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Interactions between Individual Views and Behavior*

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Abstract

We explore the inductively derived views obtained by players with partial temporal (short-term) memories. A player derives his personal view of the objective game situation from his accumulated (long-term) memories of playing, and uses it for decision making in the objective situation. A salient feature that distinguishes this paper from others on inductive game theory is partiality of a memory function of a player. This creates multiplicity of possibly derived views. Although this is a difficulty for a player in various senses, it is an essential problem of induction. Faced with multiple possible views, a player may try to resolve this multiplicity using future experiences. This creates a two-way interaction between behavior and personal views which is another distinguishing feature of the present paper.

1. Introduction

Game theory and economics are experiential sciences about individual decisions and behavior in social contexts. However, much of these disciplines has by-passed the experiential side by taking the beliefs/knowledge of a player for granted. As far as they deal with beliefs/knowledge on experiential worlds, we would meet the questions of where these basic beliefs come from and of how they emerge and change with time. Inductive game theory initiated by Kaneko-Matsui [10] and developed more systematically by Kaneko-Kline [7], [8] and Akiyama-Ishikawa-Kaneko-Kline [1] addressed the questions of the origin/development of a player's basic beliefs/knowledge about the structure of a social situation. This paper will study the interactive effects of partiality of temporal memories on personal views and behavior.

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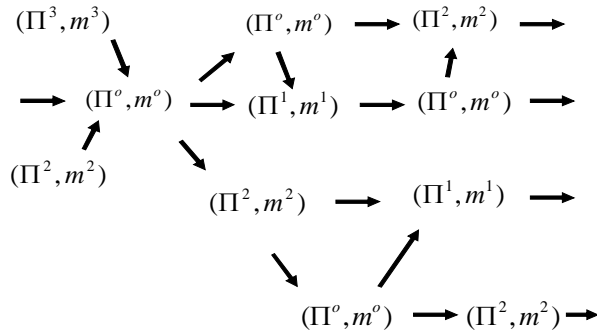


Figure 1.1: Various Social Situations

1.1. Inductive Game Theory

When a reader hears about a development of a new theory in game theory, he may ask, using the standard vocabulary of game theory, what kind of equilibrium/solution will be proposed and/or justified. However, our primary questions do not take such forms, since we do not aim to explore foundations of the extant equilibrium and/or solution concepts. Our primary question is how a person in a social context acquires beliefs/knowledge and then how he uses his beliefs/knowledge in his behavior. This requires us to rethink or to modify even very basic notions such as “information” in game theory.

One necessary modification is to the concept of an extensive game due to von Neumann-Morgenstern [16] and Kuhn [11]. The standard definition is given from the objective point of view and is not suitable for a subjective use. It describes “information” by means of information sets, but it lacks an ability to distinguish between the symbolic expression of information and its associated meaning. The emergence of beliefs/knowledge is a process of attaching a meaning to a symbolic expression. For this purpose, Kaneko-Kline [8] have developed the theory of “*information protocols*”, based on information pieces as primitives. Information protocols are used to express target social situations. In Fig.1.1, each situation such as (Π^o, m^o) is described by an information protocol Π^o together with a profile of memory functions $m^o = (m_1^o, \dots, m_n^o)$. In addition, the concept of an information protocol is used to express a subjective view of a player.

For inductive game theory, the time structure is also essential since learning about a social structure is one of our targets. In the game theory/economics literature, there are various approaches treating time structures such as the repeated game approach, the

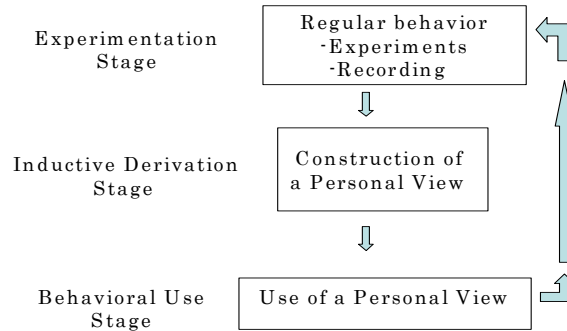


Figure 1.2: From Experimentations to Behavioral Uses

evolutionary game theory approach and behavioral economics. We do not follow these approaches, since each by-passes our basic question or avoids it by restricting attention to the behavioral consequences of decision-making¹. We will look at interactions in dynamics with learning, decision-making and behavior.

This paper will describe a more entire scenario, than in [7] and [8], moving from the stage of experimentations to that of behavioral uses of a personal view, and again to experimentations. In Fig.1.2, the scenario of these stages is illustrated. Although we touched on the stage of behavioral uses in [7], we avoided the multiplicity problem of personal views by choosing the perfect-recall memory function for a player. The focus of [8] was the introduction of an information protocol while restricting attention to the above two stages. Akiyama *et al* [1] focussed on the stage of experimentations and studied how much can be learned within a reasonable time span. This paper studies the second and third stages more extensively with emphasis on the interactions between views and behavior.

1.2. Interactions between Individual Views and Behavior

Our target is an ordinary recurrent social situation such as family affairs, business affairs, commuting, etc. The entire social system is depicted in Fig.1.1, but we focus on a particular situation, say, (Π^o, \mathbf{m}^o) . An ordinary person involved, whom we call a

¹In *ex ante* game theory, behavior results from sophisticated decision-making based on a granted view of the game itself. The repeated game approach (cf. Hart [5]) effectively follows this idea, though the interpretation associated with it may differ. In evolutionary game theory (cf., Weibull [15]) and behavioral economics (cf. Camerer, [3]), behavior is described by a specified (stochastic or non-stochastic) process within the game itself but without players thinking about the game, and limit behavior is typically analyzed.

player, spends only some of his energy and time to learn the structure of (Π^o, \mathbf{m}^o) . The information protocol Π^o and memory function \mathbf{m}_i^o will be defined in Section 2. Over some or many occurrences of the specific social situation, he is able to accumulate some of his experiences as *long-term* memories. This process of accumulation was exclusively discussed in Akiyama *et al* [1] using a specific example of Mike's bike commuting, a variant of which is discussed presently.

After the player has accumulated enough experiences, he may analyze and recombine these long-term memories to form a view. This process is the stage of inductive derivation of a personal view. Kaneko-Kline [7] and [8] concentrated on this stage, but by choosing the *self-scope perfect-recall memory function*, they found, an effectively unique view to describe those memories. A new problem in the present paper comes from the multiplicity of personal views that a player might find. When his memory function is partial, we confront a new frontier to a large untouched area involving multiplicity.

It is assumed in our theory that the player has two types of memories: (1) short-term (temporal, local) memory occurring at a point of time and lasting only for some short length of time; and (2) long-term memory lasting for a long time and serving as the source for an inductive construction of a personal view. The first type of a memory, given by a memory function \mathbf{m}_i^o , typically occurs spontaneously without conscious efforts. We regard this as a physiological functioning. Here, the player has only a worm's-eye view. In some occasions where, say, one short-term memory is repeated several times within a period of some length, it changes into a long-term memory. After he accumulates enough long-term memories, he may combine them consciously to construct his view, which becomes a bird's-eye view.

Summarizing the above argument,

- (i): short-term memories given by memory function \mathbf{m}_i^o are of a local and temporal nature;
- (ii): long-term memories are more lasting and can be used consciously for construction of his personal view on the situation (Π^o, \mathbf{m}^o) .

In fact, after (ii), he can use his view consciously to complement for the partiality of his memory function \mathbf{m}_i^o in the objective situation, as we will see in Section 6. Conversely, he may check the validity of his view with his new experiences.

The above distinction between physiological unconscious memories and long-term conscious memories becomes important when the memory function \mathbf{m}_i^o for player i involves some partiality. We postulate that physiological constraints on short-term memories can be captured by the notion of *memory-modules*. A memory-module is a unit consisting of elements and their connections. A simplest (nontrivial) memory-module consists of the immediate preceding information and the current information, with which we will define the *recall-1 memory function*. In general, we will have the *recall-k* memory function, which is based on larger memory modules. Long-term memories are a

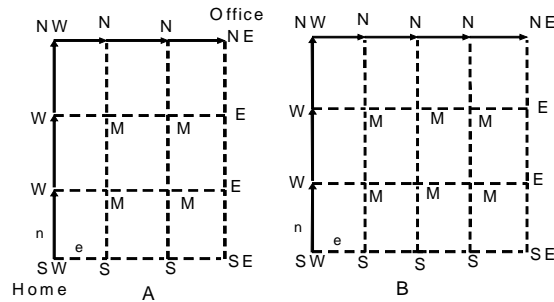


Figure 1.3: Mike's Bike Commuting

collection of memory-modules.

By the partiality of the memory function, we capture more essential problems of induction than in [7] and [8]. We may compare induction with the process of putting pieces together in a jigsaw puzzle. With the self-scope perfect-recall memory function in [7] and [8], induction was, more or less, a simple algorithm. Now, with the partial memory functions, induction is neither a simple algorithm, nor generates a unique outcome. It allows multiple views since there may be several or many ways of combining the set of modules in his long-term memories².

Let us see the above notions in one variant of “Mike’s Bike Commuting” from Akiyama et al [1]. This example will be used throughout the paper to illustrate various notions we meet in inductive game theory.

Mike’s Bike Commuting: Mike moved to the new town and started commuting from his apartment to his office by bike. The town has the lattice structure depicted in Fig.1.3.A. At each lattice point, he receives an information piece, S , W , N , E , M , SW , SE , NW , or NE . He has two possible actions “ e ” and “ n ” at SW , S , W and M . At NW and N , he must choose e , and at SE and E , he must choose n . NE is the unique endpiece. He regularly takes the route indicated by the bold arrows, which his colleague suggested to him. This forms the regular path. Occasionally, he may deviate to some other behavior and find some other path.

An objective description of this situation is given by listing all sequences from SW

²This is closer to the induction by Bacon [2] than that of Hume [6] based on similarity. Also, biology has a similar aspect of induction. A book review by A. C. Love on Hall [4] describes it as an analogy to a jigsaw puzzle: “The completion of a jigsaw puzzle brings tremendous satisfaction; however, a few missing pieces lead to considerable frustration. Having the intended picture of a puzzle on the container contributes to the satisfaction (or the frustration). How do you know if you have all the pieces? ... Such is the lot of biologists attempting to explain key evolutionary transitions in the history of life” (*Science* 317, 17, Sept.2007).

to each lattice point with actions e or n . From the player's point of view, he may receive only the information piece at each lattice point, and one possible form of his local memory is that he recalls only the previous piece and the action taken there in addition to the current piece he is receiving. For example, at the southwest M , if he comes from the west S , his local memory is just $\langle (S, n), M \rangle$. This is a memory-module of recall-1, and describes partiality in his memory.

After he has commuted many times, he may accumulate several of those small memory modules as long-term memories. His problem of induction is to combine those small modules to one picture. Even if he collects all such memory modules from commuting, he may induce an incorrect view of the town such as Fig.1.B which is larger than the correct view. The possibility of an incorrect view is a result of partiality in his memory. He might find a correct one, but there are many other possibilities that cannot be easily rejected.

In the case of the self-scope perfect-recall memory function adopted in [7], [8] and [1], Mike received the memory thread consisting of all his past received pieces and actions taken, e.g., if he reaches the southwest M from the west S , then his current local memory was $\langle (SW, e), (S, n), M \rangle$. In this case, it would be easy to construct the correct map simply by combining all his memory threads accumulated over many trips. Since we allow for more limited and partial memory in the present paper, we must consider a variety of alternative views a player might construct from the same set of accumulated memories.

After constructing a personal view, the stage of experimentations becomes important for a player again. He may test his adopted view by noting whether or not his future experiences in the objective situation correspond to his expectations based on his adopted view. If he finds some incoherency between his view and his new experiences, he may modify his view. Thus, the repetition of the cycle depicted in Fig.1.2 comes within the purview of this paper. When Mike brings his adopted view of Fig.1.3.B to his future commuting, he may notice some incoherency with his new experiences, and think about modifying his view or choosing some alternative.

In some cases, multiplicity in views can be resolved by considering only the smallest view. In other cases, however, there may be several minimal views, and a player may need to reconsider the connections between his views and experiences in order to discriminate between views. In the Mike's bike example with recall-1 given above, there are several minimal views, and in fact, each one is smaller than the correct map. Thus, focusing on minimal views might not be sufficient to find a correct one. In his journey and consideration of alternative views, he may seek out other sources of information such as the lattice structure of the town. These are the messages we will obtain in Sections 3-6.

The paper is written as follows: In Section 2, we provide the definition of information protocols, memory functions, personal views and behavior, while describing the basic

objective situation. In Section 3, we give the definition of an i.d.view (inductively derived view), and provide several basic results such as the general existence theorem and a condition for the existence of a smallest view. In Section 4, we will restrict our attention to the memory function of recall-k. We show in Section 5 that the smallest view exists uniquely under the assumption of Kuhn’s [11] distinguishability³. In Section 6, we study behavioral consequences of a view and behavioral revisions of a view. Section 7 gives concluding remarks.

2. Information Protocols, Memory, Views, and Behavior

In Section 2.1, we describe information protocols and the axioms for them, introduced in Kaneko-Kline [8]. They showed that these correspond to various forms of weakened extensive games. Section 2.2 introduces the concept of a memory function for a player, which is the interface from objective experiences to his perceptions in his mind. Then, we define an objective description (Π^o, \mathbf{m}^o) and a personal view (Π^i, \mathbf{m}^i) of player i . In Section 2.3, we give a definition of a behavior pattern (strategy configuration) for the players, and also describe a domain of accumulation for memories and finally a memory kit.

2.1. Information Protocols and Axioms

The concept of an information protocol was developed by Kaneko-Kline[8] as an alternative to extensive games⁴ due to Kuhn [11]. It deals with information pieces and actions as very basic concepts and connects a history to a new information piece and action. An *information protocol* is given as a quintuple $\Pi = (W, A, \prec, (\pi, N), (h)_{i \in N_*})$, where

IP1: W is a finite nonempty set of *information pieces*;

IP2: A is a finite nonempty set of *actions*;

IP3: \prec is a *causality relation*; formally, it is a finite nonempty subset of $\bigcup_{m=0}^{\infty} ((W \times A)^m \times W)$, where any $w \in W$ and any $a \in A$ occur in some sequence in \prec .

