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An Analysis of Discrimination in Festival Games with Limited Access

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Abstract

This paper provides an analysis of discrimination and prejudices from the perspective of inductive game theory. We model ethnic interaction as the festival game, originally given by Kaneko-Matsui. We extend the festival game to include new constraints on observability of ethnic identities and on accessible locations for players. These components enable us to study more details of the problem of discrimination. We characterize the Nash equilibrium set. With this characterization, we see a variety of segregation patterns and discriminatory behaviors. Moreover, using the characterization, we introduce a measure of discrimination and use it to analyze discriminatory behaviors. Then we discuss our results by comparing them with some sociological and social psychological studies.

1. Introduction

1.1. Motivations for this paper

Discrimination and prejudices are widespread social phenomena and can be found in any human society in the world, though their degrees of seriousness vary. Among serious and well known ones, we find discrimination against the Blacks in the US and against the untouchables in India and some other countries. A less known instance is the discrimination against the Buraku people in Japan. In addition to these "racial"

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ones, we can also find other types of discrimination such as gender discrimination, political, economic and religious discrimination. Plenty of such instances are found all over the world in different dimensions (see Marger [16] for a comprehensive survey). This has been well recognized in social sciences such as sociology, social psychology and economics. However, more study is needed on the emergence and interrelation between discrimination and prejudices, particularly at a theoretical level.

In this paper, we will further develop the inductive game theory approach to discrimination and prejudices initiated by Kaneko-Matsui [13]. The approach has two salient features:

(1): A target social situation is formulated as a (non-cooperative) festival game, where people are identical except for their symbolic group identities. Still, Nash equilibrium typically allows discrimination between groups.

(2): The approach is based on inductive game theory, rather than classical *ex ante* game theory and evolutionary game theory. No player has *a priori* knowledge of the game structure, and their behaviors are based on some beliefs acquired from experiences playing in the game. Interpretations of such experiences may generate prejudices. Recently, Kaneko-Kline [11], [12] has been developing this theory more extensively.

This paper will add two structures to the festival game of Kaneko-Matsui [13]:

(a) limited observability of ethnicities;

(b) accessibility constraint on individual players.

The first gives a limited capability to distinguish some ethnicities from others. This enables us to discuss ethnic similarity and ethnic distance, which will be illustrated in Section 1.3. The second constrains a player's trials of going to other festivals. An extreme case is a very conservative society where every player is satisfied by the present festival and nobody tries to go to another festival. With the introduction of these two structures to the festival game, the theory will become much richer than that in Kaneko-Matsui [13].

With these limitations, a variety of discriminatory phenomena are observed in Nash equilibria. We will investigate the set of Nash equilibria in Sections 3 and 4. In Section 3, we conduct a general analysis of the set of Nash equilibria. In Section 4, a more specific study of the set of Nash equilibria is conducted under the condition that the players' choices of festivals are already given. This study reveals many patterns of discriminatory behaviors.

However, some of these patterns may include degrees of discrimination in excess of that needed to sustain a given segregation pattern. We will focus on the minimum degree of discrimination needed to sustain a given pattern of festival locations, and will define the measure of discrimination to be this degree. This measure gives various interesting findings, which will be discussed in Sections 5 and 6. In Section 7, we give a brief account of personal views of the social environment derived from subjective experiences of individuals, and discuss some form of prejudices which can emerge as part of such views.

1.2. Relations to the Economic Literature

In the literatures of sociology and social psychology, there are many studies of discrimination and prejudices. These are empirical, descriptive studies and/or (non-mathematical) theories (cf. Marger [16], Brown [3]). It would be better to have also some mathematical work to have better understanding of the mechanism of discrimination and prejudices in the sense of their emergence and interrelations. In the economics literature, we find several mathematical approaches to discrimination and prejudices other than the inductive game theory approach we adopt in this paper. It would help to understand our approach itself if we look at those literatures and contrast them with ours. In this section, we give a brief survey of these literatures.

We start with the race-preference approach given by Becker [2], which is the most direct application of neoclassical economics. Some prejudicial component of an individual is expressed as a parameter (decision variable) of his utility function. A criticism is that expressing prejudices in a utility function is, more or less, equivalent to assuming what to be explained. It describes some consequences of such a prejudicial component (see Chan-Eyster [4], for example), but we cannot address the question of how the mechanism of discrimination and prejudices works.

A more sophisticated approach is the statistical discrimination theory by Arrow [1] and Phelps [19] (see also Coate-Loury [5]). In this approach, groups of people have different statistical distributions of productivities. However, an employer only has information about the average productivity of a group. Thus, for employing a worker, he avoids the person from a group with a lower average productivity. Here, the concept of prejudice means that only statistical information is taken but individual differences are ignored. In this, real or perceived substantive differences are still assumed statistically between groups.

The last approach we mention is the cultural-discrimination theory given by Lang [14]. In this approach, discrimination is caused by a transaction cost incurred by different customs and languages used by different groups. The statistical discrimination theory may be regarded as a special case of this. Nevertheless, this theory is almost a mathematical translation of what sociologists have discussed in non-mathematical terms.

We admit that the above theories have merits for understanding of discrimination and prejudices to the extent that they allow us to consider their consequences. However, these theories suffer from assuming the existence of substantive (statistical or perceptional) differences in groups in that they do not help to understand the mechanism of discrimination and prejudices. In the literature of sociology and social psychology, it has been argued that discrimination and prejudices are, often, from symbolic sources, i.e., they can exist without substantive differences (cf. Brown [3]). Our approach is based on symbolic sources for discrimination in order to better understand the emergence and interrelations of discrimination and prejudices[1].

1.3. Inductive Game Theory Approach and Some Results

In this section, we describe the basic components of our approach. Since the basic development will take various steps, here we illustrate some results from Section 6.

As already mentioned in Section 1.1, we adopt the inductive game theory approach to discrimination and prejudices. In particular, we consider a non-cooperative game called the *festival game*. The festival game is played by a population of players who are symbolically differentiated with respect to ethnicities. The ethnicity of a player has no direct effect on available actions and payoffs.

The festival game has two stages: In the first, each player independently chooses a location from the available locations. In the second, each observes the configuration of ethnicities in the location, and then chooses his attitude – he is either friendly or unfriendly to the other players in the location. The payoff for him is the mood, i.e., the number of friendly players in his festival. Fig.1.1 illustrates a festival game with 3 locations and 3 ethnic groups. When all people are behaving friendly, the height of each of the 3 rectangles represents the payoff to each player in that location.

The above situation is considered from the viewpoint of inductive game theory. The game situation is repeated and the players follow some regular behaviors but make trials and errors once in a while. These trials and errors will give a player some knowledge about the social responses to his deviant choices. This supports Nash equilibrium, though players are limited in their trials by the accessibility constraint mentioned in Section 1.1. Here, we should emphasize that Nash equilibrium is not interpreted in the standard sense of *ex ante* rational decisions (Nash [18]).

A major point of this paper is to study discrimination and prejudices. However, we need a long technical development before reaching results we provide. It would be convenient for a reader to have an idea of such results without following the rigorous development. Here we illustrate two results given formally in Section 6.

In Fig.1.1, as stated above, the height of each rectangle represents the size of population at the festival, and also represents the mood at the festival when all players are friendly at their chosen festival. Thus, a player in a smaller rectangle has an incentive to move to a bigger rectangle. As we show later, Nash equilibrium states that when a player goes to a bigger festival without coethnic players, he would face discrimination

[1]The economics literature has also acknowledged the fact that symbolic information may have behavioral consequences (Schelling [20] and Matsui [17]).

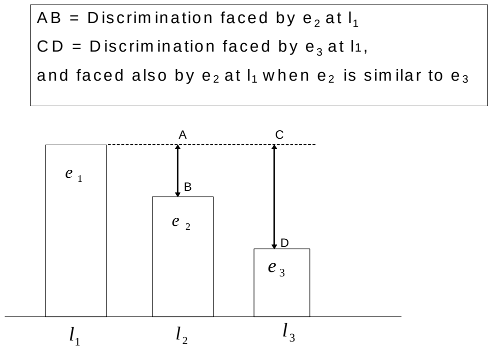

Figure 1.1: Ethnic Hierarchy

in the sense that some players at that festival change to the unfriendly action. These discriminatory responses remove the incentive to move.

One result from Section 6 is the effect of ethnic hierarchy on discrimination, and the other to be discussed in Section 5 is the effect of ethnic similarity on discrimination. The first result says that a smaller ethnic group will meet more discrimination than a larger ethnic group[2]. In Fig.1.1, anybody of ethnicity e_2 meets less discrimination, in going to location l_1, than anybody of e_3. On the other hand, the second result states that when some ethnicities are similar and cannot be distinguished by the people further away in ethnic distance, the former people are discriminated in the same way by the latter people. In Fig.1.1, when people of e_2 and e_3 cannot be distinguished by people of e_1, discrimination is the same for people of e_2 and e_3 in location l_1. Nevertheless, to keep the segregation pattern in this figure, people of e_2 and e_3 can distinguish between themselves, and people of e_2 discriminate against people of e_3.

2. Festival Game with Limited Access

We start Section 2.1 with an informal description of the festival game. In Section 2.2, a mathematical formulation of the festival game is given. In Section 2.3, we describe inductive game theory supporting our analysis of discrimination in terms of Nash equilibria.

[2] This appears not to fit some cases such as historical discrimination in South Africa. In our paper, the size of an ethnic group is interpreted as the relative group advantage. Thus, a numerically superior group like the Blacks may be less advantaged than a numerically inferior group like the Indians. See Marger [16].

2.1. Festival Game from the Inductive Viewpoint

The festival game is abstracted from social situations in order to study group formation and distribution of advantages over the groups formed. We emphasize that the people are only symbolically different and that these symbolic differences become the source for discrimination against minority groups. To focus on those, we drop the other socio-economic details including economic institutions. Here, we list main components of the festival game and their intended interpretations.

F1: *Many players of various ethnicities*: Since we are interested in social problems, the target situation involves many players, though mathematically the number of players is simply assumed to be a finite number. Since we are also interested in the emergence of discrimination, we minimize the difference among players, i.e., we consider an extreme case where the players are identical with respect to available actions and payoffs, except for symbolic differences called *ethnicities*. An ethnicity is attached with each player.

F2: *Ethnic groups*: By an ethnicity, we mean a commonly shared attribute like a race, a gender, an economic-political class, a religious group, a field in academia etc. Here, we assume no intrinsic differences in those ethnicities, that is, the players of the same ethnicity share no common values at the beginning. Instead, we would like to derive the spontaneous formation of such groups with shared values.

F3: *Locations*: There are several locations, from which each player chooses one location independently. Those locations have no *a priori* differences either. This notion is interpreted as a country, a firm, a club, a conference or a university.

F4: *Festivals*: A festival is a group of people choosing the same location. Locations should be distinguished from festivals; for example, a country in the geological sense is a location, but the people living there form a festival.

F5: *Ethnic configuration*: After each player comes to one location, he would observe the ethnic configuration of the location, which means the set of ethnicities present in the location. We assume that a player does not count the number of people at his location but observes only the ethnic configuration. This is due to the bounded cognitive abilities of players and the assumption of many players - F1.

F6: *Actions toward others*: After the observation of the ethnicity configuration at one's location, he chooses either a friendly action or an unfriendly one. The friendly action expresses cooperation with other people in his festival, with which he enjoys the festival, but the unfriendly action shows indifference to the others, with which he does not enjoy the festival.

