INSTITUTE OF POLICY AND PLANNING SCIENCES

Discussion Paper Series

No. 972

Efficient Location for a Semi-Obnoxious Facility

by

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February 2002

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February 16, 2002

Abstract *We deal with problems for the placement of a facility in a continuous plane with the twin objectives of maximizing the distance to the nearest inhabitant and minimizing the sum of distances to all the users (or the distance to the farthest user) in a unified manner. For special cases, this formulation includes 1) elliptic maximin and rectangular minisum criteria problem, and 2) rectangular maximin and minimax criteria problem. A polynomial-time algorithm for finding the efficient set and tracing out the tradeoff curve for these bicriteria models is presented.*

Keywords: *location; semi-obnoxious facility; Pareto-optimal; tradeoff curve; Voronoi diagram*

1. INTRODUCTION

It is quite evident that there is an increasing realization and awareness of the need to operational location models of semi-obnoxious facilities, which have both *desirable* and *undesirable* effects. Carrizosa and Plastria(1999) has good reviews of the existing literature on the semi-obnoxious facility location. The typical example of such facility is landfills, as pointed out by Daskin(1995). The landfills should be located from inhabitants as far as possible. At the same time, they should be located to reduce hauling vehicle miles needed to transport the waste to the landfill. Thus, decision makers of where to place the landfill face a tradeoff between push and pull objectives. Other examples are airports, incinerators, power plants, fire stations, and police stations. However, there are few analytical papers on the semi-obnoxious facilities location because the standard approaches of convex analysis and computational geometry can be of little help, as emphasized by Carrizosa and Plastria(1999).

This paper is concerned with a single facility bicriteria location problem in a continuous plane where the first objective is to locate a facility as far as possible to habitants, and the second objective is to minimize either the total distance for all users or the distance to the farthest user. The aim of the paper is to present a unifying algorithm for analytically finding the efficient set and tracing out the tradeoff curve between these conflicting objectives.

The most related paper by Ohsawa(2000) examined the location problem formulated by combining Euclidean maximin and minimax criteria. A polynomial-time algorithm is proposed to find the efficient set for this problem, based on the nearest-point and farthest-point Voronoi diagrams. The present research is a extension of this related paper by introducing general push and pull objectives. One interesting special case of our formulation is the location problem with elliptic maximin and rectangular minisum criteria, where different metrics are simultaneously used. Another is the location problem with rectangular maximin and minimax criteria, in which the efficient set may contain areas. The present model considerably extends Ohsawa(2000) such that its framework is much more suitable for analyzing real-world location decisions.

The paper is organized as follows. Section 2 presents a unifying algorithm for identifying the efficient set and the tradeoff curve. Section 3 considers the elliptic maximin and rectangular minisum criteria model. Section 4 deals with the rectangular maximin and minimax criteria model. Section 5 presents the conclusion of the paper. For convenience, all proofs are presented in the Appendix.

2. UNIFIED BICRITERIA MODEL

2.1. Push Objective

Let Ω denote a polygonal study area with $n(\geq 3)$ sides on a continuous plane. Let $\partial\Omega$ be the boundary of Ω. Let I^- be the index set of distinct inhabitants on the plane who repel the facility. Let $\mathbf{p}_i$ represent the site of the i-th inhabitant. As a *push location problem*, we choose a maximin criterion, where a facility is established within Ω in order to maximize the distance from the facility to its nearest inhabitant. This is represented as

$$\max_{\mathbf{x}\in\Omega} \qquad F(\mathbf{x}) \equiv \min_{i\in I^-} \|\mathbf{x}-\mathbf{p}_i\|, \tag{1}$$

where $\|\cdot\|$ is the distance on the continuous plane. The optimal solution is said to be an *anti-center.* We call $F(\mathbf{x})$ *push objective* in relation with Eiselt and Laporte(1995). The function $F(\mathbf{x})$ is neither convex nor concave because the contour of $F(\mathbf{x})$ is given by the set of locus equidistant from the nearest inhabitants.

The *Voronoi polygon* associated with $\mathbf{p}_i$ is defined by $V_i \equiv \bigcap_{k\in I^- \setminus\{i\}}\{\mathbf{x} \mid \|\mathbf{x}-\mathbf{p}_i\| \leq \|\mathbf{x}-\mathbf{p}_k\|\}$. Its boundary is said to be *Voronoi edges.* The anti-center has to lie on either the Voronoi edges or the boundary $\partial\Omega$, as demonstrated in Shamos and Hoey(1975). We call the $\mathbf{x}$ *singular* if and only if the contour curve of $F(\mathbf{x})$ through $\mathbf{x}$ abruptly changes direction at $\mathbf{x}$, i.e., it has different tangents at $\mathbf{x}$. Let S_F denote the set of singular points of the contours of $F(\mathbf{x})$ within Ω. It is evident from definition that $\mathbf{x}$ is singular if and only if it lies on a Voronoi edge. Therefore, S_F is given by the set of the Voronoi edges within Ω.

2.2. Pull Objective

Let I^+ be the index set of distinct users on the plane who attract the facility. Let $\mathbf{q}_i$ represent the site of the i-th users. As a *pull location problem*, we consider both minisum-type and minimax-type. In the first type, a facility is set up within the study area Ω in order to minimize the sum of distance for all the users. Its formulation is

$$\min_{\mathbf{x}\in\Omega} \qquad G(\mathbf{x}) \equiv \sum_{i\in I^+} \|\mathbf{x}-\mathbf{q}_i\|. \tag{2}$$

In the second type, a facility is set up within Ω in order to minimize the distance from the facility to the farthest user. This can be formulated as follows:

$$\min_{\mathbf{x}\in\Omega} \qquad G(\mathbf{x}) \equiv \underset{2}{} \max_{i\in I^+} \|\mathbf{x}-\mathbf{q}_i\|. \tag{3}$$

We call $G(\mathbf{x})$ *pull objective* in line with Eiselt and Laporte(1995). It is evident from (2) and (3) that the function $G(\mathbf{x})$ is convex in both types. In this paper, in order to formulate the pull location in a unified manner, these two solutions are both said to be *center.* Let S_G denote the set of singular points of the contours of $G(\mathbf{x})$.