A sequence in \prec is called a *feasible sequence*. We say that $w \in W$ is a *decision piece* iff w occurs in $[(w_1, a_1), \dots, (w_m, a_m)]$ for some feasible sequence $\langle (w_1, a_1), \dots, (w_m, a_m), w_{m+1} \rangle$ in \prec . We denote the set of all decision pieces by W^D . We define $W^E = W - W^D$, where

³This is a reformulation of Kuhn’s [11] “perfect recall” condition in terms of information pieces in an information protocol. Since the memory capability is expressed by a memory function in our approach, Kuhn’s condition is not interpreted as expressing “memory”. Thus, we use a different term for it.

⁴One difficulty with using extensive games, is that various forms of weakenings of extensive games are required from the viewpoint of a player having only partial experiences. In information protocols, these weakenings are characterized and classified by some axioms.

each piece in W^E is called an *endpiece*. Using those notions, we describe the fourth and fifth components of a protocol.

IP4 (*player assignment*): $N = \{1, \dots, n\}$ is a finite set of players, and $\pi : W \rightarrow 2^N$ is the player assignment, where $|\pi(w)| = 1$ for all $w \in W^D$ and $\pi(w) = N$ for all $w \in W^E$;

IP5 (*payoff assignment*): $h_i : W^E \rightarrow R$ for all $i \in N_*$ where $N_* \subseteq N$.

We start with two sets W and A of tangible elements listed in IP1 and IP2. Each information piece $w \in W$ may be interpreted as a pure symbolic expression like a gesture, a sentence in an ordinary language, or a formula in the sense of mathematical logic. In the example of Mike's bike commuting, $W = \{SE, W, N, E, M, SW, SE, NW, NE\}$ and $A = \{e, n\}$. The set \prec given in IP3 describes the feasible sequences of these elements occurring in some play of the game. A feasible sequence $\langle (w_1, a_1), \dots, (w_m, a_m), w \rangle$ is interpreted as meaning that in one occurrence of the protocol Π , a player first received piece w_1 and took action a_1 , then sometime later another player received w_2 and took action a_2 , so on, and now, a player receives w . It is not yet assumed that this sequence is an exhaustive history up to w . An exhaustive history will be defined presently.

We sometimes write $[(w_1, a_1), \dots, (w_m, a_m)] \prec w$ for $\langle (w_1, a_1), \dots, (w_m, a_m), w \rangle \in \prec$. We will use $\langle \xi, w \rangle$ to denote a generic element of $\bigcup_{m=0}^{\infty} ((W \times A)^m \times W)$. The set $(W \times A)^0 \times W$ is stipulated to be W and we sometimes write $\prec w$ for $\langle w \rangle \in \prec$. The set \prec is the union of a unary relation on $(W \times A)^0 \times W = W$, a binary relation on $(W \times A)^1 \times W$, a ternary relation on $(W \times A)^2 \times W, \dots$, etc. In this paper, however, we are interested only in *finite* information protocols, i.e., W, A and \prec are all finite sets. Throughout the paper, we assume $W \cap A = \emptyset$ to avoid unnecessary complications.

An information protocol is completed by adding the player assignment and the payoff assignment. The player assignment π in IP4 assigns a single player to each decision piece, and the set of all players N to each endpiece. In IP5, the payoff function h_i is specified for each player i in the set $N_* \subseteq N$. We allow N_* to differ from N to describe a view where only some players' payoffs are known to the player. In the present paper, we consider the case of either $N_* = N$ or $N_* = \{i\}$. We have left the more general requirement of $N_* \subseteq N$ for our research on social roles (Kaneko-Kline [9]), where players may learn some other players' payoffs by role playing.

We assume for simplicity that each piece $w \in W$ contains a minimal amount of information. Explicitly, each player i should be able to read the following information from looking at w :

M1: the full set $A_w = \{a \in A : [(w, a)] \prec u \text{ for some } u \in W\}$ of available actions at w if w is a decision piece;

M2: the value $\pi(w)$ of the player assignment π if w is a decision piece;

M3: his own payoff $h_i(w)$ (as a numerical value) if w is an endpiece.

In M1, the full set of available actions is written at each decision piece. Condition M2

requires w to include the information of who moves at w . Here, player i may receive (or to observe) a decision piece w at which another player j moves, but every player who observes a decision piece w agrees about who moves there. Finally, in M3, each player can read his own payoff from each endpiece.

In Mike's bike, $A_{SW} = A_S = A_W = A_M = \{e, n\}$, $A_{NW} = A_N = \{e\}$, $A_{SE} = A_E = \{n\}$ and $A_{NE} = \emptyset$. Condition M2 and M3 are rather trivial in this example⁵, since it is a 1-person problem and since the endpiece is only NE . In this example, one possible feasible sequence is a route from SW to NE . The route determined by the bold arrows is expressed as

$$\langle (SW, n), (W, n), (W, n), (NW, e), (N, e), (N, e), NE \rangle. \quad (2.1)$$

We have a total of $\binom{6}{3} = 20$ routes from SW to NE .

We use information protocols to describe both the objective situation and a personal view. The distinction between them is made by a choice of axioms they should satisfy. A protocol for the objective description should satisfy two basic axioms and three non-basic axioms. A protocol for a personal view will be required to satisfy only the two basic axioms. We give the full set of basic and non-basic axioms now.

The first basic axiom is contraction (subsequence-closed), which states that any subsequence of a feasible sequence is also feasible. For this axiom, we need a concept of a subsequence of a feasible sequence. It is defined by regarding each (v_t, a_t) or v_{k+1} as a component of $\langle (v_1, a_1), \dots, (v_m, a_m), v_{m+1} \rangle$. For example, $\langle (u_1, b_1), u_{k+1} \rangle$ and $\langle (u_2, b_2), \dots, (u_{k-1}, b_{k-1}), u_{k+1} \rangle$ are subsequences of $\langle (u_1, b_1), \dots, (u_k, b_k), u_{k+1} \rangle$, and so is $\langle u_{k+1} \rangle$ ⁶. A supersequence is defined in the dual manner.

Axiom B1 (Contraction): If $\langle \xi, v \rangle \in \prec$ and $\langle \xi', v' \rangle$ is a subsequence of $\langle \xi, v \rangle$, then $\langle \xi', v' \rangle \in \prec$.

The second basic axiom states that the decision pieces can be distinguished from the endpieces.

Axiom B2 (Weak Extension): If $\xi \prec w$ and $w \in W^D$, then there are $a \in A$ and $v \in W$ such that $[\xi, (w, a)] \prec v$.

Any protocol Π that satisfies Axioms B1 and B2 is called a *basic protocol*. So far, a feasible sequence may not be an exhaustive history. For the other three axioms, we need an exhaustive history. We say that a feasible sequence $\langle \xi, v \rangle$ is *maximal* iff \prec contains no proper feasible supersequence $\langle \eta', v' \rangle$ of $\langle \eta, v \rangle$. A *position* $\langle \xi, w \rangle$ is defined to be an

⁵In this example, we do not treat payoffs. However, if we want to treat payoffs (or preferences) in Mike's Bike, then we should include payoffs depending upon the path to NE . In this case, NE should be divided into several pieces including payoffs.

⁶Formally, we say that $\langle (u_1, b_1), \dots, (u_k, b_k), u_{k+1} \rangle$ is a *subsequence* of $\langle (v_1, a_1), \dots, (v_m, a_m), v_{k+1} \rangle$ iff $[(u_1, b_1), \dots, (u_k, b_k), (u_{k+1}, b)]$ is a subsequence of $[(v_1, a_1), \dots, (v_m, a_m), (v_{m+1}, a)]$ for some a and b .

initial segment of some maximal feasible sequence $\langle \eta, v \rangle$, where an initial segment of $\langle (w_1, a_1), \dots, (w_m, a_m), w_{m+1} \rangle$ is given as $\langle (w_1, a_1), \dots, (w_k, a_k), w_{k+1} \rangle$ for some $k \leq m$. Each position is an exhaustive history up to v in Π . We denote the set of all positions by Ξ .

We now list the three non-basic axioms.

Axiom N1 (Root): There is a distinguished element $w^0 \in W$ such that $\langle w^0 \rangle$ is an initial segment of every position.

This axiom means that all positions start with w^0 . Without this axiom, the protocol may have various starts. The next axiom states that an exhaustive history determines a unique information piece.

Axiom N2 (Determination): Let $\langle \xi, u \rangle$ and $\langle \eta, v \rangle$ be positions. If $\xi = \eta$ and it is nonempty, then $u = v$.

The last axiom states that the set of available actions at an information piece is independent of a history.

Axiom N3 (History-Independent Extension): If $\langle \xi, w \rangle$ is a position and $[(w, a)] \prec u$, then there is a $v \in W$ such that $\langle \xi, (w, a), v \rangle$ is a position.

This axiom implies that the set of available actions at any position $\langle \xi, w \rangle$ is given as the set $A_w = \{a \in A : [(w, a)] \prec u \text{ for some } u \in W\}$. If N3 is violated, the set of available actions differ at two positions ending with the same information piece.

When an information protocol Π satisfies Axioms B1, B2, N1, N2, and N3, we call it a *full protocol*. A full protocol will be used to describe an objective situation: An *objective description* is a full protocol $\Pi = (W, A, \prec, (\pi, N), (h_i)_{i \in N_*})$ satisfying Axioms B1, B2, N1, N2, N3 and $N_* = N$.

For a personal view, we require only the basic Axioms B1 and B2, and also, the payoff assignment for only the player in question. A *subjective protocol* is a basic protocol $\Pi = (W, A, \prec, (\pi, N), (h_i)_{i \in N_*})$ satisfying Axioms B1, B2 and $N_* = \{i\}$.

In Kaneko-Kline [8], it was shown that a full protocol is equivalent to an extensive game in Kuhn's [11] sense with the replacement of information sets by information pieces. Also, it is shown that the deletion of each of the non-basic Axioms N1, N2 and N3 corresponds to some weakening of the definitions for an extensive game. It was also shown that such weakenings are arising naturally as inductively derived views. In Sections 3, 5 and 6, we will encounter several examples violating some of the nonbasic Axioms N1-N3.

It is complicated to write down the causality relation \prec even in a small example of a basic protocol, since it is subsequence-closed by Axiom B1. In fact, it is enough to list the positions with endpositions rather than the entire causality relation \prec . To see

this, we partition the set of all positions Ξ into the sets:

$$\Xi^D = \{\langle \xi, w \rangle \in \Xi : w \in W^D\} \quad \text{and} \quad \Xi^E = \{\langle \xi, w \rangle \in \Xi : w \in W^E\}. \quad (2.2)$$

We call $\langle \xi, w \rangle \in \Xi^D$ a *decision position* and $\langle \xi, w \rangle \in \Xi^E$ an *endposition*.

In Mike's bike, we have 20 endpositions. The number of decision positions is 51. Fig.1.3.A is a simpler representation than the set of endpositions Ξ^E , but this is only possible in some examples.

For any set of sequences $Y \subseteq \bigcup_{m=0}^{\infty} ((W \times A)^m \times W)$, we define the operator Δ by

$$\Delta Y = \{\langle \xi, v \rangle : \langle \xi, v \rangle \text{ is a subsequence of some sequence } \langle \eta, w \rangle \in Y\}. \quad (2.3)$$

Then, we have the following lemma, which states that the endpositions represent all feasible sequences \prec .

Lemma 2.1. Let $\Pi = (W, A, \prec, (\pi, N), (h_i)_{i \in N_*})$ be a basic protocol. The set of feasible sequences \prec coincides with the set $\Delta \Xi^E$.

Proof. Let $\langle \xi, w \rangle$ be any sequence in \prec . Then there is a maximal feasible sequence $\langle \zeta, v \rangle$ in \prec such that it is a supersequence of $\langle \xi, w \rangle$. By Axiom B2, $v \in W^E$. Hence, $\langle \zeta, v \rangle \in \Xi^E$. By Axiom B1, we have $\langle \xi, w \rangle \in \Delta \Xi^E$. We obtain the converse by tracing this argument back. ■

We use this lemma to consider an example of an information protocol, which will be referred in the subsequent sections.

Example 2.1. Consider the following 1-person situation in Fig.2.1. Player 1 chooses an action twice successively. At any endnode, he receives the information piece which, we presume, is equivalent to this payoff, i.e., $h_1(0) = 0$, and $h_1(5) = 5$.

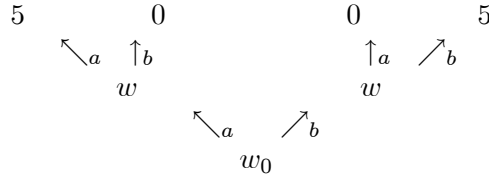


Fig.2.1

To describe this as an information protocol, we take $W = \{w_0, w, 5, 0\}$ and $A = \{a, b\}$. The set of feasible sequences \prec is quite large, but by Lemma 2.1 it suffices to list only the endpositions $\Xi^E = \{\langle (w_0, a), (w, a), 5 \rangle, \langle (w_0, a), (w, b), 0 \rangle, \langle (w_0, b), (w, a), 0 \rangle, \langle (w_0, b), (w, b), 5 \rangle\}$. This protocol is full and thus it could be interpreted as an objective situation.

Fig.2.1 may also be regarded as describing player 1's view of a 2-player interactive situation, where player 1 moves at w , and player 2 moves at w_0 . The payoff to player 1 is as in Example 2.1, and since it is his personal view, the payoff of player 2 is not listed.

2.2. Memory Functions and Views

A main problem of inductive game theory is to consider an derivation of a personal view from memories accumulated in a player's mind. The source for an inductive derivation is his memories from experiences. Therefore, a certain interface from individual experiences to memories is required. Here, we give the concept of a memory function as the description of such an interface.