F7: *Mood of a festival*: It represents a group advantage, relative to those of other festivals. Here, it is defined to be the number of friendly people at the festival. This group may include various ethnic people, but some ethnicities may not be there. Also, it may consist of people of only one ethnicity.[3]

Thus, the festival game is highly simplified by focusing only on ethnicities and aspects of group advantages and by eliminating other socio-economic aspects. On the other hand, these simplifications enable us to consider quite detailed structures of group formation together with possible discrimination occurring among ethnic groups. Again, we emphasize that no discrimination is included in the above description from F1-F7, but only symbolic differences exist between ethnicities.

The festival game up to this point was given in Kaneko-Matsui [13]. We introduce two additional structures, already mentioned in Section 1.1. The first one is limited observabilities of ethnicities. It is empirical evidence that "visibility ordinarily relates closely to the degree of inequality between the groups" (Marger [16], p.60). In fact, visibility depends upon ethnic groups, e.g., from the Whites' perspective, Blacks are all similar but from their perspective, some subgroups are still distinguishable. Limited observabilities of ethnicities are introduced to capture this kind of notion.

F8: *Limited observabilities of ethnicities*: Some ethnicities are similar or distinct for some other ethnic people. This is interpreted as reflecting also ethnic distances between ethnic groups. For similar ethnic groups, they themselves can distinguish between them, but cannot be distinguished by a group of some ethnic distance. In fact, the ethnic similarity and distance enable us to discuss various forms of discrimination, illustrated in Section 1.3.

The other additional structure is limited accessibilities to locations. To discuss this structure, we need to mention inductive game theory. Mathematically, the festival game is formulated as an extensive game Γ. Then, we apply Nash equilibrium to this game. The festival game together with Nash equilibrium describes the social situation of each point of time (or for some short period), and the entire situation is repeated. This background of our research is important for the use of the Nash equilibrium concept. Now, we enter some part of inductive game theory.

The entire situation is a recurrent situation of the festival game Γ:

$$\ldots \quad \Gamma \quad \Gamma \quad \Gamma \quad \ldots \tag{2.1}$$

We do not adopt the standard repeated game approach, where the situation is formulated as a large one-shot game and the Nash equilibrium concept is viewed as describing an *ex ante* prediction-decision. This requires the entire game structure to be known to the players, which is unsuitable to the problem we target[4]. Instead, in the inductive

[3] Although, mathematically, the mood is defined to be the number of friendly people, we interpret the mood as representing the advantage or status of the group relative to the others but not numerical superiority in some cases.

[4] See Kaneko [7] for the interpretation of the repeated game approach and its conceptual difficulty. For a more general discussion on the possible interpretations of Nash equilibrium, see Kaneko [9].

game approach, the starting assumption is that no player has *a priori* knowledge of the game structure. Experiences for each player by trial and error are the source for knowledge/belief of how he should behave. We assume that each player follows some regular behavior, i.e., the choice of a location and responses to presences of different ethnic people. Inductive game theory describes how players have constructed their beliefs about the situation.

Accordingly, we introduce limited accessible locations for player i from the location he goes regularly.

F9: *Limited accessible locations*: Recall that in the inductive interpretation mentioned above, the players have followed the regular behaviors. Then, each sometimes makes a trial deviation. Limited accessible locations restricts his trials. In this paper, such a limitation is exogenously given. It is interpreted as meaning that it may be difficult to experience all the locations, or some players may be lazy or may hesitate to go to some locations. Such limited experiences will be described by the set of accessible locations relative to the regular location configuration.

Alternatively, F9 can be interpreted as sociopolitical restrictions on mobility such as immigration barriers to countries or zoning laws within a country, e.g., the Hukou system of China. When F9 is regarded as an institutional arrangement, this may be interpreted as representing an intuitional (or active) discrimination. In this paper, we do not pursue this interpretation.

2.2. The Festival Game with Limited Observations and Limited Accessible Locations

The festival game Γ is played by n players $1, ..., n$ of various ethnicities. The set of players is denoted by $N = \{1, ..., n\}$ and the set of ethnicities is given as $\{e_1, ..., e_S\}$. Let $e(\cdot)$ be a function from N to $\{e_1, ..., e_S\}$. The value $e(i)$ is the *ethnicity* of player i. When $e = e(i) = e(j)$, we call i and j *coethnic players*. There are l_T locations available for festivals, to one of which each player will go.

Now, let us describe the structure of the festival game Γ. The game Γ has two stages:

The first stage (location choice): Each player i simultaneously chooses a location $f_i = l$ from the available locations $L_0 = \{l_1, ..., l_T\}$.

The second stage (choices of attitudes): Player i goes to location f_i, and observes the ethnicity configuration of location l. Using this observation, he chooses either *friendly action* 1 or *unfriendly action* 0.

Figure 2.1 gives an illustration of the above situation. There are three locations for festivals. The total number of players is 18, and one of three ethnicities is attached to each player. This has only 18 players, but our intention is to consider a large population for each ethnicity.

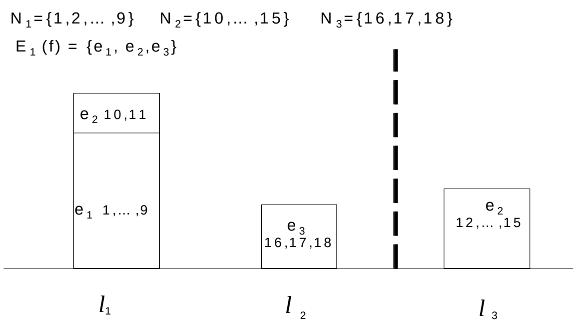

Figure 2.1: Location Choices and Ethnic Configurations

Now, let us give a formal representation of the festival game. The choice of player i in the first stage is denoted by $f_i \in L_0 := \{l_1, ..., l_T\}$. Thus, the choices of n players in the first stage are expressed by the vector $f = (f_1, ..., f_n)$, which we call a *location configuration*. In the second stage, player i observes the ethnicity configuration at location f_i, where we define the *ethnicity configuration* for player i by

$$E_i(f) = \{e(j) : f_j = f_i \text{ and } j \neq i\}. \tag{2.2}$$

That is, $E_i(f)$ is the set of ethnicities present at location f_i depending upon a location configuration $f = (f_1, ..., f_n)$. Also, we define, for $l \in L_0$,

$$E^l(f) = \{e(j) : f_j = l \text{ and } j \in N\}. \tag{2.3}$$

This is the ethnic configuration at location l from the objective view. When player i with $f_i = l$ is the only player of ethnicity $e(i)$ at location l, $E_i(f)$ differs from $E^l(f)$ because $e(i) \notin E_i(f)$ but $e(i) \in E^l(f)$.

In the second stage, player i can observe only the ethnicity configuration of location f_i for player i. This means that he neither identifies each individual player nor does he observe the number of players from each ethnicity at f_i. and as stated in F5 of Section 2.1, this assumption is due to bounded abilities of players.

In F8, we explained the additional structure called limited observabilities of ethnicities. This is formulated as follows: First, we introduce the binary relation $\sim_e$ $(e = e_1, ..., e_S)$ over the set of ethnicities $\{e_1, ..., e_S\}$. We assume, for simplicity, that $\sim_e$ is an *equivalence relation* over $\{e_1, ..., e_S\}$ for each e. The expression $e' \sim_e e''$ means that any player of ethnicity e does not distinguish between ethnicities e' and e''. For example, when the ethnic distances from e to e' and e'' are large, the players of e may treat e'

and e'' as the same ethnicities, but the players of e' and e'' may possibly distinguish between them.

In the example of Fig.2.1, suppose that ethnicities e_2 and e_3 are close and have a large ethnic distance from e_1. When the players of e_1 can distinguish between e_1 and e_2 and between e_1 and e_3 but cannot between e_2 and e_3, it holds that $e_1 \nsim_{e_1} e_2, e_1 \nsim_{e_1} e_3$ but $e_2 \sim_{e_1} e_3$. When the players of ethnicities e_2 and e_3 can distinguish between their ethnicities, it holds that $e_2 \nsim_{e_2} e_3$ and $e_2 \nsim_{e_3} e_3$.

We denote the equivalence class of $\sim_e$ including e' by $[e']_e := \{e'' : e'' \sim_e e'\}$. For any subset E of $\{e_1, ..., e_S\}$, we define the quotient set

$$E/\sim_e = \{[e']_e : e' \in E\}. \tag{2.4}$$

When player i of ethnicity e is in the festival with the ethnicity configuration $E = E_i(f)$, he effectively perceives $E/\sim_e$ since some ethnicities are treated as the same for him. From the viewpoint of game theory, what we are doing is an introduction of a coarser information partition common to the players of each ethnicity. This notion will be used for the definition of a strategy.

A *strategy* for player i of ethnicity e is given as a pair consisting of a location choice $f_i \in L_0$ and a function $r_i : 2^{\{e_1,...,e_S\}} \to \{0, 1\}$ satisfying that for all $E, E' \in 2^{\{e_1,...,e_S\}}$,

$$E/\sim_e = E'/\sim_e \text{ implies } r_i(E) = r_i(E'). \tag{2.5}$$

Here, r_i chooses friendly action $r_i(E) = 1$ or unfriendly action $r_i(E) = 0$ depending upon the ethnicity configuration $E = E_i(f)$ up to his perception of ethnicities. We will often denote a strategy (f_i, r_i) for i by σ_i. Let Σ_i be the set of all strategies for player i.

A *profile* of strategies for n players is denoted by $\sigma = (\sigma_1, ..., \sigma_n)$, where $\sigma_i \in \Sigma_i$ for all $i \in N$. We often write $\sigma = (\sigma_1, ..., \sigma_n)$ as $(f, r) = ((f_1, ..., f_n), (r_1, ..., r_n))$. We denote the set of strategy profiles by $\Sigma = \Sigma_1 \times ... \times \Sigma_n$.

When the players behave according to a profile of strategies $\sigma = (f, r) \in \Sigma$, the payoff to each player i is determined by the mood for him at f_i and his attitude determined by r_i. Under a strategy profile $\sigma = (f, r)$, the *mood* of festival l is defined as follows:

$$m_l(\sigma) = \sum_{f_j = l} r_j(E_j(f)). \tag{2.6}$$

Thus, the mood is the total number of friendly people at location l. Now we define the *payoff function* of player i by:

$$H_i(\sigma) = \begin{cases} m_l(\sigma) & \text{if } r_i(f) = 1 \\ \\ m_0 & \text{if } r_i(f) = 0, \end{cases} \tag{2.7}$$

where the *threshold utility* m_0 is a noninteger greater than 1. Definition (2.7) means that when he takes a friendly action, his payoff is the mood of his festival, but when he acts unfriendly, his payoff becomes the threshold utility m_0. The threshold m_0 may be interpreted as the utility from staying at home. The nonintegerness of m_0 is to avoid tie-situations between unfriendly and friendly actions. Beyond m_0, the choice of a friendly action is preferred to the choice of an unfriendly action.

In the example of Fig.2.1, when all the players in location l_1 take friendly actions, the mood would be 11, and is the payoff for each player there. If two players among them take unfriendly actions, the mood would be 9, but a player taking an unfriendly action would have threshold utility m_0.