The *farthest-point Voronoi polygon* associated with $\mathbf{q}_i$ is defined by $W_i \equiv \bigcap_{k \in I^+ \setminus \{i\}} \{\mathbf{x} \mid \|\mathbf{x}-\mathbf{p}_i\| \geq \|\mathbf{x}-\mathbf{p}_k\|\}$. Its boundary is said to be *farthest-point Voronoi edges.* In the minimax-type, the center has to lie on either the farthest-point Voronoi edges or the boundary $\partial\Omega$: see Shamos and Hoey(1975). Also, $\mathbf{x}$ is singular point of the contour of $G(\mathbf{x})$ if and only if it lies on a farthest-point Voronoi edge: see Ohsawa and Imai(1997). Thus, in the minimax-type, S_G is given by the set of the farthest-point Voronoi edges within Ω.

In contrast with the minimax-type, the analytical expression of the optimal solution in the Euclidean distance minisum-type cannot be known so far. Therefore, it seems to be impossible to define the set S_G in a unified manner that encompasses any kind of metric.

2.3. Efficient Location

Let us examine a bicriteria problem produced by combining the push and pull objectives. We say that for any two distinct points $\mathbf{x}, \mathbf{y} \in \Omega$, $\mathbf{x}$ *dominates* $\mathbf{y}$ if and only if $F(\mathbf{y}) \leq F(\mathbf{x})$ and $G(\mathbf{y}) \geq G(\mathbf{x})$, with at least one strict inequality. The point $\mathbf{x}$ is said to be *efficient* (*Pareto-optimal*) if and only if there does not exist $\mathbf{y} \in \Omega$ which dominates $\mathbf{x}$. The set of the efficient points, denoted by E^*, is called the *efficient set.* This set determines the options from which decision makers choose. In the objective space generated by the push and pull objectives, the locus corresponding to the efficient set is said to be the *tradeoff curve.* This curve is readily comprehensible by the decision-makers, and it provides a base for location choice decisions.

Let $\rho_F(\mathbf{x})$ and $\rho_G(\mathbf{x})$ be the radius of curvature of the contour of $F(\mathbf{x})$ and $G(\mathbf{x})$ through $\mathbf{x}$, respectively. Throughout the rest of this paper, let $\mathbf{a}^*$ be an *efficient* anti-center and $\mathbf{c}^*$ be an *efficient* center, respectively.

PROPOSITION 1 *If $\rho_F(\mathbf{x}) < \rho_G(\mathbf{x})$ for any $\mathbf{x} \in \Omega \setminus (S_F \cup S_G)$, then 1) the tradeoff curve can be defined over every value between $F(\mathbf{c}^*)$ and $F(\mathbf{a}^*)$; and 2) $E^* \subseteq S_F \cup S_G \cup \partial\Omega$.*

Note that $\Omega \setminus (S_F \cup S_G)$ indicates the set of sites within Ω except singular points. The proof of this proposition is provided in Appendix A.1.

The second claim of this proposition implies that it is sufficed to search for a quite small (possibly one-dimensional) region $S_F \cup S_G \cup \partial\Omega$. Combining this with the first claim yields the assertion that the tradeoff curve coincides with the lower envelope of the collection of the curves corresponding to $S_F \cup S_G \cup \partial\Omega$ between $F(\mathbf{c}^*)$ and $F(\mathbf{a}^*)$, where $F(\mathbf{x})$ is measured along the horizontal scale and $G(\mathbf{x})$ along the vertical. This leads directly to the procedure, which extends the technique proposed by Ohsawa(2000):

Step 1. In geographical space, construct $S_F \cup S_G \cup \partial\Omega$.

Step 2. In objective space, draw the loci $(F(\mathbf{x}), G(\mathbf{x}))$ corresponding to $S_F \cup S_G \cup \partial\Omega$.

Step 3. In the objective space, find the lower envelope of the loci between $F(\mathbf{c}^*)$ and $F(\mathbf{a}^*)$.

Step 4. In the geographical space, determine the links corresponding to this lower envelope.

In order to check that the inequality $\rho_F(\mathbf{x}) < \rho_G(\mathbf{x})$ for any $\mathbf{x}$ within $\Omega \setminus (S_F \cup S_G)$, we need a full geometrical characterization of contours of both push and pull objectives. We shall give a precise account of this inequality for two seemingly unrelated bicriteria problems.

3. ELLIPTIC MAXIMIN AND RECTANGULAR MINISUM CRITERIA

3.1. Push Objective

As pointed out in Karkazis and Papadimitriou(1992), Plastria and Carrizosa(1999), the odor and noise cannot spread according to road maps. The odor and noise come from the wind and the nuisance depends on from what direction the wind blows. For example, in seaside towns a wind blows down a mountain in the morning, but a wind blows from the sea in the afternoon. In order to express the effect of wind, we introduce the elliptic distance into the push objective, as in Plastria(1992). The elliptic distance between the facility $\mathbf{x}$ and the inhabitant $\mathbf{p}_i$ is defined as

$$\|\mathbf{x} - \mathbf{p}_i\|_e \equiv \sqrt{\kappa(x_i - x)^2 + 2\lambda(x_i - x)(y_i - y) + \mu(y_i - y)^2},$$

with $\kappa\mu > \lambda^2$. Since the three parameters κ, λ and μ specify the elongation in two direction and a rotation of an ellipse, this distance can take account of a variety of direction and velocity of the wind by changing these parameters. As a special case, if $\kappa = 1$, $\lambda = 0$ and $\mu = 1$, then the elliptic distance reduces to the Euclidean one.