A memory function describes a personal memory capability within one play of an information protocol. In other words, it describes short-term (local, temporal) memories within one play of the game. Transition from short-term memories to long-term memories needs another superstructure, which is discussed in Akiyama et al [1], and will be briefly described in Section 2.3.

Now, let Π be a basic information protocol, and let Ξ be the set of positions in Π . Let Y_i a subset of Ξ including the set $\Xi_i := \{\langle \xi, w \rangle \in \Xi : i \in \pi(w)\}$ of player i 's positions, i.e., $\Xi_i \subseteq Y_i \subseteq \Xi$.

Definition 2.2 (Memory Functions): A *memory function* \mathbf{m}_i of player i assigns, to each $\langle \xi, w \rangle \in Y_i$, a finite sequence $\langle \zeta, v \rangle = \langle (v_1, b_1), \dots, (v_m, b_m), v \rangle$ satisfying:

$$v = w; \tag{2.4}$$

$$m \geq 0 \text{ and } v_t \in W, b_t \in A_{v_t} \text{ for all } t = 1, \dots, m. \tag{2.5}$$

We call the value of a memory function $\mathbf{m}_i \langle \xi, w \rangle = \langle \zeta, v \rangle$ a *memory thread* and each component (v_t, b_t) in the thread a *memory knot*. A memory thread represents a player's short-term (local) memory about the past moves up to the position $\langle \xi, w \rangle$ within one play of Π . Condition (2.4) means that the latest piece is the one received at the current position. Except for this requirement, enough flexibility is allowed in (2.5) so as to capture forgetfulness and incorrect memories. Here, we note that the domain Y_i may contain other players' positions, in which case player i receives some other player's information piece.

A meaningful example with a small memory-module is the *recall-1* memory function, as stated in Mike's bike. In this case, the memory module is the pair of the present information piece and the previously received piece and taken action. We will consider a slightly more general class of memory functions, called "recall- k ". By "recall- k ", player i can recall back to the k latest memory knots within Y_i ; this is a limitation on the length of a memory thread (not a duration of a short-term memory). For this definition, we need the definition of the Y_i -part of a position in Y_i .

Let $\langle \xi, w \rangle = \langle (w_1, a_1), \dots, (w_m, a_m), w_{m+1} \rangle$ be any position in Y_i . We define the Y_i -part $\langle \xi, w \rangle_i$ of $\langle \xi, w \rangle$ to be the maximal subsequence $\langle (v_1, b_1), \dots, (v_s, b_s), v_{s+1} \rangle$ of $\langle \xi, w \rangle$ so that that for each $l = 1, \dots, s + 1$, the initial segment of $\langle \xi, w \rangle$ up to v_l belongs to Y_i . For

example, when $\langle \xi, w \rangle = \langle (w_1, a_1), (w_2, a_2), w_3 \rangle \in Y_i$, $\langle w_1 \rangle \in Y_i$ but $\langle (w_1, a_1), w_2 \rangle \notin Y_i$, we have $\langle \xi, w \rangle_i = \langle (w_1, a_1), w_3 \rangle$.

To define the recall- k memory function, we define: for $\langle \xi, w \rangle_i = \langle (v_1, b_1), \dots, (v_s, b_s), v_{s+1} \rangle$ and a non-negative integer k ,

$$\langle \xi, w \rangle_i^k = \begin{cases} \langle (v_{s-k+1}, b_{s-k+1}), \dots, (v_s, b_s), v_{s+1} \rangle & \text{if } k \leq s. \\ \langle \xi, w \rangle_i & \text{if } k > s \end{cases} \quad (2.6)$$

It takes the last k part of $\langle \xi, w \rangle_i$, but when k is larger than s , it takes the entire $\langle \xi, w \rangle_i$. Also, when $k = 0$, we stipulate that $\langle \xi, w \rangle_i = \langle v_{s+1} \rangle = \langle w \rangle$. In the position $\langle \xi, NE \rangle$ of (2.1), $\langle \xi, NE \rangle_1^0 = \langle NE \rangle$, $\langle \xi, NE \rangle_1^1 = \langle (N, e), NE \rangle$ and $\langle \xi, NE \rangle_1^6 = \langle \xi, NE \rangle$.

The *recall- k memory function* is formulated as:

$$\mathbf{m}_i^{Rk} \langle \xi, w \rangle = \langle \xi, w \rangle_i^k \quad \text{for each } \langle \xi, w \rangle \in Y_i. \quad (2.7)$$

When the memory bound k is zero, i.e., player i has no recall ability in short-term memories, it is called the *Markov memory function* \mathbf{m}_i^{R0} . It holds that $\mathbf{m}_i^{R0} \langle \xi, w \rangle = \langle w \rangle$ for each $\langle \xi, w \rangle \in Y_i$. This is of importance only as a reference point of our analysis.

In Mike's bike, the memory function \mathbf{m}_i^{R1} of recall 1 assigns $\mathbf{m}_i^{R1} \langle \xi, M \rangle = \langle (S, n), M \rangle$ to position $\langle \xi, M \rangle = \langle (SW, e), (S, n), M \rangle$ and $\mathbf{m}_i^{R1} \langle \xi', M \rangle = \langle (S, n), M \rangle$ to position $\langle \xi, M \rangle = \langle (SW, e), (S, e), (S, n), M \rangle$. Hence, he finds no difference in his short-term memory at the positions of these different histories.

When k is longer than the maximum depth of the protocol, the memory bound is no longer a bound. In this case, we call \mathbf{m}_i^{Rk} the *perfect-recall memory-function*⁷, denoted by \mathbf{m}_i^{PR} . It is formulated as:

$$\mathbf{m}_i^{PR} \langle \xi, v \rangle = \langle \xi, v \rangle_i \quad \text{for each } \langle \xi, v \rangle \in Y_i. \quad (2.8)$$

With the memory function \mathbf{m}_i^{PR} , player i recalls all the information pieces and actions previously observed by himself. With this function, we will define the PR-view, which will play an important role in Sections 4 and 6.

Two extreme cases should be emphasized. When Y_i coincides with the set Ξ of all positions, the memory function defined by (2.8) is called the *perfect-information memory function* and is denoted by \mathbf{m}_i^{PI} . In this case, $\mathbf{m}_i^{PI} \langle \xi, v \rangle = \langle \xi, v \rangle$ for each $\langle \xi, v \rangle \in Y_i = \Xi$. With this memory function, player i recalls the complete history within a play of Π even including the other players' pieces and actions. Another extreme case is given by $Y_i = \Xi_i$, where the memory function \mathbf{m}_i^{PR} is called the *self-scope perfect-recall memory function* and is denoted by \mathbf{m}_i^{SPR} . In this case, the player only has memories of his own information pieces and actions. This memory function was exclusively used in Kaneko-Kline [7] and [8].

⁷This differs considerably from Kuhn's [11], which will be discussed in Section 5.

The recall- k memory functions may include partiality and forgetfulness, but the memories are correct in the sense each memory thread is a subsequence of the truth. Having described an information protocol and memory functions, we now have the basic ingredients for objective descriptions and subjective personal views.

(Objective Description): A pair (Π^o, \mathbf{m}^o) is called an *objective description* iff $\Pi^o = (W^o, A^o, \prec^o, \pi^o, h^o)$ is an objective protocol, i.e., it is a full protocol, and $\mathbf{m}^o = (\mathbf{m}_1^o, \dots, \mathbf{m}_n^o)$ is an n -tuple of memory functions in Π^o .

We use the superscript o to denote the objective description. We put a superscript i to denote a personal view of player i . Just as payoffs are viewed as personal, so is the subjective memory function reconstructed by a player. Thus, a personal view of player i includes only his subjective payoff h^i and his subjective memory function \mathbf{m}^i .

(Personal View): A pair (Π^i, \mathbf{m}^i) is a *personal view* for player i iff $\Pi^i = (W^i, A^i, \prec^i, \pi^i, h^i)$ is a subjective protocol, i.e., it is a basic protocol, with a specification of player i 's payoff function h^i , and \mathbf{m}^i is a memory function for player i in Π^i .

2.3. Behavior Patterns, Closed Domains, and Memory Kits

We suppose that the game situation given by the objective description (Π^o, \mathbf{m}^o) is played repeatedly. Behavior of each player is described by the concept of a behavior pattern. Recall that Ξ_i^{oD} is the set of decision positions for player i . A function σ_i on Ξ_i^{oD} is a *behavior pattern* (strategy) of player i iff it satisfies: for all $\langle \xi, w \rangle, \langle \eta, v \rangle \in \Xi_i^{oD}$,

$$\sigma_i \langle \xi, w \rangle \in A_i^o; \quad (2.9)$$

$$\mathbf{m}_i^o \langle \xi, w \rangle = \mathbf{m}_i^o \langle \eta, v \rangle \text{ implies } \sigma_i \langle \xi, w \rangle = \sigma_i \langle \eta, v \rangle. \quad (2.10)$$

Condition (2.9) means that a behavior pattern σ_i prescribes an available action to each decision position. Condition (2.10) means that a strategy depends upon the local memory of the player moving there. We denote, by Σ_i^o , the *set of all behavior patterns* for player i in (Π^o, \mathbf{m}^o) . We say that an n -tuple of strategies $\sigma = (\sigma_1, \dots, \sigma_n)$ is a *profile of behavior patterns*.

Although a behavior pattern is defined as a complete contingent plan, we do not require that the player be fully aware of this complete plan. Rather he should be able to take an action whenever he is called upon to move. The minimal information condition M1 ensures that a player can see the available actions, and pick one, maybe, a *default* action, whenever one of his decision pieces is reached. We use the term *behavior pattern* to express the idea that the behavior of a player may initially have no strategic considerations. Once a player has gathered enough information about the game, his behavior may become strategic.

We presume that the players follow some *regular behavior patterns* $\sigma^o = (\sigma_1^o, \dots, \sigma_n^o)$. Sometimes, however, some players may deviate from these behavior patterns, which leads to new experiences and short-term memories for them. These short-term memories remain for some periods of time, but after these periods, they would disappear, except when they have occurred frequently enough to reinforce the short-term memories as lasting in his mind. When such a case occurs, a short-term memory becomes a long-term memory, and remains for longer periods. It is important to assume that objects of a short-term or long-term memory are memory threads, but not a sequence of occurrences of those memory threads.

Since there are many aspects involved in such an evolving process, there would be many possible formulations of the dynamics. Also, since the relevant time structure must be finite, limit theorems are not of interest to us at all. Therefore, we think that a computer simulation is an appropriate method to study the dynamics of accumulation of long-term memories. One simple version is given in Akiyama *et al* [1]. In the present paper, we do not give a formulation of a dynamics itself. Instead, we give a general definition of possible results of such a dynamic accumulation process, which we call a *memory kit*.

The memory kit is described in terms of some domain of accumulation. For this we start with a basic domain. We say that a subset D_i of Y_i is a *cane domain* iff for some endposition $\langle \xi, w \rangle$, D_i is given as the set $\{\langle \zeta, v \rangle \in Y_i : \langle \zeta, v \rangle \text{ is an initial segment of } \langle \xi, w \rangle\}$. Thus, D_i is the set of positions in Y_i successively continuing to the endposition $\langle \xi, w \rangle$. The regular cane domain is obtained when every player follows his regular behavior pattern σ_i^o with no deviations. A subset D_i of Y_i is said to be a *closed domain of accumulation* iff it is expressed as the union of some cane domains⁸. We focus largely on closed domains in this paper.

The *memory kit* T_{D_i} for domain D_i is defined by

$$T_{D_i} = \{\mathfrak{m}_i^o \langle \xi, w \rangle : \langle \xi, w \rangle \in D_i\}. \quad (2.11)$$

The memory kit T_{D_i} is determined by both the domain of accumulation D_i and the objective memory function \mathfrak{m}_i^o of player i . It will be the source for an inductive construction of a personal view. The set T_{D_i} of memory threads is used to construct a skeleton of the personal view.

In Mike's bike with the memory function \mathfrak{m}_1^{R1} of recall-1, if D_1 is the cane domain determined by the bold arrows, his memory kit T_{D_1} consists of 7 memory threads:

$$\langle SE \rangle, \langle (SE, n), W \rangle, \langle (W, n), W \rangle, \langle (W, n), NW \rangle, \langle (NW, e), N \rangle, \langle (N, e), N \rangle, \langle (N, e), NE \rangle \quad (2.12)$$

⁸The *active* domain and *unilateral* domain considered in Kaneko-Kline [7] and [8] are both closed domains.

If D_1 is the full domain, the memory kit T_{D_1} consists of 19 memory threads, where he has the same memory threads at some lattice points. This partiality generates a smaller view than Fig.1.A even in the case of the full domain, which will be discussed in Section 4.

In the case of a personal view (Π^i, \mathbf{m}^i) of player i , a *strategy* τ_i is defined in the same manner as that in the objective protocol except that, since Axiom N3 may be violated, (2.9) is replaced by: for any position $\langle \xi, v \rangle \in \Xi_j^{iD}$,

$$s_j \langle \xi, v \rangle \in \{a : \langle \xi, (v, a), u \rangle \text{ is a position for some } u\}. \quad (2.13)$$

We denote the set of all strategies for player i by Σ^i . One question is whether a strategy chosen in (Π^i, \mathbf{m}^i) can be brought to the objective situation (Π^o, \mathbf{m}^o) . This will be discussed in Section 6.

3. Inductive Derivations

We now start the main part of the paper. It is about the inductive construction of a personal view from a memory kit T_{D_i} of a player. Kaneko-Kline [7] and [8] adopted the self-scope perfect-recall memory function, so that a given memory kit T_{D_i} determines, more or less, a unique personal view. In this paper, the partiality in a player's memory forces us to consider multiple views for the same memory kit, which opens the theory to new types of induction. In Section 3.1, we prove the existence of an inductively derived view for each memory kit on a closed domain (Theorem 3.1). In Section 3.2, a notion of smallness is introduced as one criterion to a view.