In the rules of the festival game Γ, each player i is allowed to go to any location in $L_0 = \{l_1, ..., l_T\}$. As mentioned in F9, we restrict the location choice of player i to a nonempty subset L_i of L_0. As illustrated in (2.1), we are considering the repeated situation of the festival game Γ and a stationary state expressed as a strategy profile $\sigma = (f, r) = ((f_1, ..., f_n), (r_1, ..., r_n))$. The set L_i is interpreted as consisting of the locations that player i has experienced and is currently remembering. Since he goes regularly to the location f_i, we assume that

$$f_i \in L_i \text{ for all } i \in N \tag{2.8}$$

When (2.8) holds, we say that the location configuration $f = (f_1, ..., f_n)$ is *compatible* with an *accessibility structure* $\mathcal{L} = \{L_i\}_{i \in N}$. The point of the introduction of $\mathcal{L}$ is to limit the number of players who make trials. The following example shows a pattern of such limitations.

In the example of Fig.2.1, one possible accessibility structure is given as

$$\begin{aligned} L_1 &= ... = L_9 = \{l_1\} \\ L_{10} &= L_{11} = \{l_1, l_2, l_3\}, \ L_{12} = \{l_3, l_2\}, \text{ and } L_{13} = L_{14} = L_{15} = \{l_3\}, \\ L_{16} &= \{l_2, l_1, l_3\}, \text{ and } L_{17} = L_{18} = \{l_2\}. \end{aligned} \tag{2.9}$$

The players of ethnicity e_1 have no experiences of locations l_2 and l_3. Those of ethnicity e_2 have various types: Players 10 and 11, staying at l_1, have experiences of all locations, while only player 12 among the people staying at l_3 has explored another location l_2, but he has no effective experience of l_1. Player 16 is the only explorer among the players of ethnicity e_3.

Conceptually speaking, the accessibility structure $\mathcal{L} = \{L_i\}_{i \in N}$ is not included in the rules of the game Γ. Instead, it has been developed in the history. For the sake of convenience, however, we include $\mathcal{L}$ in the description of the festival game. That is, we will formally regard $\mathcal{L}$ as an additional structure, that is, we consider the festival game $\Gamma(\mathcal{L})$ with the accessibility structure $\mathcal{L}$.

With the introduction of $\mathcal{L}$, we should modify the strategy concept. An L_i*-strategy* for player i is given by a pair (f_i, r_i), where

$$f_i \in L_i \text{ and } r_i : 2^{\{e_1,...,e_S\}} \to \{0,1\}. \tag{2.10}$$

Recall that the first half is already given by (2.8). When $L_i = L_0$, this coincides with the definition of a strategy given above. The set of L_i-strategies for player i is denoted by $\Sigma_i(L_i)$. We say that $\sigma = (f, r)$ is an $\mathcal{L}$*-strategy profile* iff $\sigma_i = (f_i, r_i)$ is an L_i-strategy for all $i \in N$. By $\Sigma(\mathcal{L})$, we denote the set of profiles $\sigma = (f, r) = ((f_1, ..., f_n), (r_1, ..., r_n))$ with $\sigma_i = (f_i, r_i) \in \Sigma_i(L_i)$ for all $i \in N$. The payoff function H_i for player i over $\Sigma(\mathcal{L})$ is defined in the same manner as (2.7). Now, we have the festival game with limited access as the triple $\Gamma(\mathcal{L}) = \langle N, \{\Sigma_i(L_i)\}_{i\in N}, \{H_i\}_{i\in N}\rangle$.

We now define the Nash equilibrium concept in the festival game $\Gamma(\mathcal{L})$. We say that an $\mathcal{L}$-strategy profile σ is a *Nash equilibrium* in $\Gamma(\mathcal{L})$ iff for all $i \in N$,

$$H_i(\sigma) \geq H_i(\sigma_{-i}, \sigma'_i) \text{ for all } \sigma'_i \in \Sigma_i(L_i). \tag{2.11}$$

Here, we should repeat what was reported in Kaneko-Matsui [13]: We can replace (2.11) by maximization over a narrower class of strategies. That is, in a deviation, a response in the second stage does not need to take the ethnicity configuration into account.

Lemma 2.1. An $\mathcal{L}$-strategy profile σ is a Nash equilibrium in $\Gamma(\mathcal{L})$ if and only if for any $i \in N$,

$$H_i(\sigma) \geq H_i(\sigma_{-i}, (l, \delta_i)) \text{ for all } (l, \delta_i) \in L_i \times \{0,1\}. \tag{2.12}$$

Here δ_i is regarded as a constant function over $2^{\{e_1,...,e_S\}}$ taking a value from $\{0,1\}$.

Proof. First consider a Nash equilibrium σ defined by (2.11). Since the individual strategy domain in (2.11) includes that in (2.12). Hence, (2.11) implies (2.12).

We now prove that if $\sigma = (f, r)$ satisfies (2.12), then it is a Nash equilibrium. Suppose that player i takes a new strategy $\sigma'_i = (l, r'_i) \in \Sigma_i(L_i)$. Player i moves to a location l and observes the ethnicity configuration $E_i(f_{-i}, l) = E^l(f)$. In the festival l, his action is $r'_i(E_i(f_{-i}, l))$. In this case, his payoff is determined by the action $r'_i(E_i(f_{-i}, l))$; the functional structure of r'_i is irrelevant but the function value is relevant. Let $\delta_i = r'_i(E_i(f_{-i}, l))$. Then $(l, \delta_i) \in L_i \times \{0,1\}$ plays the same role as $\sigma'_i = (l, r'_i)$, and gives the same payoff. Thus, the domain $\Sigma_i(L_i)$ of player i's controllable variable in (2.11) is restricted to $L_i \times \{0,1\}$. ■

In the subsequent arguments, we will use (2.12) for the definition of Nash equilibrium. Nevertheless, we note that σ_i itself does not necessarily belong to $L_i \times \{0,1\}$. Simple behavior of taking actions is enough for deviations, but the responsive structure described in $r = (r_1, ..., r_n)$ is essential to attain the stability of a Nash equilibrium against deviators. This responsive structure may include discriminatory behavior.

We also make one more assumption on a Nash equilibrium $\sigma = (f, r)$ to avoid unnecessary complications:

Assumption M (Multiple Coethnic Players): If $f_i = l$ for some player $i \in N$, then there is a coethnic player j at location l, i.e., $f_j = l$ and $e(j) = e(i)$.

When there is a single player of ethnicity e at location l, we should take his response to the presence of a coethnic player, since he notices it because of (2.2). This will cause more considerations. However, it is a singular case for our theory, since, as stated by F1, it is intended to consider a society with many player. Thus, we will always assume Assumption M for a Nash equilibrium in the subsequent analysis.

2.3. Inductive Interpretation of Nash Equilibrium in $\Gamma(\mathcal{L})$

We adopt the Nash equilibrium concept, whose mathematical formulation is, more or less, standard (cf. Luce-Raiffa [15]). Nevertheless, we interpret Nash equilibrium from the inductive game theoretic viewpoint, which is needed for the problem of discrimination and prejudices. Here, we give a brief description of inductive game theory due to Kaneko-Matusi [13] and Kaneko-Kline [11].

Inductive game theory consists of four stages:

(i) experimentation and transformation of short-term memories into long-term memories;

(ii) inductive derivation of a personal view from the long-term memories;

(iii) use of a personal view for his own decision making; and

(iv) bringing his decision from (iii) back to (i).

As discussed in [11], a strategy profile which is stationary through those stages is a Nash equilibrium. In this paper, those four stages are compressed and only the resulting Nash equilibrium is considered. Still, some basic ideas of these stages are relevant for an understanding of what we are doing in this paper. Therefore, we give a very brief account of the relevant part.

In the recurrent situation described in (2.1), a strategy profile $\sigma = (f, r)$ is temporarily adopted by the players. Players do not know the structure of the game, but behave following their behavior patterns. They make some deviations as trial and error to get some information about the responses of other people. We assume that only a small portion of players make trial deviations at one time; specifically, we consider only unilateral deviations. Then, player i makes trial deviations on the locations in L_i. Thus, the memories from experiences are

$$\mathcal{E}_i(\sigma) = \{[(l, \delta_i), E_i(f), H_i(\sigma_{-i}, (l, \delta_i))] : l \in L_i \text{ and } \delta_i = 0, 1\}. \tag{2.13}$$

The values $E_i(f)$ and $H_i(\sigma_{-i}, (l, \delta_i))$ are listed, but they are still unknown to player i as functions.

Having the set of experiences $\mathcal{E}_i(\sigma)$, player i finds a causal relationship (or correlation) from (l, δ_i) to payoff $H_i(\sigma_{-i}, (l, \delta_i))$. Then, he chooses a (l, δ_i) from $L_i \times \{0, 1\}$ to maximize $H_i(\sigma_{-i}, (l, \delta_i))$. He modifies the corresponding part of his behavior σ_i with (l, δ_i). Now, he has the modified strategy σ_i' and brings it to the recurrent situation in (2.1). This corresponds to the stages (iii) and (iv) above. Then, the stage (i) restarts again.

A strategy profile σ which is stationary through the above revision process for all players is a Nash equilibrium in $\Gamma(\mathcal{L})$. This was already discussed in Kaneko-Matsui [13], and an inductive derivation of a personal view from $\mathcal{E}_i(\sigma)$ was the main subject of [13]. Kaneko-Kline [11] gave a full scenario of stages (i)-(iv) in a general context of extensive games.

In this paper, instead of the above described process, we start with Nash equilibrium for the festival game $\Gamma(\mathcal{L})$ with limited access. It is important to emphasize that the foundation of our approach differs from the *ex ante* game theory[5]. In the *ex ante* game theory, the game structure is assumed to be (typically, commonly) known to all players, and each player makes a rational decision before the actual play of the game. This approach is unsuitable to our target phenomena of discrimination and prejudices, since these are emerging as part of social custom and practices and are not purely from rational decisions.

The limited experiences of players prevent us from applying the concept of *subgame perfect equilibrium* introduced in Selten [21]. The relevant subgame perfection in the festival game requires payoff maximization in response to a visiting player from some other location. However, this needs a further deviation of each responder, which must have a small frequency conditional upon the original trial deviation of a visiting player. This needs too many repetitions of the game. See Kaneko-Matsui [13] for a detailed argument. They also considered subgame perfection from the viewpoint of an inductively derived view (Section 4.3 in [13]).

3. Nash Equilibria of the Festival Game $\Gamma(\mathcal{L})$ with Limited Access

In this and next sections, we analyze the set of Nash equilibria in the festival game $\Gamma(\mathcal{L})$. Here, we give a general characterization of the equilibrium set, which states that there are a great variety of location choices by players. Then, in Section 4, we fix a location configuration and study the set of equilibria relative to the location configuration. This will be used for the introduction of a measure of discrimination in Section 5.

[5] Our inductive game scenario differs from evolutionary game theory (cf. Weibull [22]) in that each player makes a decision and keeps an identity over periods in our theory, while in evolution game theory, a behavior pattern (strategy) is identical to a player and a change of a behavior is a change of a generation. In evolutionary game theory, "memory" remains and/or is accumulated in the distribution of behavior patterns; but in our theory, "memory" remains in the mind of a player.

3.1. Active Festivals and Outliers

The purpose of this section is to classify the festivals into active and inactive festivals. In a Nash equilibrium, some festivals may remain totally inactive. In the subsequent sections, we will focus only on Nash equilibria with active festivals, ignoring inactive festivals. Here, we clarify this assumption.

Let $\sigma = ((f_1, r_1), ..., (f_n, r_n))$ be a strategy profile in $\Gamma(\mathcal{L})$. We say that $l \in L_0$ is *active* iff $f_i = l$ and $r_i(E_i(f)) = 1$ for some $i \in N$. Hence, location l is *inactive* if and only if no players take friendly actions at l. Notice that the definition of an active location is not parallel to that of an inactive location, that is, in an active location, some players may possibly take unfriendly actions. However, this is not the case in equilibrium.