We consider the following maximin problem under the elliptic distance as a push objective:

$$\max_{\mathbf{x}\in\Omega} F(\mathbf{x}) \equiv \min_{i\in I^-} \|\mathbf{x} - \mathbf{p}_i\|_e, \tag{4}$$

The corresponding singular point set S_F is defined by the elliptic distance Voronoi edges within Ω. An example is presented in Figure 1, where five inhabitants $\mathbf{p}_1, \cdots, \mathbf{p}_5$ are indicated by bullets within a square study area. In this figure, the directions of the major semi-axes make 45° angle with the x-th axis, and the magnification in the direction of the major semi-axis is twice as large compared to the magnification in the direction of the minor semi-axis. As demonstrated in Scheike(1994), these edges are constructed through the method to delineate a classical Voronoi diagram and an affine transformation. This means that S_F includes $O(|I^+|)$ line segments. The anti-center $\mathbf{a}^*$ is also indicated by an asterisk in Figure 1. It is evident from the definition of the elliptic distance that any contour is identified by the boundary of the union of $|I^+|$ ellipses centered on $\mathbf{p}_i$'s, as presented in this figure.

3.2. Pull Objective

The rectangular distance is applicable to a grid of streets where travel occurs along an orthogonal set of roads. We may assume without loss of generality that the direction of travels in the rectangular distance is parallel to the coordinate axes. Hence, the rectangular distance between the facility $\mathbf{x}$ and the user $\mathbf{q}_i$ can be defined as

$$\|\mathbf{x} - \mathbf{q}_i\|_1 \equiv |x_i - x| + |y_i - y|.$$

As a pull objective, we introduce the following rectangular minisum criterion:

$$\min_{\mathbf{x}\in\Omega} G(\mathbf{x}) \equiv \sum_{i\in I^+} \|\mathbf{x} - \mathbf{q}_i\|_1. \tag{5}$$

The vertical lines through $\mathbf{q}_i$'s are numbered $1, \cdots, |I^+|$ from left to right, and the horizontal lines through $\mathbf{q}_i$'s are numbered $1, \cdots, |I^+|$ from bottom to top. These lines divide the whole region into $(|I^+|+1)^2$ blocks. This situation is shown in Figure 2, where five users $\mathbf{q}_1, \cdots, \mathbf{q}_5$ are indicated by small circles within the same study area with Figure 1. As indicated by Francis and White(1974), if $|I^+|$ is even, then the solution is identified by an intersection point of the $(n+1)/2$-th horizontal and the $(n+1)/2$-th vertical lines. Otherwise, it is defined by the block which is delimited by vertical lines $n/2$ and $n/2+1$ and horizontal lines $n/2$ and $n/2+1$. For example, the solution $\mathbf{c}^*$ is designated as an asterisk in Figure 2. Francis and White(1974) also showed that the contour of $G(\mathbf{x})$ within each block is made up of only a line, as presented in the figure. Thus, the singular point set S_G is defined by n horizontal and n vertical lines within Ω.

3.3. Efficient Location

Let us study the location problem obtained by combining the objectives (4) and (5). This model is applicable to semi-obnoxious facilities such as incinerators and landfills. Dioxin discharged by incinerators is subjected to meteorological condition. Fuel consumed by garbage vehicles is proportional to their distances travelled. Both the pollution emission and the fuel consumption should be reduced from environmental point of view.

Any elliptic curve has finite radius of curvature, while any line has infinite radius of curvature. This indicate that $\rho_F(\mathbf{x}) < \rho_G(\mathbf{x})$ for any $\mathbf{x} \in \Omega \setminus (S_F \cup S_G)$.

PROPOSITION 2 *The efficient set associated with elliptic maximin and rectangular minisum criteria can be found in $O(n|I^-||I^+|\log n|I^-||I^+|)$ time.*

See Appendix A.2 for derivation of this proposition. It should be noted that if $|I^+|$ is even, then the center is given by a block, but the efficient center is defined by one point of the boundary of the block.

The tradeoff curve and the efficient set corresponding to Figures 1 and 2 have been solved by our procedure. The tradeoff curve consists of three discontinuous parts. This is indicated by the curve joining $\mathbf{z}_1$ and $\mathbf{z}_2$; the curve joining $\mathbf{z}_3$ and $\mathbf{z}_4$; and the curve joining $\mathbf{z}_5$ and $\mathbf{z}_6$ in Figure 3 where the graphs corresponding to S_F, S_G and $\partial\Omega$ are designated by the thin, broken and thick segments in connection with Figures 1 and 2. The efficient set consists of four discontinuous pieces, as indicated by bold segments in Figure 4. It should be noted that the network in Figure 3 has the same topological structure with the network $S_F \cup S_G \cup \partial\Omega$ in Figure 4. This is due to continuity. As the decision-makers weight the relative importance of the pollution emission to the fuel consumption, the alternative to be selected moves from the center $\mathbf{c}^*$ towards the anti-center $\mathbf{a}^*$ through $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_9$. In order to confirm the process of the jumps that occur in the geographical space, Figure 4 exhibits four ellipses by dotted segments: the smallest ellipse passes through $\mathbf{v}_2$ and $\mathbf{v}_3$; two immediate ellipses pass through $\mathbf{v}_5$ and $\mathbf{v}_6$, respectively; and the largest ellipse passes through $\mathbf{v}_7$ and $\mathbf{v}_8$.