3.1. Inductively Derived Views

We relax the definition of an inductively derived view given in Kaneko-Kline [7] and [8]. In the following, we fix the objective description (Π^o, \mathbf{m}^o) . The full set of requirements for an i.d.view are as follows.

Definition 3.1 (Inductively Derived View). A personal view $(\Pi^i, \mathbf{m}^i) = ((W^i, A^i, \prec^i, \pi^i, h^i), \mathbf{m}^i)$ for player i is an *inductively derived view* from a memory kit T_{D_i} iff

ID1(Information Pieces): $W^i = \{w \in W^o : w \text{ occurs in some sequence in } T_{D_i}\}$; $W^{iD} \subseteq W^{oD}$ and $W^{iE} \subseteq W^{oE}$;

ID2(Actions): $A_w^i \subseteq A_w^o$ for each $w \in W^i$;

ID3(Feasible Sequences): $\Delta T_{D_i} \subseteq \prec^i$;

ID4(Player Assignment):

$$\pi^i(w) = \begin{cases} \pi^o(w) & \text{if } w \in W^{iD} \\ N^i & \text{if } w \in W^{iE}, \end{cases}$$

where $N^i := \{j \in N^o : j \in \pi^i(w) \text{ for some } w \in W^{iD}\}$;

ID5(Payoff Assignment): $h^i(w) = h_i^o(w)$ for all $w \in W^{iE}$;

ID6(Memory Function): \mathbf{m}^i is the perfect-information memory function \mathbf{m}^{PI} of player i for Π^i .

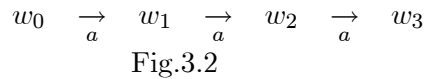
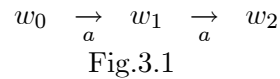
Note that since (Π^i, \mathbf{m}^i) is a personal view, it is required to satisfy the basic Axioms B1 and B2. The above definition is the same as the one in [8] except condition ID3. In [8], the corresponding condition requires equality, i.e., $\Delta T_{D_i} = \prec^i$. The same type of requirement was made in [7] for the extensive game version of an i.d.view. Nevertheless, we should discuss all of ID1 - ID6. These connect the candidate i.d.view to the original game Γ^o by making use of the minimum information conditions stated in M1, M2, and M3. Condition ID3 will be discussed after the others.

First of all, ID1 requires that the player uses only information pieces he finds in his memory kit. It follows from M1 and M3 that he distinguishes between the decision pieces and endpieces in his memory kit; thus, $W^{iD} \subseteq W^{oD}$ and $W^{iE} \subseteq W^{oE}$. Condition ID2 requires that only an objectively available action at w is available at w in the player's view. This is regarded as a consequence of M1, which requires that each player can find the full set of available actions at any of his decision pieces in his memory. In the formulations in [7] and [8], this condition is implied by some others.

Conditions ID4 and ID5 make use of M2 and M3 respectively to connect the player assignment at decision pieces and payoffs at endpieces in Π^i to those found in the objective protocol Π^o . Condition ID6 requires that the personal memory function \mathbf{m}^i is simply the perfect-information memory function \mathbf{m}^{PI} for player i in Π^i . We assume this condition since the view is in the mind of player i .

Once a personal view is specified with ID1, ID2 and ID3, the conditions ID4, ID5 and ID6 uniquely determine the player assignment, payoff and memory function. Hence, all questions about an i.d.view for a given memory kit can be answered by checking ID1 - ID3.

Now, consider condition ID3. A simple example shows the need for the weaker form of ID3 when memory is partial. Consider the recall-1 memory function in the 1-player (Π^o, \mathbf{m}^o) of Fig.3.1:



Recall-1 gives him the following memories: $\mathbf{m}_1^{R1}\langle w_0 \rangle = \langle w_0 \rangle$, $\mathbf{m}_1^{R1}\langle (w_0, a), w_1 \rangle = \langle (w_0, a), w_1 \rangle$, and $\mathbf{m}_1^{R1}\langle (w_0, a), (w_1, a), w_2 \rangle = \langle (w_1, a), w_2 \rangle$. In this protocol, the only closed domain is the full domain $D_1 = \Xi^o$. On this domain, his memory kit is

$$T_{D_1} = \{\langle w_0 \rangle, \langle (w_0, a), w_1 \rangle, \langle (w_1, a), w_2 \rangle\}.$$

While the reconstruction of a view from this memory kit might look straightforward, there is no i.d.view satisfying $\prec^i = \Delta T_{D_i}$. We have a difficulty with Axiom B2. Indeed, if there was an i.d.view with $\prec^i = \Delta T_{D_i}$, then w_1 would be a decision piece in \prec^i , but no feasible sequence in ΔT_{D_1} is an extension of $\langle (w_0, a), w_1 \rangle$, a violation of Axiom B2. To avoid this difficulty, the equivalence $\prec^i = \Delta T_{D_i}$ is weakened into $\Delta T_{D_i} \subseteq \prec^i$ in ID3.

It is easy to construct an i.d.view for this example. We simply extend the thread $\langle (w_0, a), w_1 \rangle$ by adding endpiece w_2 via action a to obtain the thread $\langle (w_0, a), (w_1, a), w_2 \rangle$. Adding this thread and its subsequences to ΔT_{D_1} , we have an i.d.view satisfying Axiom B2 and Axiom B1, which turns out to be the objective protocol Π^o . This procedure can be directly applied to the general case, which gives a simple proof of the existence of an i.d.view.

Theorem 3.1 (Existence of an i.d.view): There exists an i.d.view for each memory kit T_{D_i} obtained from any memory function on any closed domain D_i .

Proof. Define $F = \{\langle \xi, v \rangle \in T_{D_i} : \langle \xi, v \rangle \text{ is a maximal thread in } T_{D_i} \text{ and } v \in W^{oD}\}$. If F is empty, the following argument becomes trivial. For each $\langle \xi, v \rangle \in F$, we choose an action $a_{\langle \xi, v \rangle} \in A_v^o$. Define $A_F = \{a : a = a_{\langle \xi, v \rangle} \text{ for some } \langle \xi, v \rangle \in F\}$. Since D_i is closed, we have at least one $w^e \in W^{oE}$ in some thread in T_{D_i} . We extend T_{D_i} to T'_{D_i} as follows:

$$T'_{D_i} = T_{D_i} \cup \{\langle \xi, (v, a_{\langle \xi, v \rangle}), w^e \rangle : \langle \xi, v \rangle \in F\}. \quad (3.1)$$

This set T'_{D_i} is constructed so that every maximal feasible sequence ends with the endpiece w^e , i.e.,

$$\text{if } \langle \xi, v \rangle \text{ is a maximal feasible sequence in } T'_{D_i}, \text{ then } v \in W^{oE}. \quad (3.2)$$

When $F = \emptyset$, we have $T'_{D_i} = T_{D_i}$ and (3.2) holds.

We define the protocol $\Pi^i = (W^i, A^i, \prec^i)$ as follows: $W^i = \{w \in W^0 : w \text{ occurs in some sequence in } T_{D_i}\}$; $A^i = \{a \in A^0 : a \text{ occurs in some sequence in } T_{D_i}\} \cup A_F$; and $\prec^i = \Delta T'_{D_i}$. We use the information pieces occurring in T_{D_i} , the actions in T_{D_i} and newly added actions, and the set of the extended memory threads T'_{D_i} . Observe that by these definitions we ensure that ID1, ID2 and ID3 are satisfied. As remarked above, the player assignment, payoffs, and memory function are uniquely determined by ID4, ID5, and ID6.

It remains to show that this protocol is basic. By using $\prec^i = \Delta T'_{D_i}$, we have B1. Now consider Axiom B2. Let $\langle \xi, w \rangle \in \prec^i$ and $w \in W^{iD} \subseteq W^{oD}$. Then, if $\langle \xi, w \rangle$ is maximal in ΔT_{D_i} , it would be extended by (3.1) since $W^{iD} \subseteq W^{oD}$, so Axiom B2 is satisfied. Suppose that it is not maximal in ΔT_{D_i} . Then, $\langle \xi, w \rangle$ is a proper subsequence of some sequence $\langle \eta, v \rangle$ in ΔT_{D_i} with $v = w$ or $v \neq w$. In the first case, we can extend it by (3.1). In the second case, by B1 for $\Delta T'_{D_i}$, we have some extension $\langle \xi, (w, a), v \rangle$ in T'_{D_i} . Hence, there is some $\langle \eta, (w, a), v \rangle \in T'_{D_i} = \prec^i$. ■

The closedness of the domain of accumulation D_i was used in the proof of existence only to ensure that there is an endpiece w^e in the memory kit. It is essential for the existence result that the domain D_i contains at least one endpiece.

The method used in the above proof gives a quite parsimonious view. As we shall see shortly, this is just one among a great number of possible reconstructions. Consider the objective protocol of Fig.3.2 with the recall-1 memory function. The procedure in the proof constructs the protocol depicted in Fig.3.3, which violates Axiom N1 as it has two roots. The original protocol of Fig.3.2 is also an i.d.view and it can be obtained quite easily by concatenating memory threads. The multiplicity of i.d.views is an inevitable consequence of our weakening of ID3 to $\Delta T_{D_i} \subseteq \prec^i$.

$$\begin{array}{ccccccc} w_0 & \xrightarrow{a} & w_1 & \xrightarrow{a} & w_3 & & \\ & & & & & & \\ w_1 & \xrightarrow{a} & w_2 & \xrightarrow{a} & w_3 & & \end{array}$$

Fig.3.3

The multiplicity comes from various different ways of cutting and extending the memory threads in his memory kit. In fact, for each memory kit there are a countably infinite number of i.d.views. This can be seen by observing that once we have constructed one i.d.view, we can construct another by adding the same decision piece to the front of each maximal sequence in the view. This implies that great many supersets of ΔT_{D_i} will constitute i.d.views. Our next task is to find precisely what shapes they possibly take.

We say that a superset⁹ F of ΔT_{D_i} is *conservative* iff for each $\langle (w_1, a_1), \dots, (w_m, a_m), w_{m+1} \rangle \in F$, w_1, \dots, w_{m+1} occur in ΔT_{D_i} and $a_t \in A_{w_t}^o$ for $t = 1, \dots, m$. We have the following fact by ID1 and ID2 that \prec^i must be a conservative superset of ΔT_{D_i} .

Remark 3.1. If (Π^i, \mathbf{m}^i) is an i.d.view from T_{D_i} , then \prec^i is a conservative superset of ΔT_{D_i} .

Then, we have the following additional result.

Lemma 3.2. Let F be a conservative superset of ΔT_{D_i} . Then, there is at most one i.d.view from T_{D_i} with $\prec^i = F$.

Proof. Suppose that $(\Pi^i, \mathbf{m}^i) = ((W^i, A^i, \prec^i, \pi^i, h^i), \mathbf{m}^i)$ and $(\Pi'^i, \mathbf{m}'^i) = ((W'^i, A'^i, \prec'^i, \pi'^i, h'^i), \mathbf{m}'^i)$ are both i.d.views from T_{D_i} with $\prec^i = \prec'^i = F$. By IP3, $W^i = W'^i$ and $A^i = A'^i$. Since, $(W'^i, A'^i, \prec'^i) = (W^i, A^i, \prec^i)$, conditions ID4, ID5, and ID6 imply that $(\pi^i, h^i, \mathbf{m}^i) = (\pi'^i, h'^i, \mathbf{m}'^i)$. ■

⁹Strictly speaking, this set F need only to consist of the same sets of information pieces and actions as Π^o . It may contain longer feasible sequences than Π^o .

This result is in sharp contrast with Kaneko-Kline [7], where an i.d.view is defined in terms of an extensive game. There, we would meet another types of multiplicity caused by the hypothetical elements of nodes and branches. The use of an information protocol enables us to avoid this problem. The next theorem gives a necessary and sufficient condition for a conservative superset of ΔT_{D_i} to be an i.d.view. Essentially, condition (i) corresponds to Axiom B1 and condition (ii) to Axiom B2.

Theorem 3.3.(Conditions for an i.d.view): Let F be a conservative superset of ΔT_{D_i} . Then, there is an i.d.view $(\Pi^i, \mathbf{m}^i) = ((W^i, A^i, \prec^i, \pi^i, h^i), \mathbf{m}^i)$ from T_{D_i} with $\prec^i = F$ if and only if

(i): $F = \Delta F$;

(ii): $v \in W^{oE}$ for any maximal thread $\langle \xi, v \rangle \in F$.

Proof. (Only-if): Let $(\Pi^i, \mathbf{m}^i) = ((W^i, A^i, \prec^i, \pi^i, h^i), \mathbf{m}^i)$ be an i.d.view from T_{D_i} with $\prec^i = F$. Then (i) holds by Axiom B1. Consider (ii). Let $\langle \xi, v \rangle$ be a maximal thread in $F = \prec^i$. Since this is a basic protocol, $\langle \xi, v \rangle$ must be an endposition in Π^i . Hence $v \in W^{iE}$ and by ID1, $v \in W^{oE}$.

(If): Suppose that (i) and (ii) hold. Then we define the set of information pieces $W^i = \{w \in W^o : w \text{ occurs in } F\}$, the action set $A^i = \{a \in A^o : a \text{ occurs in } F\}$, and then $\prec^i = F$.

First, we show Axioms B1 and B2 for (W^i, A^i, \prec^i) . By (i), we have Axiom B1.

Consider Axiom B2. Let $\langle \xi, v \rangle \in \prec^i$ with $v \in W^{iD}$. Since F is conservative upon T_{D_i} , we have $v \in W^{oD}$. Thus, by (ii), there can be no maximal sequence in $\prec^i = F$ ending with v . Hence, \prec^i has a feasible sequence $\langle \eta, (v, c), w \rangle$ for some c and w so that this is a supersequence of $\langle \xi, (v, c), w \rangle$. By Axiom B1, $\langle \xi, (v, c), w \rangle$ is a feasible sequence. Thus, we have Axiom B2 for Π^i .