Lemma 3.1 (Polarization into Active and Inactive Festivals): Let $\sigma = ((f_1, r_1),$ $..., (f_n, r_n))$ be a Nash equilibrium in $\Gamma(\mathcal{L})$. Then

(1): if $r_i(E_i(f)) = 1$ for some $i \in N$, then $r_j(E_j(f)) = 1$ for all j with $f_j = f_i$;

(2): if $r_i(E_i(f)) = 0$ for some $i \in N$, then $r_j(E_j(f)) = 0$ for all j with $f_j = f_i$.

Proof. Let $r_i(E_i(f)) = 1$ for some $i \in N$. Then, $H_i(\sigma) = m_l(\sigma) \neq m_0$ since $m_l(\sigma)$ is an integer but the threshold m_0 is not. If $H_i(\sigma) = m_l(\sigma) < m_0$, then player i would get m_0 by switching to unfriendly action 0, which is impossible since σ is a Nash equilibrium. Hence, $H_i(\sigma) = m_l(\sigma) > m_0$. This implies that every player at l should take friendly action 1. Thus, we have proved (1), which implies (2). ■

In equilibrium, we have polarization into fully *active* festivals and *inactive* festivals; everybody at an active festival takes a friendly action, and everybody at an inactive festival takes an unfriendly action. In the latter, possibly an enough number of players are staying there, but all are keeping to unfriendly actions. We call the players in inactive festivals *outliers*. Outliers may be interpreted as "dropouts" or "homeless people" who are living in small tents, say, in river banks in Tokyo or in streets of Kolkata. They are isolated from the active festivals.

After Section 3, we will consider Nash equilibria without outliers. But since the behaviors of outliers are somewhat interesting, we will characterize the set of Nash equilibria taking them into account.

When $L_i = L_0 = \{l_1, ..., l_T\}$ for all $i \in N$, it is shown in Kaneko-Matsui [13] that all coethnic players go to a single location as far as there are active festivals involving those players. Only it is possible in some Nash equilibria that coethnic players scatter over locations having inactive festivals. In this paper, since we have a restriction on accessible locations $\mathcal{L}$, some coethnic players may be separated into different active festivals.

Theorem 3.2 (Avoidance of Outliers): Let $\sigma = ((f_1, r_1), ..., (f_n, r_n))$ be a Nash equilibrium in $\Gamma(\mathcal{L})$. Let i be an outlier, and l an active festival.

(1): If outlier i has a coethnic player in location l, then i cannot access l, i.e., $l \notin L_i$.

(2): Suppose $l \in L_i$. Then outlier i has no coethnic players in l, and the mood $m_l(\sigma_{-i}, (l, 1))$ of l induced by the presence of outlier i is lower than the threshold utility m_0.

These state that either outlier i cannot access location l with coethnic players, or he can access location l without coethnic players but if he actually goes, he would be punished and experience even a lower utility than the threshold utility.

Proof. **(1)**: Since l is an active festival and i an outlier, we have $f_i \neq l$ and $r_i(E_i(f)) = 0$ by Lemma 3.1. Thus, $H_i(\sigma) = m_0$. Since l is active, $m_l(\sigma) > m_0$. Let i have a coethnic player at l. If $l \in L_i$, then i can move to l to get a payoff $H_i(\sigma_{-i}, (l, 1)) = m_l(\sigma) + 1 > m_0 = H_i(\sigma)$. Since σ is a Nash equilibrium, we have $l \notin L_i$.

(2): Since $l \in L_i$, by (1), l cannot have a player coethnic to i. Thus if i moves to l, his presence will be observed. This will induce a decrease in the mood at l and $H_i(\sigma_{-i}, (l, 1)) = m_l(\sigma_{-i}, (l, 1))$. Since σ is a Nash equilibrium, $H_i(\sigma_{-i}, (l, 1)) \leq H_i(\sigma) = m_0$. Since $m_l(\sigma_{-i}, (l, 1))$ is an integer and m_0 is not, $m_l(\sigma_{-i}, (l, 1)) < m_0$. ■

3.2. Characterization of Nash equilibria in $\Gamma(\mathcal{L})$

When $L_i = L_0$ for all $i \in N$, the set of Nash equilibria is fully characterized in Kaneko-Matsui [13]. In this paper, we have additional structures of limited observabilities $\{\sim_e\}_e$ and limited accessible locations $\mathcal{L} = \{L_i\}_{i \in N}$. With these limitations, we find new phenomena in equilibrium. For example, some Nash equilibria allow the separation of coethnic players into various locations. To investigate such phenomena, we introduce two binary relations over locations, by which we will have a full characterization of Nash equilibria (satisfying Assumption M). In this characterization, the role of limited observabilities $\{\sim_e\}_e$ is covert, but it will be clear later.

For simplicity, we make Assumption M throughout the subsequent analysis.

Consider a location l and a strategy profile $\sigma = ((f_1, r_1), ..., (f_n, r_n))$, satisfying Assumption M, with $f_i = l$ for some i with $e(i) = e$, and also consider another location l'. Then we have two cases:

A: no players of ethnicity e at l can access l';

B: some player i with $e(i) = e$ at l can access l'; and either

B1: some coethnic player j is already at l'; or

B2: no players of ethnicity e are at l'.

Case A is irrelevant for the consideration of Nash equilibrium. Case B needs to be considered and is divided into two subcases. Subcases B1 and B2 should be treated in different manners, since the presence of a player i at l' is not observed in B1, while it is in B2.

For any $e \in \{e_1, ..., e_S\}$ and any two *distinct* $l, l' \in L_0 = \{l_1, ..., l_T\}$, B, B1 and B2 are formulated as follows:

$\hat{\mathrm{B}}$: $f_i = l$ and $l' \in L_i$ for some i with $e(i) = e$;

$\hat{\mathrm{B}}1$: $f_j = l'$ for some coethnic j of i;

$\hat{\mathrm{B}}2$: player i has no coethnic player in festival l'.

These are simply mathematical expressions of B, B1 and B2. We write $l \overset{e}{\rightarrowtail} l'$ iff $\hat{\mathrm{B}}$ and $\hat{\mathrm{B}}1$ hold, and $l \overset{e}{\rightsquigarrow} l'$ iff $\hat{\mathrm{B}}$ and $\hat{\mathrm{B}}2$ hold. Also, we write $l \rightarrowtail l'$ (respectively, $l \rightsquigarrow l'$) when $l \overset{e}{\rightarrowtail} l'$ ($l \overset{e}{\rightsquigarrow} l'$) for some $e \in \{e_1, ..., e_S\}$.

When $l \overset{e}{\rightarrowtail} l'$, player i with $e(i) = e$ can go to l' without being observed by the players at l'. On the other hand, when $l \overset{e}{\rightsquigarrow} l'$, the presence of player i may be noticed by the players at location l. Note that these relations are relative to a location configuration $f = (f_1, ..., f_n)$.

In Fig.3.1.(1), we assume that the players at l_1 can access l_2, but not the other direction, which is expressed by the wall (dotted line) between l_1 and l_2. Here we have $l_1 \overset{e_2}{\rightarrowtail} l_2$. On the other hand, since no players of e_1 are at l_2, the relation $l_1 \overset{e_1}{\rightsquigarrow} l_2$ holds. We interpret the wall as the immigration barrier to prohibit the players at location l_2 from coming to location l_1. Then, $l_1 \notin L_j$ for any j with $f_j = l_2$, for which reason, the relation $l_2 \overset{e_2}{\rightarrowtail} l_1$ does not hold.

In Fig.3.1.(2), we assume that the immigration barrier is taken away but players of e_2' at l_1 and players of e_2 at l_2 can distinguish between each other. In this case, the relations $l_1 \overset{e_2'}{\rightsquigarrow} l_2$ and $l_2 \overset{e_2}{\rightsquigarrow} l_1$ hold. It may still be possible that the people of e_1 cannot distinguish between e_2' and e_2. For example, people of e_2' are Indians living in the US and people of e_2 are Indians from India. In fact, this is described by limited observabilities $\{\sim_e\}_e$.

We have the following lemma.

Lemma 3.3 (One Direction): Let σ be a Nash equilibrium in $\Gamma(\mathcal{L})$, $e \in \{e_1, ..., e_S\}$, and $l, l' \in L_0$. If $l \overset{e}{\rightarrowtail} l'$ and at least one of l, l' is active, then

(1): l is active and $m_l(\sigma) > m_{l'}(\sigma)$;

(2)(Asymmetry): $l' \rightarrowtail l$ does not hold;

(3): $l \notin L_j$ for all j with $f_j = l'$ and $e(j) = e$.

Proof. Consider (1). Let $\sigma = ((f_1, r_1), ..., (f_n, r_n))$ be a Nash equilibrium in $\Gamma(\mathcal{L})$. Suppose that $l \overset{e}{\rightarrowtail} l'$ and at least one of l, l' is active. Then, there are some coethnic players i, j of ethnicity e such that $f_i = l$ and $f_j = l' \in L_i$. If l is inactive and l' is active, then player i can go to l' and enjoy the festival at l' since his presence is not found, which is impossible since σ be a Nash equilibrium. Hence, l must be active. By Lemma

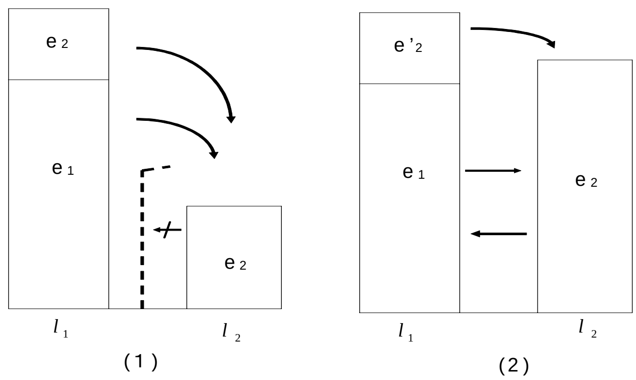

Figure 3.1: Ethnic Barrier and Discrimination by Similar Players

3.1.(1), $r_i(f_i, E_i(f)) = 1$, and thus the payoff to player i under σ is $H_i(\sigma) = m_l(\sigma)$. If i moves to l', his payoff will be $H_i(\sigma_{-i}, (l', 1)) = m_{l'}(\sigma) + 1$ and $H_i(\sigma) \geq H_i(\sigma_{-i}, (l', 1))$, i.e., $m_l(\sigma) > m_{l'}(\sigma)$.

By (1), it is impossible to have $l' \rightarrowtail l$, i.e., (2). This implies not $l' \overset{e}{\rightarrowtail} l$. Hence, we have (3). ■

Let us return to Fig.3.1.(1): If the immigration barrier prohibiting the players at l_2 from coming to l_1 was eliminated, then both $l_1 \overset{e_2}{\rightarrowtail} l_2$ and $l_2 \overset{e_2}{\rightarrowtail} l_1$ would hold. This is incompatible with Asymmetry stated in Lemma 3.3, and hence, the immigration barrier is needed to have Fig.3.1.(1) as an equilibrium location configuration.

Both $\rightarrowtail$ and $\overset{e}{\rightarrowtail}$ are irreflexive by definition. But, neither $\rightarrowtail$ nor $\overset{e}{\rightarrowtail}$ may be transitive, since $\rightarrowtail$ may depend upon e and since even if $l \overset{e}{\rightarrowtail} l'$ and $l' \overset{e}{\rightarrowtail} l''$, all players of ethnicity e at l may be prohibited to access l''.

