}(t) = k]$ are as given in Theorems 4.1 and 4.3 respectively.

$$b) \bar{H}_{(K)}(t) = \sum_{i=0}^{K-1} P[K(t) = i]$$

5. Numerical Examples

The purpose of this section is to explore numerically how the balance between N_{BOTH} and M_{PC} would affect the distribution of the number of products sold by time t , and that of sales completion time for K products. The underlying parameter values are set as given in the following table, unless specified otherwise. We also set $t = 240(\text{hours})$ and $L = N_{BOTH} + M_{PC} = 10000$, where $N_{BOTH} : M_{PC}$ is varied as $0 : 100$, $25 : 75$, $50 : 50$, $75 : 25$ and $100 : 0$.

α_1	α_2	β_1	β_2	λ_1	λ_2	τ_1	τ_2	τ_3
0.03	0.03	0.01	0.01	1/24	1/24	8	8	8

In Figure 5.1, the survival functions for $K(240)$, i.e. the number of products sold by time $t = 240$, are plotted, where the parameter α_1 is changed with 0.03, 0.06 and 0.09 from the top graph to the bottom graph respectively. In each graph, the left-most curve corresponds to $N_{BOTH} : M_{PC} = 0 : 100$ and the right-most curve represents the case of $N_{BOTH} : M_{PC} = 100 : 0$, or equivalently, the ratio N_{BOTH}/L moves from 0 to 1 from the left-most curve to the right-most curve.

We recall that α_1 is the probability of making the decision to buy the product after an access to the Internet, see also Figure 1.1. Accordingly, $K(240)$ increases stochastically as α_1 increases. This point can be observed by the fact that individual survival functions shift toward the right as α_1 increases, except the left-most curve which remains intact because no users have the mobile access. The degree of this stochastic increase becomes larger as the ratio N_{BOTH}/L moves from 0 to 1. Given α_1 , the stochastic dominance

of $K(240)$ with higher N_{BOTH}/L over $K(240)$ with lower N_{BOTH}/L can also be seen in each graph. The effect of this stochastic dominance is linear in N_{BOTH}/L as can be seen in Figure 5.2, where the expected sales volume increases linearly from 936 to 1755 for $\alpha_1 = 0.03$, from 936 to 2512 for $\alpha_1 = 0.06$, and from 936 to 3199 for $\alpha_1 = 0.09$, demonstrating the effects of the mobile access to the Internet in e-businesses.