In order to understand how wind direction and velocity affects the efficient set, we consider two other elliptic distances for the same inhabitant and user distributions: 1) the Euclidean distance, and 2) the elliptic distance where the directions of the major semi-axes make 135° angle with the x-axis, and the magnification is the same with the original distance. The corresponding efficient sets are presented by bold segments in Figures 5 and 6, respectively. Comparing these figures with Figure 4 implies that the effect of wind has a serious influence on the efficient sets from geographical point of view.

Our procedure is applicable to the elliptic maximin and minimax bicriteria problem, where the inhabitant set coincides with the user one. In this situation, S_F is defined by the elliptic farthest-point Voronoi edges. Since the elliptic distance to the farthest user is larger than the one to the nearest inhabitant, we have the inequality $\rho_F(\mathbf{x}) < \rho_G(\mathbf{x})$.

4. RECTANGULAR MAXIMIN AND MINIMAX CRITERIA

4.1. Push Objective

Instead of the elliptic distance, we introduces the rectangular distance into a push objective:

$$\max_{\mathbf{x}\in\Omega} F(\mathbf{x}) \equiv \min_{i\in I^-} \|\mathbf{x}-\mathbf{p}_i\|_1, \tag{6}$$

Therefore, the singular point set S_F is defined by the rectangular Voronoi edges. Note that if the line through two inhabitants is positioned at 45° angle with the coordinate axes, then the rectangular bisector between them is not a unique boundary but rather areas, as indicated by Lee(1980). When we use the actual inhabitants' sites, they are in general position. So we assume that the line through any two inhabitants cannot make 45° angle with the coordinate axes. As shown in Lee(1980), these edges are $O(|I^-|)$ line segments. Figure 7 provides an example of the rectangular Voronoi diagram associated with five inhabitants $\mathbf{p}_1,\cdots,\mathbf{p}_5$ within a square study area. The anti-center $\mathbf{a}^*$ is also indicated by an asterisk in this figure. Any contour of this objective function is given by the boundary of the union of $|I^+|$ squares centered on $\mathbf{p}_i$'s positioned at 45° angle with the coordinate axes, as shown in the figure.

4.2. Pull Objective

Under the rectangular pull objective, we adopts the minimax-type:

$$\min_{\mathbf{x}\in\Omega} G(\mathbf{x}) \equiv \max_{i\in I^+} \|\mathbf{x}-\mathbf{q}_i\|_1. \tag{7}$$

As examined in Elzinga and Hearn(1972), the solution to this optimization problem is identified by the center of the smallest square such that 1) it contains all the users $\mathbf{q}_i$'s; and 2) its four sides all make 45° angle with the coordinate axes. In general, the center $\mathbf{c}^*$ cannot be uniquely defined. The singular point set S_G is given by the rectangular farthest-point Voronoi edges within the study area Ω, as examined in Ohsawa and Imai(1997). Analoguous to the inhabitants' sites, we assume that the line through any two users cannot make 45° angle with the coordinate axes. It is worth noting that there are at most four rectangular farthest-point Voronoi polygons, i.e., at most five Voronoi edges. In addition, these edges are linear. Figure 8 indicates the rectangular farthest-point Voronoi diagram associated with five users $\mathbf{q}_1, \cdots, \mathbf{q}_5$ within the same study area with Figure 7. As displayed in Figure 8, any contour of the rectangular minimax criterion is a rectangle rotating 45° angle with the coordinate axes. The center is defined by a line segment joining $\mathbf{c}_1^*$ and $\mathbf{c}_2^*$, which appears as a shaded line. Two smallest enclosing squares rotating 45° from the coordinate axes centered on $\mathbf{c}_1^*$ and $\mathbf{c}_2^*$ are also drawn by dotted segments.

4.3. Efficient Location

Combining the objectives (6) and (7) yields a bicriteria model, which is rectangular version of Ohsawa(2000). This model seems to be applicable to semi-obnoxious facilities such as fire stations and police stations. Inhabitants want to be located farther from such a facility because of unpredictable traffic congestion and noise. On the other hand, the distance from a facility to the farthest user should be small from the equity viewpoint.

Since the contours of $F(\mathbf{x})$ and $G(\mathbf{x})$ are both linear, i.e., $\rho_F(\mathbf{x}) = \rho_G(\mathbf{x})$, we cannot directly apply our procedure to this model. Specifically, the procedure can trace out the exact expression of the tradeoff curve, but it can produce only a portion of the efficient set. The efficient set may consist of not only line segments but also areas. An equidistant line from their nearest inhabitant can be simultaneously equidistant from their farthest user. However, the efficient location outside $S_F \cup S_G \cup \partial\Omega$ are always located between two points within $S_F \cup S_G \cup \partial\Omega$. Therefore, we can obtain the complete expression of the efficient set by doing one more step after Steps 1-4.

Step 5. In geographical space, draw a line segment joining two distinct points which are transformed into the same point on the lower envelope.

Then, the efficient set is given by the links defined in Step 4, and the areas encompassed by these links and the line segments identified in Step 5.

PROPOSITION 3 *The efficient set associated with rectangular maximin and minimax criteria can be constructed in $O(n|I^-|\log n|I^-| + |I^+|)$ time.*

The proof of this proposition is given in Appendix A.3.