The following is an important implication of Lemma 3.3.(1).

Corollary 3.4 (No Cycle Condition): Let σ be a Nash equilibrium in $\Gamma(\mathcal{L})$. Let $\{l_1, ..., l_k\}$ a sequence of active locations and a *chain* with respect to $\rightarrowtail$, i.e., $l_t \rightarrowtail l_{t+1}$ for $t = 1, ..., k-1$. Then $m_{l_1}(\sigma) > m_{l_2}(\sigma) > ... > m_{l_k}(\sigma)$. Thus, there are no cycles with respect to $\rightarrowtail$.

The next result provides an insight into group formation in the festival game $\Gamma(\mathcal{L})$. It states that it is not the case that two coethnic players choose different locations but still can each access the other's location. We state this as the following corollary.

Corollary 3.5 (Group Formation): Consider an equilibrium location configuration $f = (f_1, ..., f_n)$. Let i and j be coethnic players. Then $f_j \in L_i$ and $f_i \in L_j$ if and only if $f_i = f_j$.

Proof. The *if* part follows from (2.8). Consider the *only-if* part. Suppose that $f_j \in L_i$ and $f_i \in L_j$. On the contrary, suppose $f_i = l \neq l' = f_j$. Let e be the ethnicity of i, j. Then, $l \overset{e}{\rightarrowtail} l'$ and $l' \overset{e}{\rightarrowtail} l$, which is impossible by Lemma 3.3.(2). Thus, $l = l'$. ■

We have been deriving necessary conditions for a Nash equilibrium in $\Gamma(\mathcal{L})$. In fact, we have a full characterization of a Nash equilibrium.

Theorem 3.6 (Characterization of Nash Equilibria): Let σ be a profile of strategies satisfying Assumption M. Then σ is a Nash equilibrium if and only if the following conditions (1), (2c) and (2n) hold: Let $l, l' \in L_0$ and $e \in \{e_1, ..., e_S\}$. Then

(1)(Active Festivals): If l is active, then $m_l(f) > m_0$.

(2c)(Coethnic Players): If $l \overset{e}{\rightarrowtail} l'$ and at least one of l, l' is active, then $m_l(\sigma) > m_{l'}(\sigma)$;

(2n)(No Coethnic Players): If $l \overset{e}{\rightsquigarrow} l'$ and i is a player of ethnicity e at l with $l' \in L_i$, then $m_l(\sigma) \geq m_{l'}(\sigma_{-i}, (l', 1))$.

Proof. (*Only-If*): Consider (1). Since there is a player i at l with $r_i(E_i(f)) = 1$, we have $m_l(f) = H_i(\sigma) \geq H_i(\sigma_{-i}, (l, 0)) = m_0$. Since m_0 is not an integer, the inequality must be strict. The other two conditions are already proved in Lemmas 3.3 and 2.1.

(*If*): Let $\sigma = ((f_1, r_1), ..., (f_n, r_n))$ be any profile of strategies satisfying Assumption M. Suppose conditions (1), (2c) and (2n). We will prove that σ is a Nash equilibrium in $\Gamma(\mathcal{L})$. By Lemma 2.1, it suffices to show that for any player i and any trial $(f_i, \delta_i) \in L_i \times \{0, 1\}$ of i, $H_i(\sigma) \geq H_i(\sigma_{-i}, (f_i, \delta_i))$. Let $e(i) = e$, $f_i = l$ and consider $l' \in L_i$ with $l' \neq l$.

Now, suppose $l \overset{e}{\rightarrowtail} l'$ with $l' \in L_i$. Then the presence of player i at l' is not observed, where Assumption M is used. If both l and l' are inactive, we have $m_l(\sigma) = m_{l'}(\sigma) = 0$, and player i's payoff is $H_i(\sigma_{-i}, (l', 1)) = 1 < m_0 = H_i(\sigma)$ and $H_i(\sigma_{-i}, (l', 0)) = m_0$. Suppose that at least one of l, l' is active. Then, $m_l(\sigma) > m_{l'}(\sigma)$ by (2c), which implies that l is active. Thus, using (1), $H_i(\sigma) = m_l(\sigma) > m_0 = H_i(\sigma_{-i}, (l', 0))$. Finally, from $m_l(\sigma) > m_{l'}(\sigma)$, we have $H_i(\sigma) = m_l(\sigma) \geq m_{l'}(\sigma) + 1 = H_i(\sigma_{-i}, (l', 1))$.

Consider the case where $l \overset{e}{\rightsquigarrow} l'$ and i is a player with $e(i) = e$ and $l' \in L_i$. Then by (2n), $H_i(\sigma) = m_l(\sigma) \geq m_{l'}(\sigma_{-i}, (l', 1)) = H_i(\sigma_{-i}, (l', 1))$. Finally, consider the deviation $(l', 0)$. If l is inactive, we have $H_i(\sigma) = m_0 = H_i(\sigma_{-i}, (l', 0))$, and if l is active, we have, using (1), $H_i(\sigma) = m_l(f) > m_0 = H_i(\sigma_{-i}, (l', 0))$. ■

Condition (1) of the theorem is part of Lemma 3.1, and (2c) is part of Lemma 3.3. Condition (2n) is a specification of the statement given in Lemma 2.1. In fact, the limited observabilities $\{\sim_e\}_e$ are involved but are covert in the abstract statement in $m_l(\sigma) \geq m_{l'}(\sigma_{-i}, (l', 1))$. In particular, the last term depends upon limited observabilities; this fact comes from (2.5) and (2.6). The inequality $m_l(\sigma) \geq m_{l'}(\sigma_{-i}, (l', 1))$ in (2n) will be analyzed more.

Theorem 3.6 shows that the Nash equilibria of the festival game may involve the partial or complete segregation of ethnicities. When a Nash equilibrium involves segregation to any degree, it is sustained by ethnic discrimination. The relation between segregation and discrimination will become clearer in the next section where we will characterize the set of Nash equilibria relative to a given location configuration.

4. Nash Equilibria with a Given Location Configuration

From the viewpoint of inductive game theory, a Nash equilibrium corresponds to a particular state of society together with the history that led to the state. The state includes a location configuration f, which is interpreted as a pattern of segregation. Our research strategy is to focus on this location configuration, and to study the multiple Nash equilibria compatible with it, which vary in the degree of discrimination. Mathematically, we characterize the set of Nash equilibria relative to a given location configuration f. In particular, we show that this set has Nash equilibria involving a minimum amount of discriminatory responses towards a visiting player. This will be used for the definition of the discrimination measure in Section 5.

To avoid complications caused by the behavior of outliers, we only consider Nash equilibria $\sigma = ((f_1, r_1), ..., (f_n, r_n))$ with a fixed f in $\Gamma(\mathcal{L})$ satisfying Assumption M and without outliers. The set of such Nash equilibria is denoted by $\Xi(f, \mathcal{L})$. In those equilibria, the mood $m_l(\sigma)$ at each location l is invariant over all Nash equilibria σ in $\Xi(f, \mathcal{L})$. Defining $m_l(f) := |\{j : f_j = l\}|$ (the cardinality of this set) for $l \in L_0$, we have

$$m_l(f) = m_l(\sigma) \text{ for any } \sigma \in \Xi(f, \mathcal{L}). \tag{4.1}$$

Note that if we allow outliers, it may be the case that some location has only unfriendly players in one equilibrium and all friendly in another equilibrium, so the invariance result (4.1) does not hold.

We first give a criterion for the nonemptiness of $\Xi(f, \mathcal{L})$.

Lemma 4.1 (Nonemptiness Criterion): Let $f = (f_1, ..., f_n)$ be any location configuration compatible with $\mathcal{L}$ and satisfying Assumption M. Then $\Xi(f, \mathcal{L})$ is nonempty if and only if

(1): for any $l, l' \in L_0$, $l' \overset{e}{\rightarrowtail} l$ implies $m_{l'}(f) > m_l(f)$;

(2): for any $l, l' \in L_0$, $l' \overset{e}{\rightsquigarrow} l$ implies that the original mood at l' is larger than the number of players at l who cannot perceive the presence of e, i.e.,

$$m_{l'}(f) > \left|\{j : f_j = l \text{ and } (E_j(f) \cup \{e\})/\sim_{e(j)} = E_j(f)/\sim_{e(j)}\}\right| \tag{4.2}$$

Recall that the quotient set relative to $\sim_{e(j)}$ is defined in (2.5). Without the introduction of limited observabilities $\{\sim_e\}_e$, every player at l could observe the presence of ethnicity e, and thus (2) would be redundant. However, with $\{\sim_e\}_e$, many players may not perceive the presence of e, and hence we need (2).

Proof of Lemma 4.1. (*Only If*): Let $\Xi(f, \mathcal{L}) \neq \emptyset$. Consider $\sigma \in \Xi(f, \mathcal{L})$. Since there are no outliers in σ and all players act friendly in equilibrium, any nonempty location is active.

Consider (1) and let $l, l' \in L_0$ with $l \overset{e}{\rightarrowtail} l'$. Since both l and l' are active, we have $m_l(f) > m_{l'}(f)$ by Lemma 3.3.(1).

Consider (2). The right term of (4.2) is the number of players finding no difference in ethnicities at l with the presence of e and keeping friendly actions to the presence of e. This is the minimal mood possibly induced by e. If this was larger than or equal to $m_{l'}(f)$, we cannot prevent a player i at l' with $l \in L_i$ and $e(i) = e$ from going to location l, and thus $\Xi(f, \mathcal{L})$ is empty. Hence, when $\Xi(f, \mathcal{L}) \neq \emptyset$, we have (4.2).

(*If*): We prove $\Xi(f, \mathcal{L}) \neq \emptyset$ under (1) and (2). For all $i \in N$, we define $r_i : 2^{\{e_1,\dots,e_S\}} \to \{0, 1\}$ by

$$r_i(E) = \begin{cases} 1 & \text{if } E/\sim_{e(i)} = E_i(f)/\sim_{e(i)} \\ 0 & \text{otherwise.} \end{cases} \tag{4.3}$$

It suffices to show that $\sigma = (f, r)$ is a Nash equilibrium in $\Gamma(\mathcal{L})$. There are two possible cases with respect to a player i coming to location l: either (i) he has a coethnic player at l or (ii) he has no coethnic player present at l.

In Case (i), we have $l' \overset{e}{\rightarrowtail} l$ with $f_i = l'$, $l \in L_i$ and $e = e(i)$. By (1), we have $m_{l'}(f) > m_l(f)$. Thus, it is better to stay at his original location l'. Consider Case (ii). By (4.3), all players at l who finds the presence of player i act unfriendly. The mood induced at l is the right term of (4.2), possibly plus 1, and does not exceed $m_{l'}(f)$. Hence, σ is a Nash equilibrium and $\Xi(f, \mathcal{L}) \neq \emptyset$. ■

When $\Xi(f, \mathcal{L})$ is nonempty, we call $f = (f_1, ..., f_n)$ an *equilibrium location configuration.* This can be interpreted as a stable pattern of segregation.

