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A Comment on Section 4 of D.P.1002

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A comment on Section 4 of D. P. 1002.

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Abstract. In this paper the author considers the same problem as that in Section 4 of D. P. 1002 (Nogami(2002)) and shows an improved procedure of testing the hypothesis $H_0:\theta=\theta_0$ and the alternative hypothesis $H_1:\theta\neq\theta_0$ with some constant θ_0.

Comment. Let us consider the same problem as that in Section 4 of Nogami(2002). Let $X_1, \ldots, X_n$ be a sample of size n taken from (1) of Nogami(2002). Let $\theta_0=\theta_1+\theta_2$ and $c=\theta_2-\theta_1(>0)$ as in p. 2 of Nogami(2002). Let $X_{(i)}$ be the i-th smallest observation of $X_1, \ldots, X_n$ and define $Y=2^{-1}(X_{(1)}+X_{(n)}-\theta_0)$.

Instead of T in (6) of Nogami(2002) we use

$$S=(Y-\theta)/\{(n+1)(n-1)^{-1}Z/\sqrt{2(n+1)(n+2)}\} \tag{1}$$

where $(n+1)(n-1)^{-1}Z$ is an unbiased estimate for c. Making a variable transformation $S=\sqrt{2(n+1)(n+2)}T$ for $h_T(t)$ in p. 7 of Nogami(2002) we obtain

$$h_S(s)=\sqrt{(n+1)/\{2(n+2)\}}\{(n-1)^{-1}\sqrt{2(n+1)/(n+2)}|s|+1\}^{-n}, \qquad \text{for } 0\leq|s|<\infty.$$

Let α be a real number such that $0<\alpha<1$. We call (U_1,U_2) a $(1-\alpha)$ interval estimate for the parameter ν if $P_\nu[U_1<\nu<U_2]=1-\alpha$. To get the conditional (or restricted) minimum-length $(1-\alpha)$ interval estimate for θ we shall find real numbers r_1 and r_2 $(r_1<r_2)$ which minimize r_2-r_1 subject to

$$P[r_1<S<r_2]=\int_{r_1}^{r_2} h_S(s)\,ds=1-\alpha. \tag{2}$$

Letting λ be a real number we define

$$L = r_2 - r_1 - \lambda\{\int_{r_1}^{r_2} h_S(s)\,ds - 1 + \alpha\}.$$

By Lagrange's method, $\partial L/\partial r_1 = 0 = \partial L/\partial r_2$, which leads to

(3) $\quad h_S(r_1) = h_S(r_2)\,(=\lambda^{-1})$.

Since $\partial L/\partial\lambda = 0$ is equivalent to (2), (2) and (3) leads to $r_2 = -r_1\,(=r)$. Taking

$$\int_r^{\infty} h_S(s)\,ds = \alpha/2$$

we have

$$r = (n-1)\sqrt{(n+2)/\{2(n+1)\}}\,(\alpha^{-1/(n-1)} - 1).$$

Thus, in view of (1) and (2) the conditional minimum-length $(1-\alpha)$ interval estimate for θ is as follows:

(4) $\quad (Y - r[\sqrt{n+1}\;Z/\{(n-1)\sqrt{2(n+2)}\}],\ Y + r[\sqrt{n+1}\;Z/\{(n-1)\sqrt{2(n+2)}\}])$.

Therefore, to test the hypothesis $H_0:\theta=\theta_0$ versus the alternative hypothesis $H_1:\theta\neq\theta_0$ we invert (4) with respect to $\theta=\theta_0$ and get the following acceptance region of our two-sided test.

(5) $\quad -r < (Y-\theta_0)/[\sqrt{n+1}\;Z/\{(n-1)\sqrt{2(n+2)}\}] < r.$

Here, since $\lim_{x\to\infty}\sqrt{(x+3)/\{2(x+2)\}} = 2^{-1/2}$ and $\lim_{x\to\infty} x(\alpha^{-1/x} - 1) = -\log_e \alpha$, it follows that as $n\to\infty$ $r \to -2^{-1/2}\log_e \alpha$. Hence, the acceptance region (5) is more natural than that with t_0 appeared on the 5-th line from the bottom in p. 7 of Nogami(2002).

REFERENCES.

Nogami, Y. (2002). Hypothesis testing based on Lagrange's method: Application to the uniform distribution (II)., Discussion Paper Series No. 1002, Institute of Policy and Planning Sciences, University of Tsukuba, August.