The tradeoff curve and the efficient set corresponding to Figures 7 and 8 have been obtained by use of our procedure. The tradeoff locus is formed at one point and three discontinuous segments. This is illustrated in Figure 9 where the plots corresponding to

S_F, S_G and $\partial\Omega$ are designated by the thin, broken and thick segments with reference with Figures 7 and 8. The efficient set consists of the center $\mathbf{c}^*$ and three areas, which appear as shaded areas, denoted as A_1, A_2 and A_3 in Figure 10. These areas A_1, A_2 and A_3 are delimited by the segment between $\mathbf{u}_1$ and $\mathbf{u}_2$, the segment between $\mathbf{v}_1$ and $\mathbf{v}_2$ and the segment between $\mathbf{w}_1$ and $\mathbf{w}_2$, respectively. As the decision-makers weights the relative importance of the desirable characteristic to the undesirable one, the alternative to be selected moves from the center $\mathbf{c}^*$ towards the anti-center $\mathbf{a}^*$ through three regions A_1, A_2 and A_3. To confirm the process of the jumps occurred in the geographical space, four squares are indicated by dotted segments in Figure 10: the smallest square passes through $\mathbf{c}^*$, $\mathbf{u}_1$ and $\mathbf{u}_2$; two immediate squares pass through $\mathbf{u}_6$ and $\mathbf{v}_1$, respectively; and two largest squares pass through $\mathbf{v}_3$ and $\mathbf{w}_1$, respectively.

Three other remarks peculiar to this rectangular model are presented. First, the tradeoff curve is made up of only the lines of a slope +1 in the objective space: see Figure 9. This means that both the tradeoff curve and the efficient set can be readily traced by use of broadly available computer codes. Second, the line segment which makes 45° angle with the coordinate axes in the geographical space can be expressed as a point in the objective space. For example, the line segment joining $\mathbf{u}_3$ and $\mathbf{u}_4$ in Figure 10 is plotted by $\mathbf{z}_2$ in Figure 9. This is because both the nearest and the farthest distances of all points on the segment are the same. Finally, several loci may overlap each other in the objective space. For example, the path from $\mathbf{u}_2$ to $\mathbf{u}_6$ through $\mathbf{u}_5$, and the path from $\mathbf{u}_1$ to $\mathbf{u}_6$ through $\mathbf{u}_3$ and $\mathbf{u}_4$ in Figure 10 are both expressed as the same segment connecting $\mathbf{z}_1$ with $\mathbf{z}_3$ in Figure 9. In addition, the segment connecting $\mathbf{w}_1$ with $\mathbf{a}^*$, and the one connecting $\mathbf{w}_2$ with $\mathbf{a}^*$ are both expressed as the same segment joining $\mathbf{z}_4$ and $\mathbf{z}_5$.

The weighted scalarization problems are examined in Mehrez et al.(1985) and Drezner et al.(1986). Mehrez et al.(1985) proved that if the inhabitant set coincides with the user set, then the solution to the weighted problems lies at an intersection point of the rectangular bisectors between any pair of users. Drezner et al.(1986) demonstrated that if both objective functions (6) and (7) are assigned by an equal weight, the solution lies either at a nearest-point or a farthest-point Voronoï edge. Thus, our finding is consistent with their results.

Our procedure from Steps 1 to 5 is applicable to the rectangular maximin and minisum bicriteria problem, where the efficient set may contains an area. Konfurty and Tamir(1997) examined both the rectilinear minisum and minimax problems with minimum rectilinear distance constraints. Theoretically speaking, the efficient set is given by the solution to all possible restricted problems obtained by changing the minimum threshold. However, their proposed solution method is specialized to the problems for a fixed minimum distance constraint. Our procedure can identify all at once not only the efficient set but also the tradeoff curve different from their work.

5. CONCLUDING REMARKS

Location models are constantly evolving to accommodate the requirements of specific applications. In addition, location for semi-obnoxious facility that provides both services and some damage is certainly an exciting field of research.

In this paper, we first presented a unifying technique for tracing out the tradeoff curve and then finding the efficient set associated with such facility in a continuous plane. Second we examined the efficient set and the tradeoff curve for the elliptic maximin and rectangular minisum bicriteria location problem, and the rectangular maximin and minimax bicriteria location problem.

ACKNOWLEDGMENT

This research has been supported by the Research Grant of the Ministry of Education, Science, Sports and Culture of Japan, Grant No. 12680430(2000,2001). The authors are grateful to Takeshi Koshizuka, Hiroshi Kotoh and Osamu Kurita.

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Mathematical Appendix

A.1. Proof of Proposition 1

For specified values $\alpha(\geq 0)$ and $\beta(\geq 0)$, define $L_F(\alpha)$ and $L_G(\beta)$ by $L_F(\alpha) \equiv \{\mathbf{x} \in \Omega | F(\mathbf{x}) = \alpha\}$ and $L_G(\beta) \equiv \{\mathbf{x} \in \Omega | G(\mathbf{x}) = \beta\}$, respectively. Thus, $L_F(\alpha)$ and $L_G(\beta)$ symbolize a contour of $F(\mathbf{x})$ and $G(\mathbf{x})$, respectively. Consider a restricted problem in which the objective is to minimize $G(\mathbf{x})$ subject to the constraint $\mathbf{x} \in L_F(\alpha)$. Let $\mathbf{s}^*(\alpha)$ denote the solution of this restricted problem.