For the subsequent analysis, we introduce the notion of conditional mood $m_l(\sigma \mid e) = m_l((f, r) \mid e)$ when a player i of ethnicity $e = e(i)$ with $f_i \neq l$ comes to l. This is given as

$$m_l(\sigma \mid e) = \begin{cases} m_l(\sigma_{-i}, (l, 1)) & \text{if } e \notin E^l(f) \\ m_l(\sigma) + 1 & \text{if } e \in E^l(f). \end{cases} \tag{4.4}$$

Recall $E^l(f) = \{e(j) : f_j = l\}$ defined by (2.3). In the second case, some coethnic player of i is already in location l and player i with $e(i) = e$ comes to l; thus the last $+1$ counts his presence at l. Using this notation, when player i with $e(i) = e$ and $l \in L_i$ goes to location l, his payoff is represented by $m_l(\sigma \mid e)$, i.e.,

$$H_i(\sigma_{-i}, (l, 1)) = m_l(\sigma \mid e). \tag{4.5}$$

This notation will simplify our subsequent analysis.

The set $\Xi(f, \mathcal{L})$ consists of many Nash equilibria, and its multiplicity corresponds to the degree of discriminatory responses to a visiting player. Our present aim is to measure this degree. For this, we first focus on a particular location l and on the players who possibly visit l. Using the binary relations $\overset{e}{\rightarrowtail}$ and $\overset{e}{\rightsquigarrow}$ introduced in Section 3, the visitors fall into the two cases:

(**Coethnic Players**): $l' \overset{e}{\rightarrowtail} l$;

(**No Coethnic Players**): $l' \overset{e}{\rightsquigarrow} l$.

In these two cases, we would like to measure the degree of discrimination induced by a player from the outside. To do this, we first need to examine the induced mood at l. We note that we always make Assumption M without mentioning in the following.

The first lemma states that in the case of $l' \overset{e}{\rightarrowtail} l$, no discrimination is induced, and the mood is invariant over all Nash equilibria in $\Xi(f, \mathcal{L})$.

Lemma 4.2 (No Discrimination to Coethnic Players): Let $l' \overset{e}{\rightarrowtail} l$. Then, $m_l(\sigma \mid e) = m_l(\sigma' \mid e) = m_l(\sigma) + 1$ for any $\sigma, \sigma' \in \Xi(f, \mathcal{L})$.

Proof. By (4.4), $m_l(\sigma \mid e) = m_l(\sigma) + 1 = m_l(f) + 1 = m_l(\sigma') + 1 = m_l(\sigma' \mid e)$. ■

The next result is actually more important for Section 5. Recall that a Nash equilibrium consists of location-response choices for all players in the entire game Γ. However, the discrimination degree against a visiting player at a location can be local to his location, rather than global to the entire game. The next result guarantees that we can restrict ourselves to a single location and a single ethnicity, independent of the remaining part.

Theorem 4.3 (Localization): Let $\sigma = (f, r) \in \Xi(f, \mathcal{L})$. Let e be an ethnicity with $e \notin E^l(f)$. Suppose that any $\{r_j^o(E^l(f) \cup \{e\})\}_{f_j = l}$ satisfies

$$m_{l'}(f) \geq \sum_{f_j = l} r_j^o(E^l(f) \cup \{e\}) + 1 \text{ for all } l' \text{ with } l' \overset{e}{\rightsquigarrow} l. \tag{4.6}$$

Then, we define r' by replacing the part $\{r_j(E^l(f) \cup \{e\})\}_{f_j = l}$ of r by $\{r_j^o(E^l(f) \cup \{e\})\}_{f_j = l}$. Then $\sigma' = (f, r') \in \Xi(f, \mathcal{L})$.

Proof. We prove that the newly defined $\sigma' = (f, r')$ is a Nash equilibrium. Note that it differs from $\sigma = (f, r)$ only in the domain of r where a player i with $f_i = l' \overset{e}{\rightsquigarrow} l$

and $e(i) = e$ i goes to location l. Consider a player i with $f_i = l' \overset{e}{\rightsquigarrow} l$ and $e(i) = e$. If player i goes to location l, his payoff becomes $\sum_{f_j=l} r_j^o(E^l(f) \cup \{e\}) + 1$, which is smaller than or equal to his present payoff $m_{l'}(f)$ by (4.6). Player i's deviation not to go to l would meet the response described by the original r, and hence, does not induce a higher payoff. Finally, If $l' \overset{e}{\rightsquigarrow} l$ does not hold or $e(i) \neq e$, then any deviation of player i would meet the responses described by the original r. Thus, his deviation does not give a higher payoff than $m_{l'}(f)$. ■

Theorem 4.3 enables us to focus on discriminatory responses at the local level. Then, we can combine the responses from those local levels into one equilibrium. The following corollary illustrates this in the context of minimal discrimination, and is used in Section 5.

Corollary 4.4.(Minimal Degrees of Discrimination): Let $\Xi(f, \mathcal{L}) \neq \emptyset$. There is a Nash equilibrium σ in $\Xi(f, \mathcal{L})$ such that for any l' with $l' \overset{e}{\rightarrowtail} l$ or $l' \overset{e}{\rightsquigarrow} l$, $m_l(\sigma \mid e) \geq m_l(\sigma' \mid e)$ for any σ' in $\Xi(f, \mathcal{L})$.

Proof. Let σ be a Nash equilibrium in $\Xi(f, \mathcal{L})$. By Lemma 4.2, we do not need to think about the case of $l' \overset{e}{\rightarrowtail} l$. Now, consider $l' \overset{e}{\rightsquigarrow} l$. Suppose that $m_l(\sigma \mid e) < m_l(\sigma' \mid e)$ for some σ' in $\Xi(f, \mathcal{L})$ and this $m_l(\sigma' \mid e)$ is the maximum for (l, e). Then, we replace the part $\{r_j(E^l(f) \cup \{e\}\}_{f_j=l}$ of r by $\{r'_j(E^l(f) \cup \{e\})\}_{f_j=l}$, and we have the new σ^1. This σ^1 is a Nash equilibrium in $\Xi(f, \mathcal{L})$ by Theorem 4.3. We continue this replacement process until no new pair (l, e) is found. ■

Corollary 4.4 asserts the existence of a Nash equilibrium in $\Xi(f, \mathcal{L})$ having the minimum amount of discriminatory reactions against any visiting player. This describes the necessary degree of discrimination needed to maintain the given location configuration. This will be used in the definition of the measure of discrimination in Section 5.

5. Measure of Discrimination

In this section, we define a measure of discrimination relative to a given location configuration f, and then analyze the properties of this measure[6]. There are many Nash equilibria with respect to a given location configuration f. Accordingly, the degree of discrimination depends upon a Nash equilibrium. Nevertheless, it would be useful to have a unidimensional measure indicating the degree of discrimination necessarily involved. We define our measure so that it gives the minimum degree of discrimination needed to sustain f as an equilibrium configuration. What we intend to capture by this measure will be explained presently.

[6]The term "measure" is technically unrelated to the notion of a "measure" in mathematics. Conceptually, both mean that they measure amounts, quantities or degrees of something or some states.

Let $f = (f_1, ..., f_n)$ be an equilibrium location configuration in $\Gamma(\mathcal{L})$ satisfying Assumption M, and $l \in L_0$. Some ethnicity e may irrelevant for the consideration of discrimination at location l; for example, no player of e has ever been present at l or all players of e are only found in l. We define the set of relevant ethnicities for location l by

$$E^l(f, \mathcal{L}) = \{e : l' \overset{e}{\rightarrowtail} l \text{ for some } l'\} \cup \{e : l' \overset{e}{\rightsquigarrow} l \text{ for some } l'\}. \tag{5.1}$$

These are the ethnicities so that some of them may visit location l. The discrimination measure $d_f(e \mid l)$ is defined over this set.

Now, we define the *discrimination measure* $d_f(e \mid l)$ as follows: for any $l \in L_0$ and any $e \in E^l(f, \mathcal{L})$,

$$d_f(e \mid l) = \min_{\sigma \in \Xi(f, \mathcal{L})} [m_l(f) - (m_l(\sigma \mid e) - 1)]. \tag{5.2}$$

Recall that $m_l(\sigma \mid e)$, defined by (4.4), is the total mood conditional upon the presence of a player of ethnicity e, which counts that player. The -1 term in (5.2) eliminates the effect of that player on the mood at l. Thus, the difference $m_l(f) - (m_l(\sigma \mid e) - 1)$ is the pure change in the mood at location l caused by ethnicity e. We take the minimum of such differences over all possible Nash equilibria in $\Xi(f, \mathcal{L})$. In sum, $d_f(e \mid l)$ expresses the degree of discrimination that will inevitably occur at location l against ethnicity e.

Now, let us look at an example, which suggests what questions we should address. In Fig.1.1, we assume that all ethnicities are observably distinct. The location configuration presented in the figure can be sustained by many Nash equilibria. For example, in one equilibrium, all players at l_1 may discriminate against a visiting player from l_2. In another equilibrium, only some players at l_1 may discriminate. The discrimination measure $d_f(e_2 \mid l_1)$ is the minimum amount of discrimination needed to keep e_2 away from l_1, which is seen to be the height AB. Similarly, the discrimination measure $d_f(e_3 \mid l_1)$ is seen to be CD. As mentioned in Section 1.1, the ethnicity from a smaller festival faces more discrimination.

The minimum discrimination degree $d_f(e \mid l)$ is the minimum welfare loss associated with discriminatory behavior needed to sustain the segregation pattern f. In this paper, we do not address an analysis of partial and/or total welfare losses associated with segregation, though it is an important problem.

Although the above argument seems clear, a rigorous derivation of the discrimination measure needs to find an equilibrium supporting the minimum degree of discrimination. This is a complex task in a general setting rather than an example. The method of calculating $d_f(e \mid l)$ needs to be developed.

We start with a simple case and proceed to more general cases.

Lemma 5.1 (Calculation 1)(1): Let $l, l' \in L_0$ and $l' \overset{e'}{\rightarrowtail} l$, then $d_f(e' \mid l) = 0$.

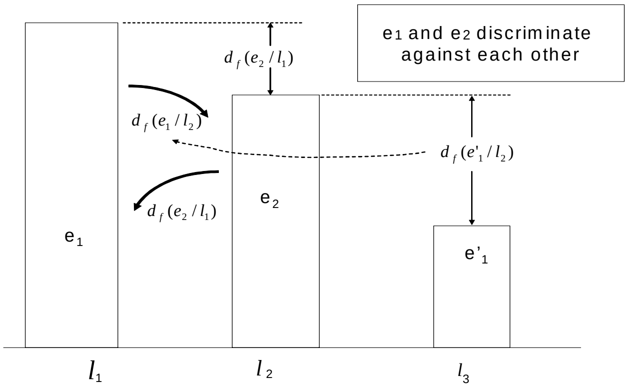

Figure 5.1: Effect of a Similar Ethnicity

(2): Let $l, l' \in L_0$, $m_l(f) > m_{l'}(f) > 0$ and $l' \overset{e'}{\rightsquigarrow} l$. Then, $d_f(e' \mid l) \geq m_l(f) - (m_{l'}(f) - 1)$.

The first claim simply states that a visiting player of ethnicity e will face no discrimination (with respect the minimum degree) at l if he has coethnic players there. The second states that when a player i is a visitor from a smaller festival l' to a larger festival l, the minimum discrimination he faces is the difference between the regular moods of the festivals. The last term -1 is to remove the effect of player i's contribution to l'. When we talk about interpretations of some results, we often ignore the this term such as the above comparisons of the rectangles of Fig.1.1.

Proof.(1): Let $l' \overset{e'}{\rightarrowtail} l$. By Lemma 4.2, $m_l(\sigma \mid e') = m_l(\sigma) + 1 = m_l(f) + 1$ does not depend upon $\sigma \in \Xi(f, \mathcal{L})$. Hence, $d_f(e' \mid l) = 0$.