Next, we show that the conditions ID1 to ID5 are satisfied. The first part of ID1 follows from the supposition that F is conservative upon T_{D_i} . It follows from (ii) and B2 that $W^{iD} \subseteq W^{oD}$ and $W^{iE} \subseteq W^{oE}$. Condition ID2 follows from conservativeness. Condition ID3 follows from $F \supseteq \Delta T_{D_i}$.

Since ID1, ID2 and ID3 are satisfied, the remaining parts π^i, h^i , and \mathbf{m}^i are uniquely determined by ID4, ID5, and ID6. ■

By the above theorem, we have a direct way to check whether or not a conservative superset F of T_{D_i} will form an i.d.view.

As a final result for this section, we show that the general existence result can be extended, under some relatively weak additional condition, to obtain an i.d.view that satisfies Axioms N1, N2 and N3. The additional condition is that the domain D_i has at least one decision position with at least two actions. In the theorem, we weaken the closedness condition to the condition of the domain containing at least one endpiece which we already remarked is a sufficient condition for existence of an i.d.view. The

theorem will be proved in the Appendix.

Theorem 3.4 (Existence of a Full I.D.view): Let D_i be a domain of accumulation satisfying that $D_i \cap \Xi^{oE} \neq \emptyset$ and $|A_v^o| \geq 2$ for some $\langle \xi, v \rangle \in D_i \cap \Xi^{oD}$. There is an i.d.view from T_{D_i} that satisfies Axioms N1, N2 and N3.

The next example exemplifies Theorems 3.1 and 3.4.

Example 3.1. Consider the 1-person protocol of Example 2.1 with the recall-1 memory function $\mathbf{m}_1^o = \mathbf{m}_1^{R1}$. The protocol of Fig.2.1 itself is an i.d.view. A protocol obtained by the procedure of the proof of Theorem 3.1 is depicted in Fig.3.4, which lists up all the endpositions in Ξ^{1E} . This satisfies neither of Axioms N1, N2 and N3.

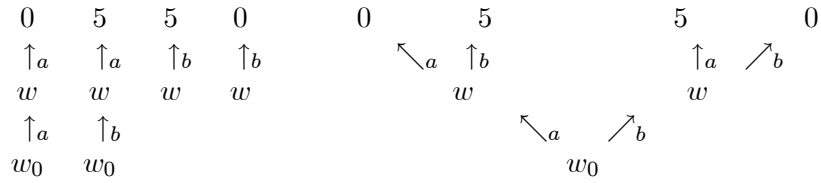


Fig.3.4

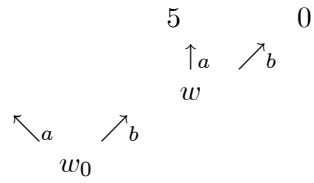


Fig.3.5

Fig.3.5 is obtained from Fig.3.4 by connecting the third and fourth w with w_0 via actions a and b , respectively. This is an i.d.view with Axioms B1-B2 and N1-N3, but it differs from the objective situation of Fig.3.1, which is itself another i.d.view satisfying all the axioms.

3.2. Minimal and Smallest Views

Theorem 3.1 claims the existence of an i.d.view for any given memory kit T_{D_i} . Nevertheless, our definition of an i.d.view allows us to have a countably infinite number of i.d.views. A player often has a different source of information in addition to T_{D_i} which he may use to discriminate between views. One source is his criterion of the economy of thought, i.e., to choose a small view. Here, we consider “smallness” of i.d.views. In Section 6, we will consider some other sources for discriminating between views.

We say that the i.d.view (Π^i, \mathbf{m}^i) is *smaller than* the i.d.view $(\Pi^{i'}, \mathbf{m}^{i'})$ iff $\prec^i \subseteq \prec^{i'}$. This notion is based on the idea of not using more sequences than what are needed. An i.d.view (Π^i, \mathbf{m}^i) is *minimal* iff no i.d.view $(\Pi^{i'}, \mathbf{m}^{i'})$ is strictly smaller than (Π^i, \mathbf{m}^i) . Since an i.d.view is finite, it follows from Theorem 3.1 that there exists a minimal i.d.view for any given memory kit T_{D_i} . An i.d.view (Π^i, \mathbf{m}^i) is the *smallest* iff it is smaller than every i.d.view. If the smallest view exists, it is unique, but as we shall see, the smallest view may not exist.

There are some clear-cut cases when we will have a smallest i.d.view. One is when we get an i.d.view where the set of feasible sequences $\prec^i = \Delta T_{D_i}$. We state this fact as

the next lemma.

Lemma 3.5. Let T_{D_i} be a memory kit, and $(\Pi^i, \mathbf{m}^i) = ((W^i, A^i, \prec^i, \pi^i, h^i), \mathbf{m}^i)$ an i.d.view from T_{D_i} . If $\prec^i = \Delta T_{D_i}$, then (Π^i, \mathbf{m}^i) is the smallest i.d.view for T_{D_i} .

A necessary and sufficient condition for the existence of such an i.d.view is given in the following corollary.

Corollary 3.6. Let T_{D_i} be a memory kit. There is an i.d.view for T_{D_i} with $\prec^i = \Delta T_{D_i}$ if and only if for any maximal thread $\langle \xi, v \rangle$ in T_{D_i} , the piece v appears in W^{oE} .

Proof. This follows from Theorem 3.3. Indeed, if there is an i.d.view for T_{D_i} with $\prec^i = \Delta T_{D_i}$, then condition (ii) of Theorem 3.3 is the latter statement. Conversely, if the latter holds, then by taking $F = \Delta T_{D_i}$ for Theorem 3.3, we have an i.d.view for T_{D_i} with $\prec^i = \Delta T_{D_i}$. ■

As mentioned earlier, Kaneko-Kline [7] and [8] focused on the self-scope perfect-recall memory function and used the strict definition $\prec^i = \Delta T_{D_i}$ for an i.d.view. In the present context, a perfect-recall memory function, which may have others in his scope, determines the smallest i.d.view.

Corollary 3.7. Let T_{D_i} be the memory kit of player i obtained from a perfect-recall memory function on a closed domain D_i . There is a smallest i.d.view from T_{D_i} and it satisfies $\prec^i = \Delta T_{D_i}$.

Proof. It suffices to show the if-part of Corollary 3.6 holds. Since the domain is closed, for each endposition $\langle \xi, v \rangle$ in Π^o , the value $\mathbf{m}_i^o \langle \xi, v \rangle = \langle \eta, w \rangle$ satisfies $w = v$. Also, since \mathbf{m}_i^o is perfect-recall, any memory thread in T_{D_i} is a subsequence of the value of \mathbf{m}_i^o at some endposition $\langle \xi, v \rangle$. Hence, for any maximal $\langle \eta, w \rangle \in T_{D_i}$, $w \in W^{oE}$. ■

Now we turn to examples where no view exists satisfying $\prec^i = \Delta T_{D_i}$. One is the example of one player objective view of Fig.3.1 with the recall-1 memory function. As we saw, in this example there is no i.d.view satisfying $\prec^i = \Delta T_{D_i}$. While there are a countably infinite number of other views for this memory kit, the view of Fig.3.1 is indeed the smallest one. On the other hand, the smallest view from the objective situation of Fig.3.1 with recall-1 is given as Fig.3.3.

When we consider partiality in memory in slightly more complicated examples, we would typically find multiple minimal views. These views themselves are small, but the problem is that a player might not be sure about which one to choose. Recall the recall-1 memory function on the objective protocol of Example 2.1. In this case, Fig.3.4 is a minimal view. However, we have another minimal i.d.view by changing the connection between w_0 and w , e.g., the first connection is replaced with the third one. Thus, the notion of smallness does not always resolve the multiplicity problem.

4. Reconstruction of a View from a Recall- k Memory Function

In this section, we explore the recall- k memory function of a player and the associated i.d.views. With this memory function, a player can recall only the last k information pieces and actions taken. If his memory ability is very weak, e.g., $k = 0$ or $k = 1$, then we might expect a great multiplicity of minimal i.d.views. However, as his ability gets stronger, the number of minimal i.d.views decreases. For large enough k , we know from Corollary 3.6 that there is a unique smallest view. First, we give some basic results for recall- k memory functions and focus on a particular i.d.view called the PR-view. Then, we exemplify the present considerations with the example of “Mike’s Bike Commuting” of Akiyama *et.al* [1].

Recall that the memory function of a player has a domain Y_i of positions including player i ’s positions $\Xi_i^o = \{\langle \xi, v \rangle \in \Xi^o : i \in \pi^o(v)\}$. We now fix the domain Y_i of a player i and also his domain of accumulation D_i . We are interested in how the i.d.views change when the player’s memory function increases from recall- k to higher levels of recall. We have the following result that as a player’s recall ability rises, the number of views he might derive declines.

Theorem 4.1 (Higher Recall Reduces Possibilities): Let D_i be a closed domain for player i and let T_{D_i} and T'_{D_i} denote the memory kits obtained from the recall k and recall k' memory functions respectively. If $k > k'$, then every i.d.view for T_{D_i} is an i.d.view for T'_{D_i} .

Proof. Let (Π^i, \mathbf{m}^i) be an i.d.view for T_{D_i} . We show that (Π^i, \mathbf{m}^i) is also an i.d.view for T'_{D_i} . Since $\prec^i \supseteq \Delta T_{D_i}$ by ID3 for T_{D_i} and $\Delta T_{D_i} \supseteq \Delta T'_{D_i}$ by $k > k'$, we have $\prec^i \supseteq \Delta T'_{D_i}$, i.e., ID3 for T'_{D_i} . Since D_i is closed, the set $\{w \in W^o : w \text{ occurs in } T_{D_i}\}$ coincides with the set $\{w \in W^o : w \text{ occurs in } T'_{D_i}\}$. Thus, ID1, ID2, ID4, ID5, and ID6 for T'_{D_i} follow directly from the corresponding conditions for T_{D_i} . ■

The converse may not hold. Indeed, for the recall-1 memory function defined on the protocol of Fig.2.1, the protocols of Fig.3.4 is a minimal view and also there are some others. However, for the recall-2 memory function, the view given by Fig.2.1 itself is the smallest one.

The smallest i.d.view for a perfect-recall memory function was given in Corollary 3.7. By Theorem 4.1, this view is also a view for any level of recall. To state this fact formally, we refer to this i.d.view as the *PR-view for D_i* denoted by $(\Pi^{RR}, \mathbf{m}^{PI})$, where it is defined by the set of feasible sequences $\Delta\{\langle \xi, v \rangle_i : \langle \xi, v \rangle \in D_i\}$.

Corollary 4.2 (PR-View is an i.d.view for any Recall- k Memory Function): Let the objective memory function \mathbf{m}_i^o be the recall- k memory function \mathbf{m}_i^{Rk} ($k \geq 0$) on a closed domain D_i for player i . The PR-view $(\Pi^{RP}, \mathbf{m}^{PI})$ for D_i is an i.d.view for T_{D_i} .

This guarantees that the PR-view for D_i is an i.d.view for any recall- k memory

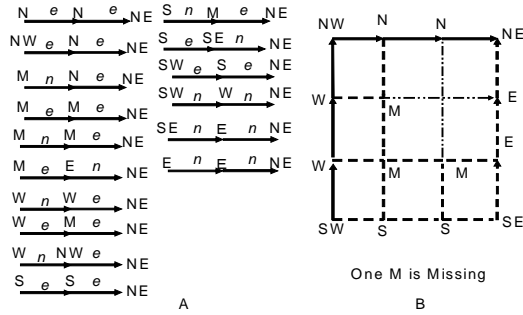


Figure 4.1: Various I.D.Views for Mike's Bike

function. It is one candidate view that we might concentrate on for a player with recall- k memory. However, the PR-view may not be a minimal view as we have seen for the case of Fig.2.1 with the recall-1 memory.

Mike's Bike Commuting: Let us return to the example of Mike's Bike with the full domain. Here, we consider the possible i.d.views when he has the memory function of recall- k for small k . Suppose that Mike has recall-1. First, the true map (Fig.1.3.A) and the larger one (Fig.1.3.B) are possible i.d.views. Now, there are several minimal views, which are obtained by the procedure given in the proof of Theorem 3.1: One is depicted in Fig.4.1.A. Each of these consists of 16 maximal sequences of length 3 to the endpoint NE . The differences are in attached actions at S, W , and M to NE . Each minimal view violates the non-basic axioms N1 and N3.

If we allow him a stronger memory, say recall- k but $k \leq 4$, then there is still a minimal i.d.view smaller than Fig.1.3.A. If he has memory function of recall-5 or higher (perfect-recall), then his smallest view is Fig.1.3.A.

Now, return to the case with recall-1. Let us restrict our attention to minimal views that satisfy N1-N3. Even with this restriction, we find that Fig.1.3.A is not yet a minimal one, since Fig.4.1.B is a strictly smaller view satisfying N1-N3, where one M is missing. In this case, however, recall-2 is enough to guarantee that Fig.1.3.A is the smallest view.

Thus we see that additional requirements (or information) often help the player obtain a better view. In this example, the root condition N1 looks natural for a player since he always starts from the root SW as his home.

5. Kuhn's Distinguishability Condition

In the theory of extensive games, Kuhn [11] gave a mathematical condition on information sets interpreted as “perfect recall”. In our context, his interpretation is of a memory function, but his mathematical condition is an attribute of information pieces. The condition is formulated as follows: We say that an information protocol Π satisfies the *distinguishability condition* for player i iff for any $\langle \xi, v \rangle, \langle \eta, w \rangle \in Y_i$,

$$\langle \xi, v \rangle_i \neq \langle \eta, w \rangle_i \text{ implies } v \neq w. \quad (5.1)$$

That is, when two positions have different personal histories up to Y_i , some difference must be expressed in the current pieces¹⁰. This means that the player can distinguish between the pieces. It does not mean that by looking at a current piece, he would be able to articulate the difference in the history, nor does it mean that he has perfect recall about what he has observed in the past.