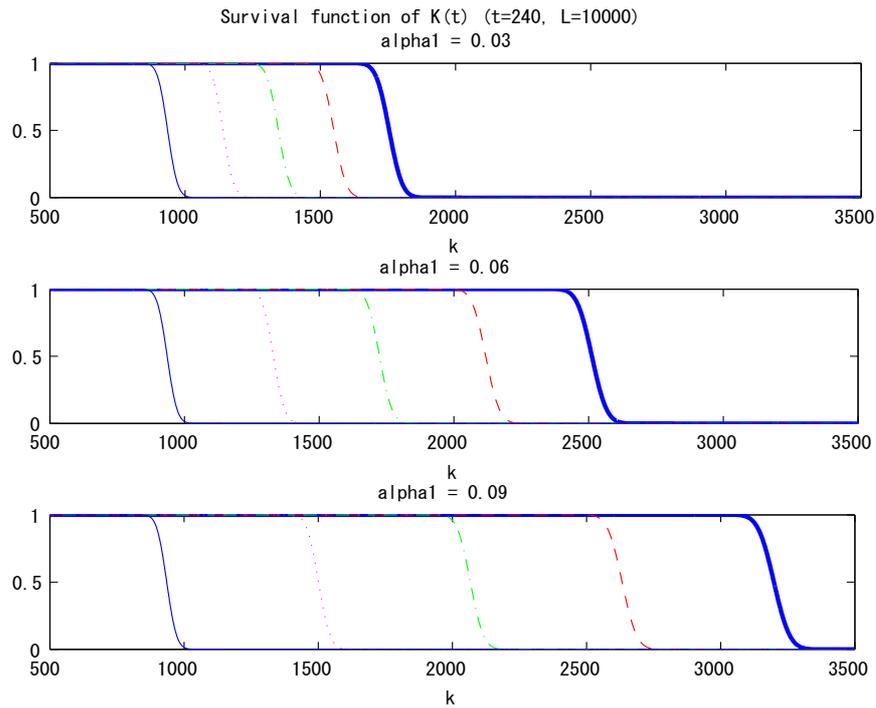


Figure 5.1. Survival function of $K(t)$

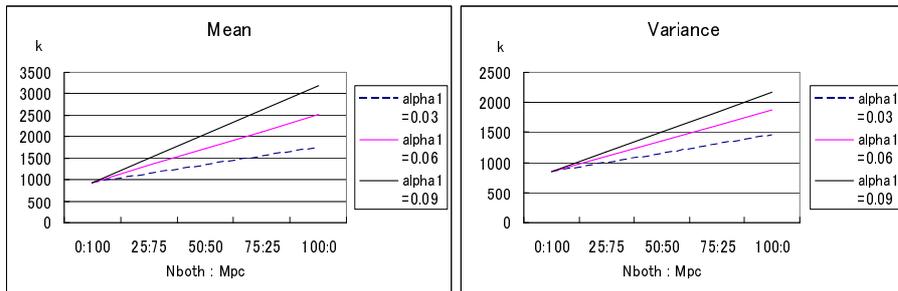


Figure 5.2. Mean and variance of $K(t)$

Figures 5.3 and 5.4 provide the counterparts of Figures 5.1 and 5.2 for the survival function for $T(2000)$, i.e. the sales completion time for $K = 2000$ products, except that the left-most curve now corresponds to $N_{BOTH} : M_{PC} = 100 : 0$ and the right-most curve represents the case of $N_{BOTH} : M_{PC} = 0 : 100$, or equivalently, the ratio N_{BOTH}/L moves from 1 to 0 from the left-most curve to the right-most curve. We observe that $T(2000)$ decreases stochastically as α_1 increases or the ratio N_{BOTH}/L increases. The stochastic change of $T(2000)$ in N_{BOTH}/L is, however, no longer linear as can be seen in Figure 5.2. The expected sales completion time decreases from 558 to 275 for $\alpha_1 = 0.03$, from 558 to 177 for $\alpha_1 = 0.06$, and from 558 to 128 for $\alpha_1 = 0.09$, again demonstrating the effects of the mobile access to the Internet in e-businesses.

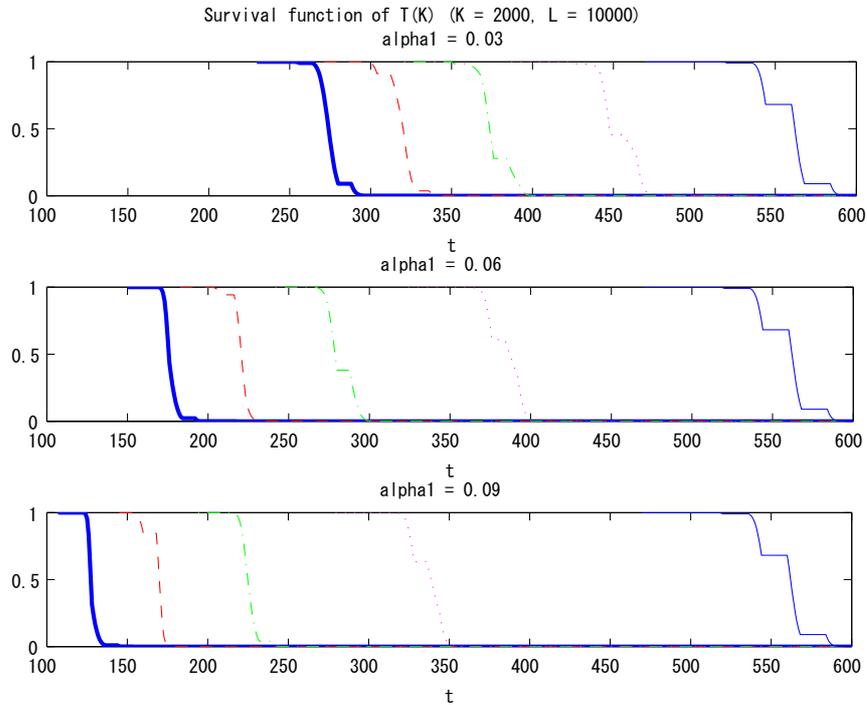


Figure 5.3. Survival function of $T(K)$

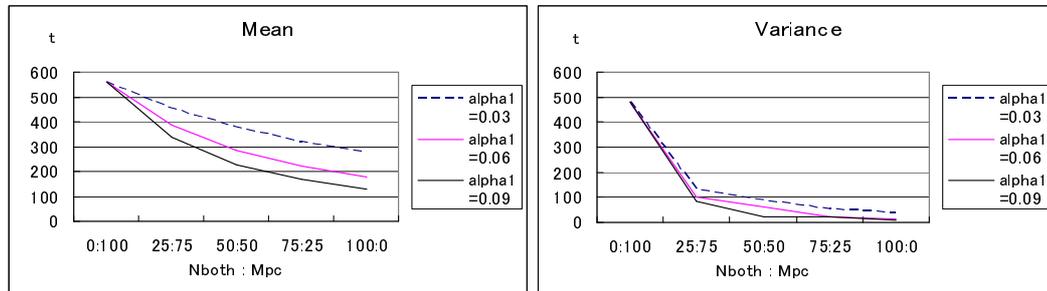


Figure 5.4. Mean and variance of $T(K)$

References

- [1] V. Roto, Browsing on Mobile Phones, Nokia Research Center, 2005.
- [2] S.H. Kim, Impact of Mobile-Commerce: Benefits, Technological and Strategic Issues and Implementation, *Journal of Applied Sciences* 6 (12) (2006) 2523–2531.
- [3] M. Chae and J. Kim, What 's So Different About the Mobile Internet?, *Communications of the ACM*, 2003.
- [4] P. Barwise, TV, PC, or Mobile? Future Media for Consumer e-Commerce, *Business Strategy Review*, Vol.12 (2001) 35–42.
- [5] K. Hammond, B2C e-Commerce 2000-2010: What Experts Predict, *Business Strategy Review*, 2001.
- [6] J.H. Wu and T.L. Hisa, Analysis of E-commerce innovation and impact: a hypercube model, *Electronic Commerce Research and Applications* 3 (2004) 389–404.
- [7] K. Siau and H. Sheng and F. Nah, The Value of Mobile Commerce to Customers, *Proceedings of the Third Annual Workshop on HCI Research in MIS*, 2004.
- [8] Y.H. Park and P.S. Fader, Modeling Browsing Behavior at Multiple Websites, *Marketing Science* 23 (3) (2004) 280–303.
- [9] G. Büyüközkan, Determining the mobile commerce user requirements using an analytic approach, *Computer Standards & Interfaces* 31 (2009) 144–152.