First, we prove that $E^* = \{\mathbf{s}^*(\alpha) | F(\mathbf{c}^*) \leq \alpha \leq F(\mathbf{a}^*)\}$, leading to the first claim. It is straightforward to see that $E^* \subseteq \{\mathbf{s}^*(\alpha) | 0 \leq \alpha < \infty\}$. For all α with $0 \leq \alpha < F(\mathbf{c}^*)$, $\mathbf{s}^*(\alpha) \prec \mathbf{c}^*$ since $F(\mathbf{s}^*(\alpha)) = \alpha < F(\mathbf{c}^*)$ and $G(\mathbf{s}^*(\alpha)) > G(\mathbf{c}^*)$. In addition, for all α with $F(\mathbf{a}^*) < \alpha$, $\mathbf{s}^*(\alpha)$ can not be defined since $L_F(\alpha) = \phi$. Therefore, we get $E^* \subseteq \{\mathbf{s}^*(\alpha) | F(\mathbf{c}^*) \leq \alpha \leq F(\mathbf{a}^*)\}$. On the other hand, for $F(\mathbf{c}^*) \leq \alpha < \beta < \gamma \leq F(\mathbf{a}^*)$, $F(\mathbf{x}) < F(\mathbf{s}^*(\beta))$ for any $\mathbf{x} \in L_F(\alpha)$. This means that $\mathbf{s}^*(\beta)$ cannot be dominated by $\mathbf{x}$. Since $G(\mathbf{x})$ is quasi-convex, $G(\mathbf{s}^*(\beta)) < G(\mathbf{s}^*(\gamma))$, indicating that $G(\mathbf{s}^*(\beta)) < G(\mathbf{x})$ for any $\mathbf{x} \in L_F(\gamma)$, i.e., $\mathbf{s}^*(\beta)$ cannot be dominated by $\mathbf{x}$. This means that $E^* \supseteq \{\mathbf{s}^*(\alpha) | F(\mathbf{c}^*) \leq \alpha \leq F(\mathbf{a}^*)\}$.

Next, we demonstrate that $\{\mathbf{s}^*(\alpha) | F(\mathbf{c}^*) \leq \alpha \leq F(\mathbf{a}^*)\} \subseteq S_F \cup S_G \cup \partial\Omega$. Suppose now that $\mathbf{s}^*(\alpha) \notin S_F \cup S_G \cup \partial\Omega$. The contour $L_G(G(\mathbf{s}^*(\alpha)))$ has to be inscribed in the contour $L_F(\alpha)$ due to the quasi-convexity of $G(\mathbf{x})$. However, this contradicts $\rho_F(\mathbf{x}) < \rho_G(\mathbf{x})$. In conclusion, $\mathbf{s}^*(\alpha) \in S_F \cup S_G \cup \partial\Omega$. Combining $E^* = \{\mathbf{s}^*(\alpha) | F(\mathbf{c}^*) \leq \alpha \leq F(\mathbf{a}^*)\}$ with this yields $E^* \subseteq S_F \cup S_G \cup \partial\Omega$, i.e., the second claim. □

A.2. Proof of Proposition 2

The elliptic Voronoi diagram can be constructed in $O(|I^-|\log|I^-|)$ time: see Scheike(1994). The vertical and horizontal lines can be build in $O(|I^+|)$ time. Therefore, Step 1 can be done in $O(|I^-|\log|I^-| + |I^+|)$ time. Since each Voronoi edge intersects these lines at $O(|I^+|)$ time, there are $O(n|I^-||I^+|)$ links within the network $S_F \cup S_G \cup \partial\Omega$. Thus, Step 2 requires $O(n|I^-||I^+|)$ operations. As we shall see later, the loci corresponding to any two links of $S_F \cup S_G \cup \partial\Omega$ intersect each other at most twice in objective space. Hence, Step 3 requires $O(n|I^-||I^+|\log n|I^-||I^+|)$ time in the worst case, as shown in Boissonnat and Yvinec(1998). Since the number of the loci on the lower envelope is $O(n|I^-||I^+|)$, Step 4 can be carried out in $O(n|I^-||I^+|)$ time. Therefore, the maximum time complexity, which equals the time complexity of Step 3, is $O(n|I^-||I^+|\log n|I^-||I^+|)$.

Let us now return to prove that the loci corresponding to any two links of $S_F \cup S_G \cup \partial\Omega$ intersect each other at most twice. Consider a link l such that $l \in V_i \cap B_{j,k}$. Assume that l is on a liner equation $\alpha x + \beta y = \gamma$. Any point $\mathbf{x}$ on l fulfills the following three equations:

$$\begin{aligned} F(\mathbf{x}) &= \kappa(x_i - x)^2 + 2\lambda(x_i - x)(y_i - y) + \mu(y_i - y)^2; && (8) \\ G(\mathbf{x}) &= \sum_{z\in\{z|x_z>x_j\}} x - x_z + \sum_{z\in\{z|x_z<x_j\}} x_z - x + \sum_{z\in\{z|y_z>y_k\}} y - y_z + \sum_{z\in\{z|y_z<y_k\}} y_z - y && (9) \\ \beta y &= -\alpha x + \gamma. && (10) \end{aligned}$$

It follows from this systems that the loci corresponding to any two links intersect each other at most twice. □

A.3. Proof of Proposition 3

The rectangular nearest-point and farthest-point Voronoi diagrams can be in $O(|I^-|\log|I^-|)$ and $O(|I^+|)$ time. So, Step 1 can be carried out in $O(|I^-|\log|I^-|+|I^+|)$ time. The network $S_F \cup S_G \cup \partial\Omega$ contains $O(n|I^-|)$ links, Step 2 can be done in $O(n|I^-|)$ time. As we shall see later, the loci corresponding to any two links of $S_F \cup S_G \cup \partial\Omega$ intersect each other at once. Therefore, Step 3 requires $O(n|I^-|\log n|I^-|)$ time in the worst case. Since the number of the loci on the lower envelope is $O(n|I^-|)$, Step 4 can be carried out in $O(n|I^-|)$ time. Step 5 can be done in $O(n|I^-|)$ time. Therefore, the total complexity is given by $O(n|I^-|\log n|I^-|+|I^+|)$.