Since the above is an important point in inductive game theory, we give one example showing how the condition might be satisfied, and then we will comment on why (5.1) should not be regarded as expressing “perfect recall”.

Consider the example of Mike's Bike with recall-1. Here, (5.1) is violated, since he receives the same piece at several lattice points. Let us change the example so that in addition to each information piece such as SW , S , W , etc., he receives the information of the distance from his home - - his bike has now the distance meter. In fact, we need to assume that the town is slightly skewed: For example, each horizontal block has evenly $1,000m$, and the length of each of the 3 vertical blocks, from the west end, is $1,000m$, $1,001m$, $1,004m$ and $1,013m$ (hence the total length from SW to NW is $3,000m$ but that from SE to NE is $3,039m$). At the northeast M through the history $\langle (SW, e), (S, n), (M, e), (M, n), M \rangle$, the distance meter indicates $4,005m$ ($= 1,000m + 1,001m + 1,000m + 1,004m$), that is, he receives the new information piece:

$$M \wedge (d = 4,005). \quad (5.2)$$

If he chooses a different path to the same lattice point, he now receives a different information piece. Thus, this example satisfies (5.1).

His memory of recall- k with $k \geq 1$ gives him some hints to the past history. In Fig.1.3.A, without a distance meter, the information M is too coarse and the hints in his recall-1 memory do not allow him to construct the true map as a smallest one. On the other hand, if his information pieces are distinguishable, then those hints in

¹⁰A reader may find some analogy between this idea and the Eve-hypothesis in the recent biological anthropology. That hypothesis is based on the assumption that some different anthropological histories inherited through women can be distinguished by some differences in their current mitochondria. See Mithen [12].

his recall-1 memory are enough. The fact holds generally up to the PR-view, which is stated in Theorem 5.1 and proved in the end of this section.

Theorem 5.1 (Smallest Under the Distinguishability Condition): Let Π^o satisfy the distinguishability condition for player i , and $\mathbf{m}_i^o = \mathbf{m}_i^{Rk}$ for $k \geq 1$. Let D_i be a closed domain of accumulation for player i . The PR-view $(\Pi^{PR}, \mathbf{m}^{PI})$ is the smallest i.d.view for T_{D_i} among the class of i.d.views that satisfy the distinguishability condition for player i .

In Mike’s Bike, if he collects his memories including the information from the distance meter over the whole town, then he may succeed to connect the links (or hints) in his recall-1 memory modules to construct a unique smallest map which is the actual map!

Now, we should consider condition (5.1) from the viewpoint of Kuhn’s interpretation of “perfect recall”. The example by (5.2) implies that (5.1) does not necessarily represent Kuhn’s interpretation. Then, consider the following example: In Mike’s Bike, we replace the information piece at each lattice point by the piece describing the complete history to it; for example, if Mike reaches the northeast M via his choices e at SW , n at the west S , e at the southwest M and n at the southeast M , then his new piece is

$$(SW, e) \wedge (S, n) \wedge (M, e) \wedge (M, n) \wedge M. \quad (5.3)$$

In this case, each piece contains all information about his previous moves, and this describes Kuhn’s interpretation of “perfect recall”.

Although (5.2) and (5.3) satisfy (5.1), they are entirely different. A faithful interpretation of (5.2) does not contain “perfect recall” at all, which implies that (5.1) does not have the specific meaning of “perfect recall”. The Example of (5.3) is only a possibility for (5.1). Strictly speaking, in Kuhn’s theory of extensive games, the information partitions induced by (5.2) and (5.3) are identical, i.e., (5.2) and (5.3) cannot be distinguished¹¹. This indistinguishability may explain why “perfect recall” has remained the dominant interpretation of Kuhn’s mathematical condition.

From our theoretical point of view, we regard (5.3) as an unnatural description in that a memory capability is not separated from information received: If one wants to describe “perfect recall” in our theory, it should be described by a memory function.

Before proving this theorem, we consider one more example to show how smallness is used in the theorem. Consider the 2-player protocol of Fig.5.1 which is a slight variant on the 1-player protocol of Fig.2.1. In the new objective description, player 1 moves at w_1 and w_2 and player 2 moves at the root piece w_0 . Also the payoffs are changed to satisfy distinguishability. Player 1 has the perfect-recall memory

¹¹One relevant remark is that when we talk about the equivalence between an information protocol and an extensive game such as in Kaneko-Kline [8], we should not forget the qualification for “equivalence up to the induced information partitions”: Otherwise, these theories are different.

function on his domain of accumulation $D_1 = Y_1 = \Xi_1^o$. Player 1's memory kit is $T_{D_1} = \{\langle (w_1, a), 5 \rangle, \langle (w_1, b), 0 \rangle, \langle (w_2, a), 1 \rangle, \langle (w_2, b), 2 \rangle\}$.

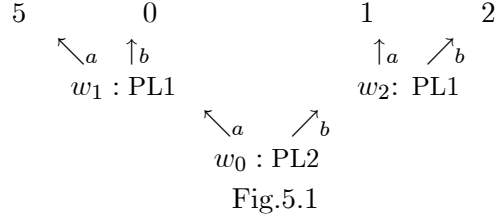


Fig.5.1

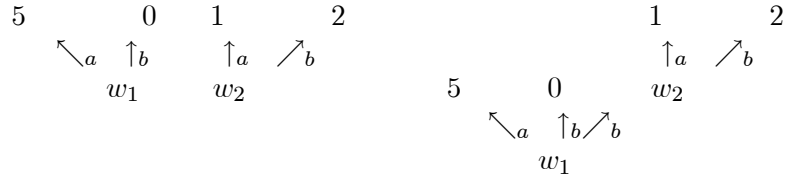


Fig.5.2

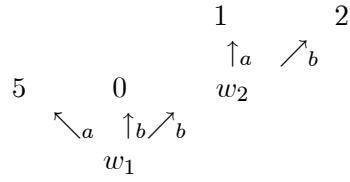


Fig.5.3

The PR-view is described in Fig.5.2. By Theorem 5.1, this view is the smallest among the class of i.d.views with distinguishability. Moreover, this view is the smallest even among the class with and without distinguishability. The personal view of Fig.5.3 is also an i.d.view for the same memory kit that satisfies Kuhn's distinguishability condition, but it is larger than the PR-view.

We will use the following lemmas in the proof of Theorem 5.1.

Lemma 5.2. Let (Γ^o, \mathbf{m}^o) be an objective situation, let D_i be a domain of player i , and let Π^{PR} be the PR-view for D_i . If Π^o satisfies the distinguishability condition for player i , then so does Π^{PR} .

Proof. Let $\langle \xi, v \rangle, \langle \eta, w \rangle \in \Xi^d$ with $\langle \xi, v \rangle \neq \langle \eta, w \rangle$. Since this is the PR-view, there must be two positions in the objective view $\langle \xi, v \rangle^o, \langle \eta, u \rangle^o \in \Xi^o$ such that $\langle \xi, v \rangle_i^o = \langle \xi, v \rangle$ and $\langle \eta, u \rangle_i^o = \langle \eta, u \rangle$. By distinguishability on Π^o and $\langle \xi, v \rangle_i^o \neq \langle \eta, w \rangle_i^o$, we have $u \neq v$. ■

Lemma 5.3. Let (Γ^o, \mathbf{m}^o) be an objective situation, let D_i be a domain of player i , and (Π^i, \mathbf{m}^i) a personal view of player i . Suppose the distinguishability condition on the set of all positions in Π^i . The function φ defined by $\varphi(\xi, v) = v$ for all $\langle \xi, v \rangle \in \Xi^i$ is a bijection to W^i .

Proof. By IP3 for Π^i , φ is a surjection. Let $\langle \xi, v \rangle, \langle \eta, w \rangle \in \Xi^i$ with $\langle \xi, v \rangle \neq \langle \eta, w \rangle$. By the distinguishability condition, we have $v \neq w$. ■

Proof of Theorem 5.1. Recall that D_i be a closed domain and $\mathbf{m}_i^o = \mathbf{m}_i^{Rk}$ for $k \geq 1$.

By Theorem 4.1, $(\Pi^{PR}, \mathbf{m}^{PI})$ is an i.d.view for T_{D_i} , and by Lemma 5.2, Π^{PR} satisfies the distinguishability condition.

Consider any i.d.view (Π^i, \mathbf{m}^i) for T_{D_i} satisfying the distinguishability condition. We will show that Π^d is smaller than Π^i .

Since Π^i and Π^{PR} both are i.d.views for T_{D_i} , it follows from condition ID1 that $W^i = W^{PR}$. By Lemma 5.3, there is exactly one position for each information piece. Now, we show by induction on the length of positions that each position $\langle \xi, v \rangle \in \Xi^{PR}$ is a subsequence of a position $\langle \eta, v \rangle \in \Xi^i$ for the same current piece v . From this and Lemma 2.1, it follows that $\prec^d = \Delta \Xi^{PR} \subseteq \Delta \Xi^i = \prec^i$.

For the base case, let $\langle \xi, v \rangle = \langle v \rangle \in \Xi^{PR}$, i.e., it is a position of length 1 in Ξ^{PR} . Since $W^i = W^{PR}$ is the set of pieces occurring in T_{D_i} , v occurs in \prec^i . There is one maximal sequence in Π^i including v . Then the initial segment $\langle \eta, v \rangle$ of it is a position in Π^i .

Next, let $\langle \xi, v \rangle \in \Xi^{PR}$ be a position of length $m > 1$, and suppose that any position from Ξ^{PR} of length less than m is a subsequence of some position in Ξ^i . We will show that $\langle \xi, v \rangle$ is a subsequence of a position $\langle \eta, v \rangle \in \Xi^i$.

Let $\langle \xi, v \rangle = \langle (w_1, a_1), \dots, (w_{m-1}, a_{m-1}), v \rangle$. Then by hypothesis, $\langle (w_1, a_1), \dots, (w_{m-2}, a_{m-2}), w_{m-1} \rangle$ is a subsequence of a position $\langle \eta_{m-1}, w_{m-1} \rangle$ in Ξ^i . Since player i has recall- k ($k \geq 1$) on a closed domain D_i , $\langle (w_{m-1}, a_{m-1}), v \rangle \in \Delta T_{D_i} \subseteq \prec^i$. By Lemma 5.3, $\langle \eta_{m-1}, (w_{m-1}, a_{m-1}), v \rangle$ must be a subsequence of $\langle \eta, v \rangle$. Since $\langle \xi, v \rangle$ is a subsequence of $\langle \eta_{m-1}, (w_{m-1}, a_{m-1}), v \rangle$, we have that $\langle \xi, v \rangle$ is a subsequence of $\langle \eta, v \rangle$ as desired. ■

6. Behavioral Uses of I.D.Views

After a player derives an i.d.view from his long-term memories, he uses the view for decision making. Kaneko-Kline [7] discussed behavioral uses of an i.d.view, while focussing on the self-scope perfect-recall memory function and small restricted domains of accumulation. These restrictions enabled the player to obtain a unique i.d.view and to succeed in making a subjective decision in his i.d.view. There were neither technical nor conceptual difficulties with these behavioral uses, though Nash equilibrium needed to be redefined in a more restricted manner. On the contrary, when memory involves partiality, we would meet some new difficult but essential problems in induction. These are discussed in this section.

6.1. Multiplicity of I.D.Views and its Possible Resolution

When the memory function is partial, there may be multiple i.d.views for a player even if he focuses on minimal i.d.views. Multiplicity of i.d.views could be a serious problem if they suggest different behavior. In this case, he may start looking for more clues

to discriminate between those views. Here, we consider how he might use his new experiences to reject or accept some views.

We assume that he keeps his regular behavior and makes new trials within the domain D_i of accumulation. His memory is now aided and expanded by his view Π^i : At a position $\langle \xi, w \rangle$ in (Π^o, \mathbf{m}^o) , he experiences his temporal memory $\mathbf{m}_i^o \langle \xi, w \rangle$ and considers its relation to his view Π^i . He tries to identify each of his experiences with a position in his subjective view Π^i . Also, he checks successive positions in Π^i with successive experiences. In this process of successive checking, he may find some incoherency between his view Π^i and experiences. If no such incoherency exists between them, he continues to keep (Π^i, \mathbf{m}^i) .

To describe this idea of successive checking, we define immediate successorship relations in Π^o and Π^i . We define the relation $\langle \xi, w \rangle <_a^{oI} \langle \eta, v \rangle$ in Π^o iff $\langle \eta, v \rangle$ is an immediate successor of $\langle \xi, w \rangle$ in D_i with the choice of action a at w . Likewise, $\langle \xi', w' \rangle <_a^{iI} \langle \eta', v' \rangle$ is defined in Π^i , in which case, $\langle \xi', w' \rangle$ is, directly, the immediate predecessor position of $\langle \eta', v' \rangle$ with the choice a at $\langle \xi', w' \rangle$.

We say that player i cannot *falsify* (Π^i, \mathbf{m}^i) with his experiences iff there is a function ψ from D_i to the set of positions Ξ^i in Π^i such that

F0: ψ is a surjection;

F1: for any $\langle \xi, w \rangle$ in D_i , if $\psi \langle \xi, w \rangle = \langle \eta, u \rangle$, then $w = u$;

F2: for any $\langle \xi, w \rangle, \langle \zeta, v \rangle$ in D_i , $\langle \xi, w \rangle <_a^{oI} \langle \zeta, v \rangle$ if and only if $\psi \langle \xi, w \rangle <_a^{iI} \psi \langle \zeta, v \rangle$.

The existence of the function ψ is required from the objective point of view, since player i does not know the structure of D_i . Nevertheless, conditions F0, F1 and F2 describe the stability of an i.d.view against player i having the ability of effectively falsifying Π^i by his experiences. If F0 is violated, then he could realize after some time that some position in Π^i never occurs. Condition F1 means that he identifies his currently received piece u with some position ending with u in Π^i . Condition F2 is the requirement of player i 's successive checking of his current and next positions in the objective Π^o and in his view Π^i .