(2): Suppose that $d_f(e' \mid l) < m_l(f) - m_{l'}(f) + 1$. There is a Nash equilibrium $\sigma \in \Xi(f)$ such that $d_f(e' \mid l) = m_l(f) - (m_l(\sigma \mid e) - 1) < m_l(f) - m_{l'}(f) + 1$. Let i be a player at l' with $e(i) = e'$ and $l \in L_i$. Then, $H_i(\sigma_{-i}, (l, 1)) = m_l(\sigma \mid e) > m_{l'}(f) = H_i(\sigma)$. This is a contradiction to that σ is a Nash equilibrium. ■

Here, we introduce one definition to save mathematical expressions. Let E be a set of ethnicities and e an ethnicity. We say that e is *ethnically distinguishable from E* at location l iff for all $e'' \in E^l(f)$,

$$e \not\sim_{e''} e' \text{ for all } e' \in E. \tag{5.3}$$

Using this definition, we have the following theorem.

Theorem 5.2 (Calculation 2): Let $l \in L_0$ and $e \in E^l(f, \mathcal{L})$. Assume that e is ethnically distinguishable from the set $\{e' : e' \neq e,\ l' \overset{e'}{\rightsquigarrow} l$ and $m_{l'}(f) < m_l(f)$ for some

$l'\}$ at location l. Then we have the following.

(1): Suppose that $m_{l'}(f) \geq m_l(f)$ for all l' with $e \in E^{l'}(f)$. Then $d_f(e \mid l) = 0$.

(2): Suppose that $l' \overset{e}{\rightsquigarrow} l$ and $m_{l'}(f) < m_l(f)$ for some l'. Then $d_f(e \mid l)$ is given as

$$d_f(e \mid l) = m_l(f) - \min_{l' \overset{e}{\rightsquigarrow} l} m_{l'}(f) + 1. \tag{5.4}$$

The assumption of ethnic distinguishability of e at l is needed for these results, since otherwise, some players at l may respond to e in the same way as to other ethnicities. When this assumption does not hold, even in (1), ethnicity e may be confused with some other ethnicity e' at location l and be discriminated against; hence, the value $d_f(e \mid l)$ could be positive. In (2), for a similar reason, the value $d_f(e \mid l)$ could be larger than that given in (5.4). Thus, our calculation method is restrictive, but the above results may still be used.

Let us look at the above results in an example. In Fig.5.1, the players at locations l_1, l_2, l_3 can access the other locations. The players of ethnicity e_1 can distinguish all between e_1', e_1 and e_2, and the players of e_2 can distinguish themselves from e_1 and e_1', but not between e_1 and e_1'. The ethnic distinguishability assumption of the theorem holds for e_2 at location l_1, and for e_1' at location l_2. Hence, when a player comes from l_2 to l_1, or from l_3 to l_2, we have (2), i.e., he would meet the minimum discrimination $d_f(e_2 \mid l_1) = m_{l_1}(f) - m_{l_2}(f) + 1$, or $d_f(e_1' \mid l_2) = m_{l_2}(f) - m_{l_3}(f) + 1$, described in Fig.5.1. Furthermore, when a player comes from l_3 to l_1, we have the minimum discrimination $d_f(e_1' \mid l_1) = m_{l_1}(f) - m_{l_3}(f) + 1$. Those are all obtained by (2).

A player from either l_1 or l_2 meets the 0-minimum discrimination, i.e., $d_f(e_1 \mid l_3) = d_f(e_2 \mid l_3) = 0$, which is (1).

Now, suppose that the players of ethnicities e_1 and e_1' are similar and cannot be distinguished by the players of ethnicity e_2. When player i from l_1 visits l_2, the players at l_2 cannot distinguish him from e_1' people, and thus player i meets at least $d_f(e_1' \mid l_2)$ amount of discrimination. Here, we have mutual discrimination between ethnicities e_1 and e_2 even in the sense of minimum discrimination. This may capture a certain aspect of conflicts between different ethnicities.

Certainly, it would be important to analyze the behavior of the discrimination measure without the assumption of ethnic distinguishability. Some results will be given in Section 6.2.

Proof of Theorem 5.2.(1): Consider a Nash equilibrium σ given in Corollary 4.4. Let i be a player with $f_i = l'$ and $e(i) = e$. In that Nash equilibrium σ, it must hold that

$$m_l(\sigma \mid e) = m_l(\sigma_{-i}, (l, 1)) = m_l(f) + 1. \tag{5.5}$$

This implies $d_f(e \mid l) = 0$ by (5.2). Let us see the reason for (5.5). Since $m_{l'}(f) \geq m_l(f)$ for all l' with $e \in E^{l'}(f)$, any player of e has no incentive to go to location l, even if no discrimination is expected at l. Only the ethnicities in the set $\{e' : l' \overset{e'}{\rightsquigarrow} l$ and $m_{l'}(f) < m_l(f)$ for some $l'\}$ are relevant for discriminatory responses. But e is ethnically distinguished from those ethnicities. Hence, no discriminatory responses to the presence of e are required at l to sustain a Nash equilibrium. Hence, in the maximum Nash equilibrium, no discrimination occurs if e comes to location l. This is (5.5).

(2): Let $m_{l^o}(f) = \min\limits_{l' \overset{e}{\rightsquigarrow} l} m_{l'}(f)$ and $l^o \overset{e}{\rightsquigarrow} l$. Let σ be any Nash equilibrium in $\Xi(f, \mathcal{L})$. Then, since l is accessible for some player j in l^o with $e(i) = e$, we have $m_l(\sigma \mid e) \leq m_{l^o}(f)$. Since σ is arbitrary in $\Xi(f, \mathcal{L})$, we have $\max\limits_{\sigma \in \Xi(f,\mathcal{L})} m_l(\sigma \mid e) \leq m_{l^o}(f)$. Any ethnicity e' at a location l' with a lower mood than $m_{l^o}(f)$ can be ethnically distinguishable from e by the assumption of the theorem. Hence, it suffices to decrease the mood at location l to $m_{l^o}(f)$ so as to prevent the visit of player i to l. Thus, the minimum degree of discrimination is $m_{l^o}(f)$, and we can find a Nash equilibrium σ^o by Corollary 5.4 so that $m_l(\sigma^o \mid e) = m_{l^o}(f)$. Thus, $\max\limits_{\sigma \in \Xi(f,\mathcal{L})} m_l(\sigma \mid e) = m_{l^o}(f)$. Now, we have

$$\max_{l' \overset{e}{\rightsquigarrow} l} [m_l(f) - m_{l'}(f) + 1] = m_l(f) - m_{l^o}(f) + 1 = m_l(f) - \max_{\sigma \in \Xi(f,\mathcal{L})} m_l(\sigma \mid e) + 1 = d_f(e \mid l).$$

■

When any players at l cannot distinguish between e' and e'', they are treated as the same ethnicities at least at location l. The following is the result about these similar players as targets.

Theorem 5.3 (Similar Players as Targets): Let $\Xi(f, \mathcal{L}) \neq \emptyset$. Consider an location l and two ethnicities $e', e'' \in E^l(f, \mathcal{L})$. Suppose that $e' \sim_e e''$ for all $e \in E^l(f)$. Then, $d_f(e' \mid l) = d_f(e'' \mid l)$.

Proof. Suppose $l' \overset{e'}{\rightarrowtail} l$ or $l' \overset{e'}{\rightsquigarrow} l$. Then, all the players at l cannot distinguish between e' and e''. Hence, their responses to the presence of e'' are the same. Hence $d_f(e' \mid l) = d_f(e'' \mid l)$. ■

We have seen how to calculate the discrimination measure under the assumption of ethnic distinguishability. Once calculation becomes possible, we use it for comparisons, for example, of the discrimination faced by different ethnicities at a location. In the definition of $d_f(\cdot \mid \cdot)$, however, minimization is taken over the set $\Xi(f, \mathcal{L})$ for a given location l and a given ethnicity e. Thus, discrimination faced by different ethnicities at a location may be supported by different Nash equilibria, which represent different realizations of the social state. However, the next result states that we can avoid this complication, which follows from Corollary 4.4.

Theorem 5.4 (Uniformity). Let $\Xi(f, \mathcal{L}) \neq \emptyset$. Then there is a Nash equilibrium

$\sigma^o \in \Xi(f, \mathcal{L})$ such that for all $l \in L_0$ and all $e \in E^l(f, \mathcal{L})$,

$$d_f(e \mid l) = m_l(f) + 1 - m_l(\sigma^o \mid e). \tag{5.6}$$

This result allows us to compare the discrimination faced by different ethnic groups in a single Nash equilibrium. This will be helpful in the next section, where we provide two applications of this measure.

6. Ethnic Hierarchy, Similarity, and Distances

In Section 5, we have defined a measure of discrimination and provided a method of calculating the measure. This section demonstrates the applicability of this measure by using it to investigate discrimination in an ethnic hierarchy and discrimination between groups perceived as similar to one another. It should be pointed out that these phenomena are not mutually exclusive and may be observed in the same situation. It should also be reemphasized that there are many other discriminatory phenomena which can be observed in the festival game. We have focused on these cases because they are enough to make the applicability of the discrimination measure clear and to show that our model generates patterns of discrimination consistent with evidence from sociology and social psychology.

6.1. Ethnic Hierarchy

In Section 1.3, we gave an idea of ethnic hierarchy and its effect on discrimination using Fig.1.1. Also, Theorem 5.2 already includes some ethnic hierarchy, but we would like to make it more explicit.

Theorem 6.1 (Ethnic Hierarchy): Let $\Xi(f, \mathcal{L}) \neq \emptyset$ and $l^1 \in L^0$. Let $l^2, ..., l^k \in L^0$ be the enumerated locations whose moods are less than that of l^1. Consider ethnicities $e^2, ..., e^k$, and suppose that for $t = 2, ..., k$,

(a): e^t is found only in location l^t among $l^2, ..., l^k$ and $l^t \overset{e^t}{\rightsquigarrow} l^1$;

(b): e^t is ethnically distinguishable from $\{e : e \neq e^t \text{ and } l \overset{e}{\rightsquigarrow} l^1 \text{ for some } l = l^2, ..., l^k\}$ at l^1.

Then, $d_f(e^t \mid l^1) = m_{l^1}(f) - m_{l^t}(f) + 1$ for any $t = 2, ..., k$. When $m_{l^1}(f) > m_{l^2}(f) > ... > m_{l^k}(f)$, we have

$$d_f(e^2 \mid l^1) < ... < d_f(e^k \mid l^1). \tag{6.1}$$

Proof. By (b), the ethnic distinguishability assumption of Theorem 5.2 holds. By (a), $l^t \overset{e^t}{\rightsquigarrow} l^1$ and e^t is in l^t among $l^1, ..., l^k$. Hence, $d_f(e^t \mid l^1) = m_{l^1}(f) - m_{l^t}(f) + 1$. Thus, we have (6.1). ■

Thus, (6.1) shows a hierarchy of ethnicities at location l^1. People of lower ranked ethnic groups would meet severer discrimination against them than those of higher ranked groups. Also, it follows from Lemma 5.1 that people of higher ranked groups would not meet any discrimination against at location l^1, unless they have similar players in lower ranked locations. This hierarchical structure is found rather universally for any groups unless the ethnic confusions occur.[7]

Our result can be interpreted as showing the effect of *ethnic stratification* on discrimination (cf., Marger [16]). A specific instance is discussed in Hagendoorn *et al* [6], who conducted an empirical study of ethnic relations in the former Soviet union. They showed that discrimination follows a hierarchical structure in that the magnitude of discrimination is inversely related to the social position of a group in the ethnic hierarchy. This is closer to and quite consistent with the above result.