The process of successive checking might go as follows. When he receives the first piece w in Π^o , he finds the minimal position $\langle w \rangle$ in Π^i . When he receives the next piece v after action a at w , he finds the immediate successor $\langle (w, a), v \rangle$ of $\langle w \rangle$ in Π^i . He continues this process, and if F0-F2 are satisfied, he finds no difficulties, and otherwise, he would find something wrong with his present view.

In Mike's bike without the distance meter he might construct the minimal view of Fig.4.1.A from his recall-1 memory on the full domain of Fig.1.3.A. Suppose that he follows his regular behavior and checks his view using his his new local memories in the objective situation. At the start SW , he has the local objective memory thread $\mathbf{m}_1^o \langle SW \rangle = \langle SW \rangle$. He checks his view of Fig.4.1.A and finds the corresponding position in his view. He then follows his regular behavior of heading north "n".

Next, he receives the information piece W and the local memory $m_1^o\langle(SW, n), W\rangle = \langle(SW, n), W\rangle$. He successfully finds the corresponding position in his view as an extension of his previous position. Following his regular behavior, he next obtains the memory $m_1^o\langle(SW, n), (W, n), W\rangle = \langle(W, n), W\rangle$. Now, he cannot find the corresponding position in his view that is an extension of his previous position. His view suggest that he should be at NE now, but he finds himself at W . Thus he doubts his view and move on to another one.

The next theorem states that under the assumption of Y_i -correctness on the memory function, the PR- view is the only i.d.view that cannot be falsified, which will be proved in the end of this subsection.

Theorem 6.1 (Falsification and the PR-View). Let D_i be a closed domain and let m_i^o be a Y_i -correct memory function, i.e., $m_i^o\langle\xi, w\rangle$ is a subsequence of $\langle\xi, w\rangle_i$ for all $\langle\xi, w\rangle \in D_i$. Let (Π^i, m^i) be an i.d.view from a memory kit T_{D_i} . Then (Π^i, m^i) cannot be falsified with experiences if and only if (Π^i, m^i) is the PR-view.

We now consider one important implication of the above theorem. Suppose that player i considers his possible i.d.views from his memory kit and proceeds in the following way:

P1: Player i enumerates his i.d.views $(\Pi^{i1}, m^{i1}), (\Pi^{i2}, m^{i2}), \dots$,¹²

P2: If he brings the i.d.view (Π^{ik}, m^{ik}) with him to the objective situation and finds some incoherency in it with experiences, then he brings the next view $(\Pi^{i,k+1}, m^{i,k+1})$.

If F0-F2 are able to be applied without errors, it would be a consequence of Theorem 6.1 that the above process will always terminate with the PR-view. Nevertheless, the process of falsification may have some difficulties and may fail.

As far as (Π^i, m^i) is an i.d.view from the memory kit T_{D_i} , we can find a function ψ satisfying the requirement F1. Hence, we restrict our attention to a function ψ satisfying F1. Thus, falsification itself is characterized by the negation of F0 or F2. Although the checking of each of F0 and F2 may take long time, the violation of F2 is clear-cut: While he has received two successive memory threads $m_i^o\langle\xi, w\rangle$ and $m_i^o\langle\zeta, v\rangle$ with action a at w , $\psi\langle\xi, w\rangle$ and $\psi\langle\zeta, v\rangle$ do not successively occur in Π^i . Falsification of F0 is uncertain and needs a decision as he may not be sure if he has waited long enough.

Trial-error has stochastic components, as described in Akiyama *et al* [1]. Therefore, after many repetitions of the situation, it is not completely certain for player i that some position in Π^{ik} will never happen. Here, he needs to make a doxastic decision (cf. Plato [13]) or a statistical decision to reject the present view (Π^{ik}, m^{ik}) . There may be two types of errors as in statistical inference (cf., Rohatig [14], p.708). A Type I error

¹²Note that he does not need to enumerate all of these views before this process. Instead, he needs only some algorithm to have a “next” candidate from the present one. Here, he is also successively checking each view.

occurs when player i waits for every position in Π^{ik} to occur and incorrectly does not reject the present (incorrect) view, and a Type II error occurs if he does not wait long enough for some position in $(\Pi^{ik}, \mathbf{m}^{ik})$ and incorrectly rejects the (correct) PR-view.

In Mike's bike, it will be almost immediate to reject Fig.1.3.A. But when Fig.1.3.B becomes a candidate in his mind, it would take quite some time to find that one M is missing. On the other hand, even when the true one (Fig.1.3.A) comes to his mind, he may reject it because some point may not occur to him and he cannot wait any longer.

Proof of Theorem 6.1. (*If*): Let (Π^i, \mathbf{m}^i) be the PR-view. Then, it is generated by the perfect-recall memory function $\mathbf{m}_i^o = \mathbf{m}_i^{PR}$ over the closed domain D_i . Then \prec^i is given as $\Delta\{\langle \xi, w \rangle_i : \langle \xi, w \rangle \in D_i\}$, equivalently, the set of positions in Π^i is $\Xi^i = \{\langle \xi, w \rangle_i : \langle \xi, w \rangle \in D_i\}$. We define ψ by

$$\psi\langle \xi, w \rangle = \langle \xi, w \rangle_i \text{ for all } \langle \xi, w \rangle \in D_i. \quad (6.1)$$

Then, ψ is a surjection, which is F0. Since $\mathbf{m}_i^o\langle \xi, w \rangle = \mathbf{m}_i^{PR}\langle \xi, w \rangle = \langle \xi, w \rangle_i$, we have condition F1. Finally, consider F2. Suppose that $\langle \xi, w \rangle, \langle \eta, v \rangle$ in D_i and $\langle \xi, w \rangle <_a^{oI} \langle \eta, v \rangle$. Then, $\langle \xi, w \rangle_i, \langle \eta, v \rangle_i$ are positions in Π^i and $\psi\langle \xi, w \rangle = \langle \xi, w \rangle_i <_a^{oi} \langle \eta, v \rangle_i = \psi\langle \eta, v \rangle$. The converse can be seen by tracing back this argument.

(*Only-If*): It is enough to show (6.1). We prove this assertion from the minimal sequences in D_i .

Let $\langle \xi, w \rangle$ be a minimal position in D_i , i.e., D_i has no proper initial segment of $\langle \xi, w \rangle$. Then, $\langle \xi, w \rangle_i = \langle w \rangle$ since D_i is closed (specifically, downward closedness). Thus, $\mathbf{m}_i^o\langle \xi, w \rangle = \langle w \rangle$ by Y_i -correctness. By F1, $\psi\langle \xi, w \rangle = \langle (w_1, a_1), \dots, (w_m, a_m), u \rangle$ satisfies $u = w$. Now, suppose that $m \geq 1$, i.e., $\psi\langle \xi, w \rangle \neq \langle w \rangle$. Since ψ is a surjection from D_i to the set of all positions Ξ^i of Π^i by F0, there is a $\langle \eta, v \rangle$ in D_i such that $\psi\langle \eta, v \rangle = \langle (w_1, a_1), \dots, (w_{m-1}, a_{m-1}), w_m \rangle$. Then, $\psi\langle \eta, v \rangle <_{a_m}^{iI} \psi\langle \xi, w \rangle$. Hence, by F2, we have $\langle \eta, v \rangle <_{a_m}^{oI} \langle \xi, w \rangle$, which contradicts the assumption that $\langle \xi, w \rangle$ is a minimal position in D_i . Hence, we have $\psi\langle \xi, w \rangle = \langle w \rangle = \langle \xi, w \rangle_i$.

Now, we suppose the inductive hypothesis that $\psi\langle \xi, w \rangle = \langle \xi, w \rangle_i$. Let $\langle \eta, v \rangle$ be the next position in D_i with action a at w . Thus, $\langle \xi, w \rangle <_a^{oI} \langle \eta, v \rangle$. By F2, we have $\psi\langle \xi, w \rangle <_a^{iI} \psi\langle \eta, v \rangle$. By the inductive hypothesis $\psi\langle \xi, w \rangle = \langle \xi, w \rangle_i$ and F1, we have $\psi\langle \eta, v \rangle = \langle \xi_i, (w, a), v \rangle = \langle \eta, v \rangle_i$, where ξ_i is the first part of $\langle \xi, w \rangle_i$. ■

6.2. Violations of N1-N3 and their Effects on Decision Making with a View

It is an implication from the discussion of Section 6.1 that it could take a long time to reach the PR-view or a player may even fail to reach it. If player i makes a doxastic decision that his PR-view is not falsified, it would be stable. Nevertheless, the PR-view or the one he settles on may neither be a full information protocol nor help his decision-making. In this section, we discuss these problems.

Now, suppose that player i finds an i.d.view (Π^i, m^i) by some method and decides to use it for his decision making. Then, this subjective view Π^i may violate Axioms N1-N3 even if it is the PR-view. We consider the problems arising from each violation:

Violation of N1(Root): The view has several trees;

Violation of N2(Determination): An exhaustive history does not determine a unique present information piece;

Violation of N3(History-Independent Extension): Some available actions stated in a piece are not in the view.

Since the violations are caused for different reasons, we should connect difficulties in decision making with the original situations causing the violations.

The violation of N1 may be caused by partial memory and the ignorance of another player. The violations in Fig.3.3 and Fig.3.4 are caused by partial memory, and that in Fig.5.2 is caused by the ignorance of player 2.

The violation of Axiom N2 comes also from the ignorance of another player. For example, in Fig.5.1, if $w_1 = w_2 = w$, then a protocol of Fig.6.1 is the smallest view for player 1 and it violates Axiom N2. In this case, this protocol is also the PR-view.

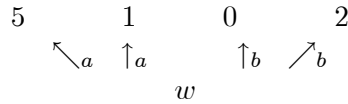


Fig.6.1

Finally, consider the causes of the violation of N3. The main reason is the partiality of the domain D_i and one extreme example is that D_i is the cane domain while each piece states the availability of multiple actions. The partiality of D_i was extensively discussed in Kaneko-Kline [7]. Note that the violation of N3 may also come from partiality in memory as in Fig.3.4.

Now, let us turn to the potential difficulty with his decision making by his view. The simplest case is the violation of N3: Even though an information piece tells him some actions are available, his view does not give consequences of those actions. In this case, the player should ignore those actions, and does not meet a serious problem with his decision making.

The violation of N2 is more serious as seen in Fig.6.1. Player 1 has a difficulty to make a choice of a or b . While the violation of N1 may appear to be of a similar nature, the analysis below will show that a violation of N1 creates no serious problem with decision making .

Recall that a strategy s_j in a personal protocol (Π^i, m^i) is defined by (2.13) and (2.10). Since (Π^i, m^i) is a subjective view, we use a different letter to denote a strategy. Let N^i be the player set of Π^i . Now, we denote a profile of strategies for N^i by $s =$

$(s_j)_{j \in N^i}$. Then, we say that a position $\langle \xi, (v_k, a_k), \dots, (v_m, a_m), v_{m+1} \rangle$ is *s-compatible* with a position $\langle \xi, v_k \rangle$ iff

$$s_j \langle \xi, (v_k, a_k), \dots, (v_{t-1}, a_{t-1}), v_t \rangle = a_t \text{ for all } t = k, \dots, m \text{ with } \pi^i(v_t) = j. \quad (6.2)$$

Axiom N2 guarantees that for each position $\langle \xi, v_k \rangle$ and each profile s , there is a unique *s-compatible* endposition.

Lemma 6.2 (Strategy-Determinancy). Let (Π^i, \mathbf{m}^i) be an i.d.view satisfying Axiom N2, and $s = (s_j)_{j \in N^i}$ a strategy profile. Then, any given position $\langle \xi, v_k \rangle$ uniquely determines an endposition which is *s-compatible* with $\langle \xi, v_k \rangle$.

Proof. It holds by Axiom N2 that for any $t = k, \dots, m$, $\langle \xi, (v_k, a_k), \dots, (v_{t-1}, a_{t-1}), v_t \rangle$ and a_t determine the unique v_{t+1} . For $t = m$, this statement determines the unique endposition. ■

For a position $\langle \xi, v \rangle$ and strategy profile $s = (s_j)_{j \in N^i}$, we define the *conditional payoff* $H_{i, \langle \xi, v \rangle}(s)$ to be the set of payoffs for player i given at the endpositions that are *s-compatible* with $\langle \xi, v \rangle$. In the example of Fig.6.1, $s_1(w) = a$ gives $H_{1, \langle w \rangle}(s) = \{5, 1\}$, and $s'_1(w) = b$ gives $H_{1, \langle w \rangle}(s') = \{0, 2\}$.

Suppose that s_{-i} is fixed. We say that a strategy s_i is *unambiguously optimal* at a position $\langle \xi, v \rangle$ iff for any strategy s'_i for player i ,

$$\alpha \in H_{i, \langle \xi, v \rangle}(s_i, s_{-i}) \text{ and } \alpha' \in H_{i, \langle \xi, v \rangle}(s'_i, s_{-i}) \text{ imply } \alpha \geq \alpha'. \quad (6.3)$$

We say that s_i is *unambiguously optimal* iff it is unambiguously optimal at *all* decision positions $\langle \xi, v \rangle$ for player i in Π^i . These are relative concepts to the given s_{-i} . In other words, at any decision position of player i , the worst payoff from his given strategy is at least as good as the best from any alternative. In the example of Fig.6.1, no strategy is unambiguously optimal. Nevertheless, we have a guarantee that such a strategy exists in any i.d.view (Π^i, \mathbf{m}^i) with Axiom N2.

Theorem 6.3 (Unambiguous Optimality with Axiom N2). Let (Π^i, \mathbf{m}^i) be an i.d.view that satisfies Axiom N2, and let s_{-i} be a profile of other players' strategies. Then, there is an unambiguously optimal strategy s_i for player i .

Proof. First, we construct a strategy $s_i = \{s_i \langle \xi, v \rangle : \langle \xi, v \rangle \in \Xi_i^{iD}\}$ by backward induction over the positions for player i . Then we prove that it is unambiguously optimal at each decision position $\langle \xi, v \rangle$ for him.

For the base case, let $\langle \xi, v \rangle \in \Xi_i^{iD}$ and suppose that there is no $\langle \xi', v' \rangle \in \Xi_i^{iD}$ such that $\langle \xi, v \rangle$ is a proper initial segment of $\langle \xi', v' \rangle$. By Lemma 6.2, the conditional payoff $H_{i, \langle \xi, v \rangle}(s'_i, s_{-i})$ is a singleton for any strategy s'_i . Hence, we can find an action $s_i \langle \xi, v \rangle$ that is locally optimal at $\langle \xi, v \rangle$. Since \mathbf{m}^i is the perfect information memory function \mathbf{m}^{PI} in Π^i , the action $s_i \langle \xi, v \rangle$ at $\langle \xi, v \rangle$ can be taken without any restriction by an action

at any other $\langle \xi', v' \rangle \in \Xi_i^{iD}$. Hence, we can choose $s_i \langle \xi, v \rangle$ to be unambiguously optimal at $\langle \xi, v \rangle \in \Xi_i^{iD}$.

Next, for the inductive step, consider any $\langle \xi, v \rangle \in \Xi_i^{iD}$. Let $\langle \xi^1, v^1 \rangle, \dots, \langle \xi^k, v^k \rangle \in \Xi_i^{iD}$ be the positions so that $\langle \xi, v \rangle$ is an immediate initial segment of each of them. Suppose that for each $t = 1, \dots, k$, $\{s_i \langle \xi'', v'' \rangle : \langle \xi^t, v^t \rangle\}$ is an initial segment of $\langle \xi'', v'' \rangle \in \Xi_i^{iD}$ is unambiguously optimal at $\langle \xi^t, v^t \rangle$. We then show that there is an action $s_i \langle \xi, v \rangle$ such that

$$\{s_i \langle \xi, v \rangle\} \cup \bigcup_t \{s_i \langle \xi'', v'' \rangle : \langle \xi^t, v^t \rangle\} \text{ is an initial segment of } \langle \xi'', v'' \rangle \in \Xi_i^{iD}$$

is unambiguously optimal at $\langle \xi, v \rangle$. If $\langle \xi^t, v^t \rangle$ is reached, the payoff is uniquely determined. Hence, we choose one action $s_i \langle \xi, v \rangle$ leading such a $\langle \xi^t, v^t \rangle$. The set is unambiguously optimal at $\langle \xi, v \rangle$.

The above inductive construction tells us that the constructive $s_i = \{s_i \langle \xi, v \rangle : \langle \xi, v \rangle \in \Xi_i^{iD}\}$ is unambiguously optimal over all $\langle \xi, v \rangle$ in Ξ_i^{iD} . ■

We remark that the theorem uses the fact that the subjective memory function \mathbf{m}^i is the perfect-information memory function. As mentioned earlier, since the player has this view in his mind, the perfect-information memory function makes sense. The violation of Axiom N2 still presents potential problems in this case.

If the player has a difficulty in decision making with his subjective view violating Axiom N2, he may try to overcome it in various ways. He might modify his view (such as in Theorem 3.4). Alternatively, he might use a weaker optimality criterion such as maximin optimality, i.e., he compares the worst payoffs compatible with each strategy, which gives always an optimal strategy even when N2 is violated. Another possibility is that he looks outside his memory kit for some source of this indeterminacy, e.g., the move of some other unobserved player.

We close this section by pointing out two problems when a player brings a subjectively optimal strategy to the objective situation.

First, the behavior suggested by his view may not be objectively optimal. As described in Section 6.1, once an i.d.view (Π^i, \mathbf{m}^i) is adopted, player i can compare it with his experiences. Let us see this comparison in Example 2.1 with the recall-1 memory function $\mathbf{m}_i^o = \mathbf{m}_i^{R1}$. Suppose that he adopts the i.d.view of Fig.3.5. Then, according to this view, he should receive payoff 0 after his moves (w_0, a) and (w, a) , but actually, he receives 5. Thus, an incorrect view may suggest an objectively non-optimal strategy as an optimal strategy.

Second, it may be necessary to bring his view in addition to the optimal strategy suggested by it. We continue with Example 2.1, but now suppose, for simplicity, that player 1's objective memory \mathbf{m}_1^o is the Markov memory function. Suppose that player 1 adopts the PR-view, i.e., the protocol of Fig.2.1 itself together with the perfect-

information memory function \mathbf{m}^{PI} . This view has the optimal strategies σ_1 and σ'_1 :

$$\begin{aligned}\sigma_1\langle w_0 \rangle &= a, \quad \sigma_1\langle (w_0, a), w \rangle = a \quad \text{and} \quad \sigma_1\langle (w_0, b), w \rangle = b \\ \sigma'_1\langle w_0 \rangle &= b, \quad \sigma'_1\langle (w_0, a), w \rangle = a \quad \text{and} \quad \sigma'_1\langle (w_0, b), w \rangle = b.\end{aligned}$$

The use of either strategy requires player 1 to memorize his first action in his view, since his objective memory function \mathbf{m}_1^o does not tell him which action he chose at the piece w . Thus, he needs to bring this view to complement the forgetfulness of his objective memory function.

7. Conclusions

7.1. Summary

Since our discourse is long having various steps, we give an overall summary by highlighting the main findings.

Highlight 1: In the definition in Kaneko-Kline [7] and [8], an inductively derived view is effectively the same as the memory kit. This paper generalized the definition of an inductively derived view to allow a view to have a larger set of feasible sequences than the accumulated memory kit - - ID3. This enabled us to consider partiality in the objective memory function \mathbf{m}_i^o .

Highlight 2: The generalized definition of an i.d.view allows general existence of an i.d.view. However, there are typically multiple i.d.views. On the one hand, multiplicity may be regarded as a cost in that the analysis has become much more complicated. On the other hand, multiplicity as well as generality leads us to a new frontier of inductive game theory.

Highlight 3: We have considered minimal/smallest i.d.views. Minimality avoids large redundant views. However, there may be multiple minimal views. When the memory function \mathbf{m}_i^o is subject to partiality, minimal views may not capture essential structures, since they may be too small.

Highlight 4: Even when the memory function is partial, a player may have some different ways to improve or correct his view. When Kuhn's distinguishability condition is satisfied, i.e., each information piece contains something to distinguish between different histories, he may reach the PR-view as the smallest view. However, Kuhn's distinguishability condition is a demanding requirement for an information piece, and also the player is required to have the ability to analyze the hints hidden in each piece. In this sense, the result is not necessarily regarded as a resolution of multiplicity.

Highlight 5: The next step is to check an i.d.view with new experiences in the objective situation. If he is fortunate, he may reach the PR-view and it becomes stable in the

sense that he does not notice any incoherency between his view and his experiences. However, it could take a long time to reach the PR-view or he might even reject it or fail to reach it.

Highlight 6: Even if he takes a view as stable, e.g., the PR-view, he might meet some difficulties in his decision making. This is caused by the violations of Axioms N1-N3 for his view. The violation of Axiom N2 is more serious than the others: As long as Axiom N2 is satisfied, he can use his view for his payoff maximization.

This paper has intended to develop the discourse on inductive game theory on individual experiences, inductive derivations of his views and their uses for decision-making/improving behavior in the objective situation. We already have many results on each step of the discourse. Nevertheless, there are still many open problems. For example, what happens with the considerations of the latter part of this paper when the objective memory function has more incorrect components. Some computer simulation studies may help the consideration of checking views with experiences in the objective situation.

7.2. Thinking of Other Players

As yet we have touched a lot of problems and difficulties with a player's own experiences, a view, and its use in the discourse of inductive game theory. A natural continuation is to study a player's thought on the activities of other players. In the literature of game theory, the origin/source of these thoughts has never been discussed. We look for the origin/source also in individual experiences. Since a player does not directly experience other players' subjective thinking and/or payoffs, he needs some way to get inside the heads of other players. Communication may help in this regard.

A more direct method is to have the experiences of others by stepping inside their shoes. Society has various social roles, and people take on different social roles from time to time throughout their lives. The concept of a "player" in a game may be regarded as a social role rather than a fixed individual. By switching roles, a player can gain experiences of others and even their views. Then, he may combine these experiences of different roles to obtain a better view of society including his and others' activities and thoughts.

This study may separate cooperative behavior from noncooperative behavior. When players switch roles regularly and share the same experiences, they learn each other's behavior, available actions and payoffs. This may allow each to extend his view. These extended views may or may not cause the players to change their behavior. In particular, if their domains are too narrow, they may find no better outcomes. However, if the domains include each other's deviations and the players find some beneficial joint activities, switching roles may facilitate cooperation.

We will discuss the above problem of experiential sources for other players' thoughts in a separate paper. Nevertheless, treatments of individual experiences as well as individual views discussed in this paper and in the other papers [7], [8] are basic for the new theory of other players' thoughts. In the present paper, we found that interactions between a player's view and his behavior is essential. For the experiential consideration of others' thoughts, we need to consider also interactions between various players' views and behavior. We anticipate these explorations will lead to many new insights on human behavior and thought in society.

8. Appendix

Proof of Theorem 3.4. Let us start with a sketch of the proof. Theorem 3.1 gives an i.d.view (Π^i, \mathbf{m}^i) from T_{D_i} . However, this i.d.view may violate Axioms N1, N2 and N3. In Step 1 of the proof we attack both N1 and N2 by attaching a decision piece with at least two actions at the beginning of each position. This guarantees N1, and we can guarantee N2 by using the same decision piece over again if necessary. In Step 2, we attack N3 by attaching an endpiece after each occurrence of a violation of N3.

Step 1: Extending the protocol to satisfy N1 and N2. Choose v satisfying $|A_v^o| \geq 2$ for some $\langle \xi, v \rangle \in D_i \cap \Xi^{oD}$. Let $A_v^o = \{a_{v1}, \dots, a_{vl}\}$. First, we enumerate the endpositions in Π^i as $\langle \xi_1, v_1 \rangle, \dots, \langle \xi_k, v_k \rangle$. We attach the root v before each of those positions, i.e., the first one is $\langle (v, a_{v1}), \xi_1, v_1 \rangle$, and the second one is $\langle (v, a_{v2}), \xi_2, v_2 \rangle$. However, when $k > l$, we meet a shortage of actions. Hence, at the last action a_{vl} , we attach v and continues the same process, which is depicted in Fig.8.1.

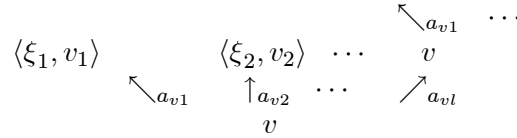


Fig.8.1

In general, we choose the natural number $r \geq 0$ so that $r(l-1) < k \leq r(l-1) + l$. We repeat the above process of last-action-continuation r times. If the number of endpositions k is not more than the number of actions in A_v^o , then $r = 0$. Extension from v is repeated only after the last action in A_v^o .

We write $(v, a_{vl})^t$ for the t times repetitions of v and the last action (v, a_{vl}) . Then, we extend $\langle \xi_t, v_t \rangle$ for $s(l-1) < t < s(l-1) + l$ and $s < r$ by

$$\langle (v, a_{vl})^s, (v, a_{v1}), \xi_1, v_1 \rangle, \dots, \langle (v, a_{vl})^s, (v, a_{v(l-1)}), \xi_t, v_t \rangle$$

and extend $\langle \xi_t, v_t \rangle$ for $r(l-1) < t \leq k$,

$$\langle (v, a_{vl})^r, (v, a_{v1}), \xi_1, v_1 \rangle, \dots, \langle (v, a_{vl})^r, (v, a_{v(k-r(l-1))}), \xi_t, v_t \rangle.$$

We denote, by Ξ , the set of all of these extended sequences.

Let Ξ be the set of endpositions obtained in this way and let $F = \Delta\Xi$. By Lemma 3.2, there is at most one i.d.view (Π^i, \mathbf{m}^i) from T_{D_i} with $\prec^i = F$ with the root v . By construction and Theorem 3.3, this set of endpositions is seen to satisfy Axioms B1 and B2. Finally, let us see that this satisfies Axiom N2. Indeed, consider two positions $\langle \xi, u \rangle$ and $\langle \xi, w \rangle$ in Π^i . If ξ consists of only the new additions, then it determines the same endposition, say, $\langle \xi_t, v_t \rangle$, i.e., $u = w$. If it consists of only some part of the new additions, then $u = w = v$. If it is longer than the new addition, $\langle \xi, u \rangle$ and $\langle \xi, w \rangle$ are the initial segment of the same endposition, i.e., $u = w$. Thus, Π^i satisfies N2.

Step 2: Extending the protocol to satisfy N3. Let (Π^i, \mathbf{m}^i) be an i.d.view for T_{D_i} satisfying N1 and N2. Now, we can choose some $\langle \xi, w^e \rangle \in D_i \cap \Xi^{oE}$, since $D_i \cap \Xi^{oE} \neq \emptyset$. Let Ξ'' denote the set of positions from Ξ' that violate N3, i.e., if $\langle \xi, w \rangle \in \Xi''$, then there is an action $a \in A_w^o$ such that for no $v' \in W'$, $\langle \xi, (w, a), v' \rangle$ is a position for in Ξ' . We cure this problem by adding the position $\langle \xi, (w, a), w^e \rangle$ to the set of positions in Ξ' . Each such new position will be an endposition. Violation of N3 is cured while preserving B1, B2 N1 and N2. Let Ξ^{*i} denote the set of positions obtained after this process. Let $\prec^{*i} = \Delta\Xi^{*i}$, $W^{*i} = W^i$ and $A^{*i} = A^i$. Then, Π^{*i} is a full information protocol. The player assignment, payoff assignment, and memory function function are uniquely determined by ID4, ID5, and ID6. This personal view is an i.d.view for T_{D_i} . ■

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