6.2. Ethnic Distance and Similarity

With limited observabilities $\{\sim_e\}_e$, we can find phenomena quite different from Theorem 6.1. When an ethnic group e far away from two other similar groups e' and e'' in ethnic distance, the group e may regard the groups e' and e'' as the same. But those groups e' and e'' themselves can distinguish between themselves by finding physical or cultural differences (see Marger [16], Chap.2). Some discrimination may happen between them though they are almost regarded as identical from the others' perspective. For instance, Chinese Australian can distinguish new Chinese immigrants from themselves, but others typically cannot. The former may discriminate against the latter. We have an abundant of such examples. The following result is a possible formulation of this observation.

Theorem 6.2 (Ethnic Distance and Discrimination by Similar Players): Let $\Xi(f, \mathcal{L}) \neq \emptyset$, $l, l' \in L_0$, and let $e^1 \in E^l(f)$, $e^2 \in E^{l'}(f)$ with $l' \overset{e^2}{\rightsquigarrow} l$. Then, suppose that $e^1 \sim_e e^2$ for all $e \in E^l(f) \backslash \{e^1\}$ and $e^1 \nsim_{e^1} e^2$. Then,

(1): in any equilibrium in $\Xi(f, \mathcal{L})$, any discriminator against e^2 at l is a player of e^1;

(2): $\left|\{i : e(i) = e^1 \text{ and } f_i = l\}\right| \geq d_f(e^2 \mid l)$.

Proof.(1): When a player of ethnicity e^2 visits location l, his presence is observed only by the players of e^1, and hence only those players are possible discriminators against a player of e^2.

(2): Only the discriminators against e^2 at l are e^1 people. The value $d_f(e^2 \mid l)$ is

[7] While it is generally true that a superior group discriminates more against a subordinate group of low position in the ethnic hierarchy than against one of relatively higher position, there is also some evidence from social psychology that a superior group may exhibit magnanimity when the status differential with the subordinate group is perceived as unbreachably wide. We do not consider this possibility.

sustained by the discrimination by those people. Hence, there are at least $d_f(e^2 \mid l)$ number of e^1 people in location l. ■

The supposition of the theorem means that e^1 and e^2 are close in ethnic distance and similar for all other ethnicities at l. However, they can distinguish between each other. Then, all discriminators against e_2 are of ethnicity e_1; only akin people discriminate. The number of e^1 players has to be greater than or equal to $d_f(e^2 \mid l)$; otherwise f would not be an equilibrium configuration - see Lemma 4.1.

In conclusion, we have shown certain possible applications of our approach. There are many phenomena which we may discuss by our approach by modifying and/or extending it. Those should belong to future research agenda.

7. Prejudices and Personal Views

Up to the previous section, the limited observabilities of ethnicities expressed by $\{\sim_e\}_e$ have played a more explicit role in the study of discrimination than limited accessible locations $\mathcal{L} = \{L_i\}_{i\in N}$. On the other hand, prejudice is a (negative) view of other ethnicities and is often related to passive experiences. Limited accessible locations $\mathcal{L}$ may classify individual experiences into active and passive ones. This will add a large scope of research into prejudices to the approach of Kaneko-Matsui [13]. In this section, we give a very brief discussion on a possible study of prejudices in the present framework.

Let us illustrate the new phenomena captured in the present framework, using Fig.7.1. Consider the location configuration described in it. This is an equilibrium configuration with various limited accessible locations $\mathcal{L}$. In [13], it is assumed that $L_i^E = \{l_1, l_2, l_3\}$ for all $i \in N$; every player i of ethnicity e_t $(t = 1, 2, 3)$ stays regularly at l_t and sometimes goes to the other locations. He is an *explorer* having active experiences, and has also passive experiences caused by a visiting player from the other locations. The players at location l_t have those experiences uniformly, while possibly some players respond differently when a visiting player comes to l_t. Thus, only two types of players at l_t. They may develop a simple explanatory view of those experiences, and some others may construct a more complicated explanatory view for them ([13], Section 6).

Contrary to the above, we have much more freedom for $\mathcal{L} = \{L_i\}_{i\in N}$. The extreme opposite case is: $L_i^C = \{l_t\}$ for all players i with $e(i) = e_t$ and $t = 1, 2, 3$, i.e., all are *conservative* and nobody makes a trial deviation. In this case, each player has no experiences other than his own festival. Player i at l_t may have no idea about the other locations and no explanation is required. In this case, it is a possible personal view that location l_t is a sole world for him. In fact, any personal view with an imaginary structure about the other world is compatible with his experience.

The above is a poor and uninteresting case. In fact, however, there are many intermediate cases between $\mathcal{L}^E = \{L_i^E\}_{i\in N}$ and $\mathcal{L}^C = \{L_i^C\}_{i\in N}$. In the following, we

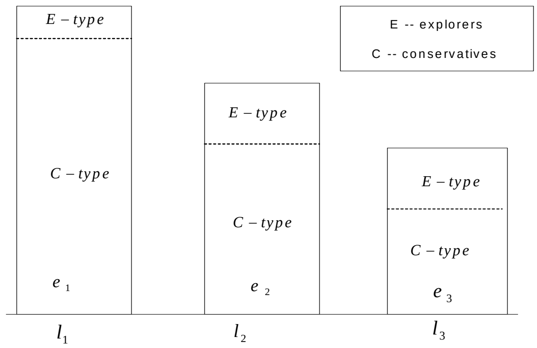

Figure 7.1: Variety of Players of Different Types

consider an intermediate one $\mathcal{L}^M = \{L_i^M\}_{i \in N}$, where at each location l_t, a majority of players are conservative (C-type), and the remaining minority consists of explorers (E-type). This situation is described in Fig.7.1.

Suppose that player i at location l_t, say, $l_t = l_2$, is of C-type. Then, he has only passive experiences to have visiting players from l_1 and l_3. In the following, we assume that a visiting player to l_2 takes only a friendly action. In the minimum discrimination equilibrium described by Corollary 4.4, player i's has the following three experiences:

(1): $[(l_2, 1), \{e_2, e_1\}, m_{l_2}(f) + 1]$ – a visitor comes from location l_1;

(2): $[(l_2, 1), \{e_2\}, m_{l_2}(f)]$ – his regular stay at l_2;

(3n): $[(l_2, 1), \{e_2, e_3\}, m_{l_2}(f) - d_f(e_3 \mid l_2) + 1]$ – a visitor comes from l_3, induces discriminatory responses, but player i is not a discriminator;

(3d): $[(l_2, 1), \{e_2, e_3\}, m_0]$ – a visitor comes from l_3 and he behaves as a discriminator.

Since the number of players is large as stated by F1 in Section 2.1, we assume that the additional $+1$ is ignored.

Consider case (1,2,3n). Then, player i is indifferent about the presence of a visiting player of e_1, i.e., the utility value for (1) is (approximately) the same as that for (2). However, his utility value decreases a lot with the presence of a visiting player of e_3. Hence, he needs to explain this fact. One simple explanation for this pattern is the naive hedonistic view (Kaneko-Matsui [13]). Player i has a preference against e_3 but is indifferent about e_1. In the case of (1,2,3d), an explanation is similar, though his response to the presence of e_3 becomes unfriendly.

When player i is of E-type, he has the following two experiences in addition to (1)-(3) in the minimum discrimination equilibrium:

(4): $[(l_1, 1), E_i(\sigma_{-i}, (l_1, 1)), H_i(\sigma_{-i}, (l_1, 1))] = [(l_1, 1), \{e_1\}, m_{l_2}(f)]$ – player i goes to location l_1;

(5): $[(l_3, 1), E_i(\sigma_{-i}, (l_3, 1)), H_i(\sigma_{-i}, (l_3, 1))] = [(l_3, 1), \{e_3\}, m_{l_3}(f) + 1]$ – player i goes to location l_3.

Observe that (4) is similar to (1). That is, the utility values in (4) and in (1) are (approximately) the same. He may think that this same utility value is caused by the common component e_1 in $[(l_2, 1), \{e_2, e_1\}]$ and $[(l_1, 1), \{e_1\}]$. Hence, (1) and (4) may be explained in the same manner. In the same manner, (3n) and (5) may be explained. Thus, player i does not need to extend his native hedonistic explanation.

The above argument relies upon the minimum discrimination equilibrium of Corollary 4.4. In a different equilibrium, a E-type player needs to develop a more sophisticated personal view than a C-type player.

The purpose of this section is simply to point out that our approach has more potential in the consideration of prejudices. When player i has various different types of coethnic players, he may communicate others to have others' views. In this respect, our approach gives a framework for communication. Also, if he has more knowledge about the entire situation, he could think about more sophisticated views. To consider such a possibility, we would take the epistemic logic approach more seriously, which has been developed but still some distance to our purpose (cf., Kaneko [8], Kaneko-Suzuki [10]).

8. Conclusions

This paper provided an analysis of discrimination and the associated phenomenon of segregation from the perspective of inductive game theory. Social interaction was modeled as the festival game of Kaneko-Matsui [13], which was extended in this paper by introducing additional constraints on the observability of ethnic identities and on locations accessible by the players. These additional components enabled us to study the problem of discrimination in greater detail.

More specifically, we provided a complete characterization of the Nash equilibrium set of the festival game with limited access, adopting the inductive game theory perspective developed in [13] and Kaneko-Kline [11], [12]. We subsequently characterized the set of Nash equilibria relative to a given location configuration. This allowed us to introduce a measure of discrimination, interpreted as the minimum degree of discrimination needed to sustain a given location configuration. We then provided a few applications of this measure.

As mentioned before, the dual constraints of limited observability and limited access lead to a much richer structure of the Nash equilibrium set than is seen in the festival game of Kaneko-Matsui. The modified game environment reveals a greater variety of discriminatory behavior and stable patterns of segregation sustained by such discriminatory behavior. In Sections 5 and 6, we singled out some of these phenomena using the measure of discrimination. However, there are much more phenomena than we have been able to consider in this paper and an exploration of these will be a part of future research.

We have briefly pointed out in Section 7 that the additional structures imposed on the festival game have great potential for the study of prejudices. While a detailed study of prejudice from the perspective of inductive game theory will be given in a future paper, we believe we have taken an initial step in this direction and more generally, in the direction of applying inductive game theory to the study of social phenomena.

Conceptually, an analysis of the emergence of prejudices is important for the continuation of our research on discrimination. In this paper, we have only considered "passive" discrimination in the sense that when a player of a different ethnicity visits a festival, the players have discriminatory responses, but there is no "active" (or institutional) discrimination in the sense that they do not organize political campaigns or develop institutional arrangements to intensify discrimination against some ethnic groups. Active discrimination may occur when prejudices associated with passive discrimination is developed and a trigger is pulled. It will be possible to consider this when passive discrimination and the emergence of prejudices are fully analyzed.

In conclusion, the festival game captures social interaction in a highly abstract manner, concentrating on discriminatory behaviors that arise as a part of group formation and eliminating other social-economic components. Due to this abstraction, we are able to study various forms of segregation patterns and discriminatory behaviors. These are suggestive for empirical studies of intergroup relations. Nevertheless, we admit that our theory cannot directly be connected to empirical studies; both because it is a highly simplified and focused theory on discrimination and prejudices and because in reality, institutional backgrounds such as colonialism may be too significant to be ignored. Our theory is suitable to a heuristic use for the study of discrimination and prejudices.

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