We must now return to verify that the loci corresponding to any two links of $S_F \cup S_G \cup \partial\Omega$ intersect each other at once. If 1) $\mathbf{x}$ lies on the link l with $\alpha x + \beta y = \gamma$, 2) $\mathbf{p}_i$ is the nearest habitant from $\mathbf{x}$, and 3) $\mathbf{q}_j$ is the nearest user, then we have

$$\begin{aligned} F(\mathbf{x}) &= |x_i - x| + |y_i - y|; \\ G(\mathbf{x}) &= |x_j - x| + |y_j - y|; \\ \beta y &= -\alpha x + \gamma. \end{aligned}$$

It follows from this simultaneous linear equations that $F(\mathbf{x})$ is linear with respect to $G(\mathbf{x})$. This means that the loci corresponding to any two links intersect each other at once □

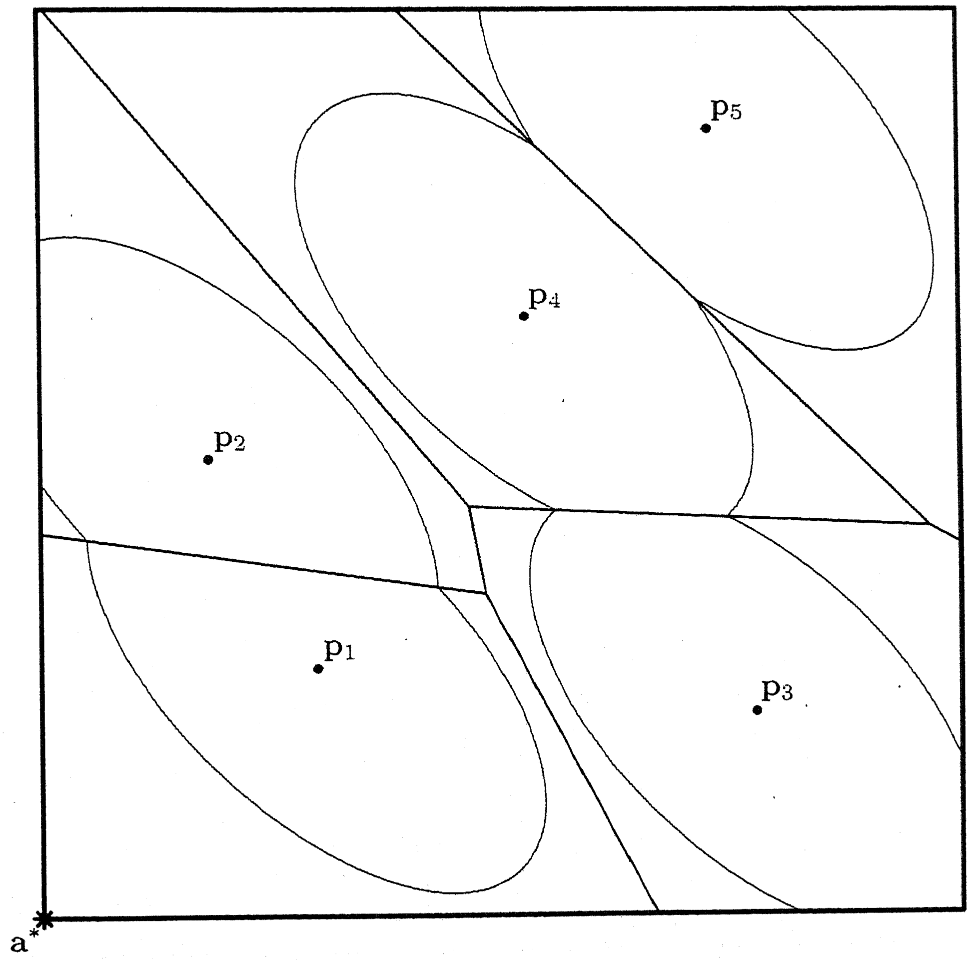

Figure 1: Elliptic Voronoi diagram, and contour of maximin criterion

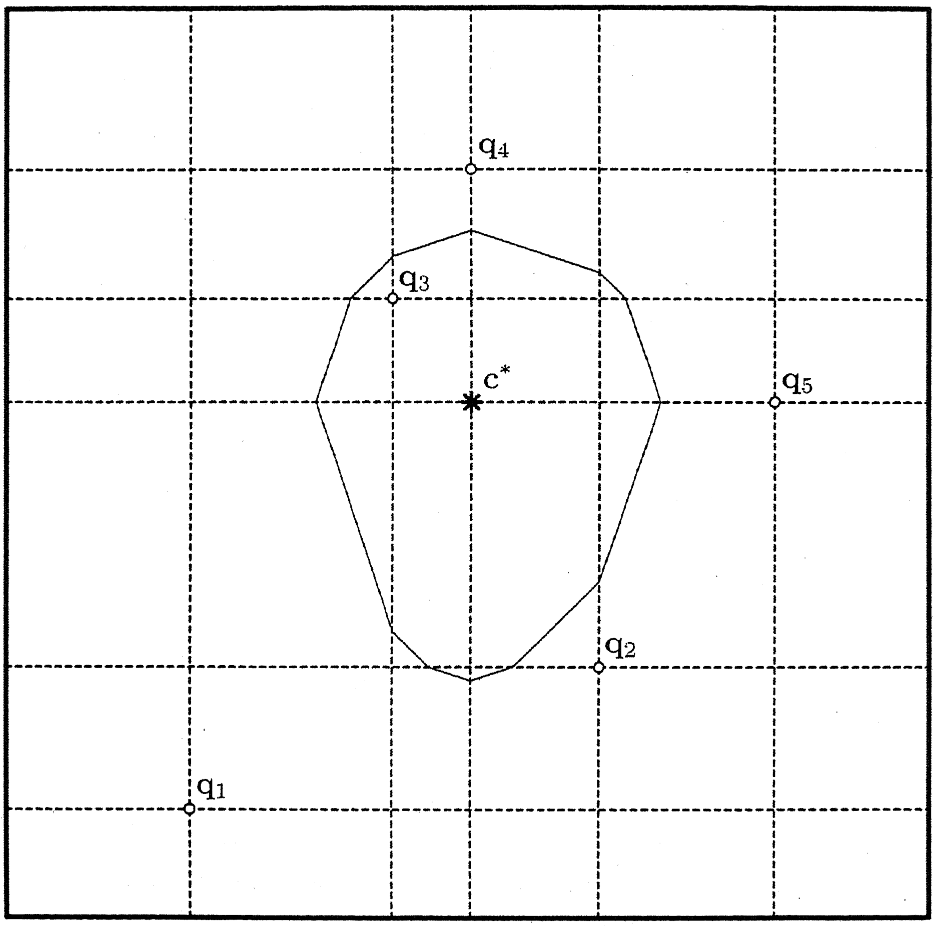

Figure 2: Parallels, and contour of rectangular minisum criterion

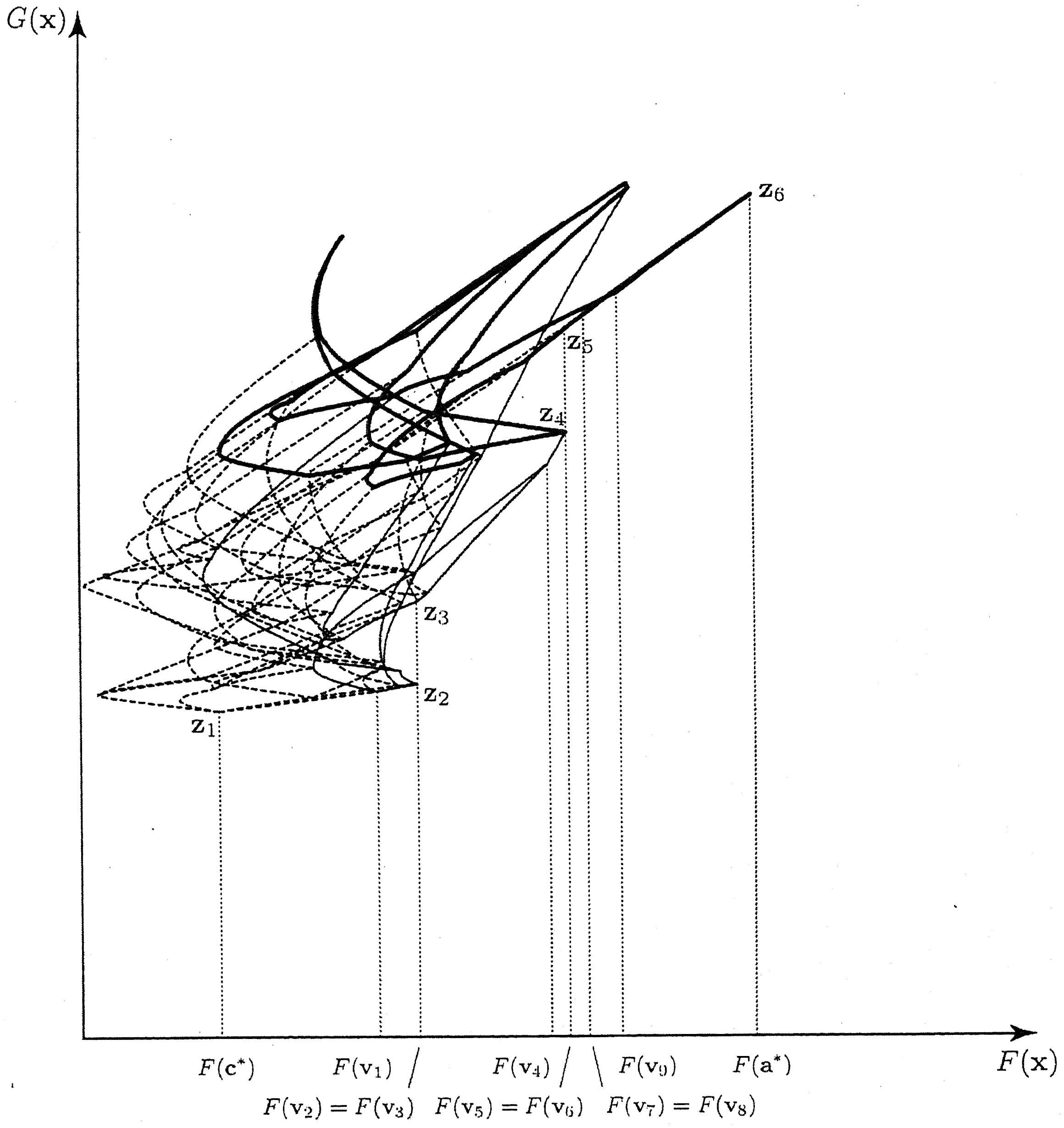

Figure 3: Tradeoff curve of elliptic maximin and rectangular minisum criteria

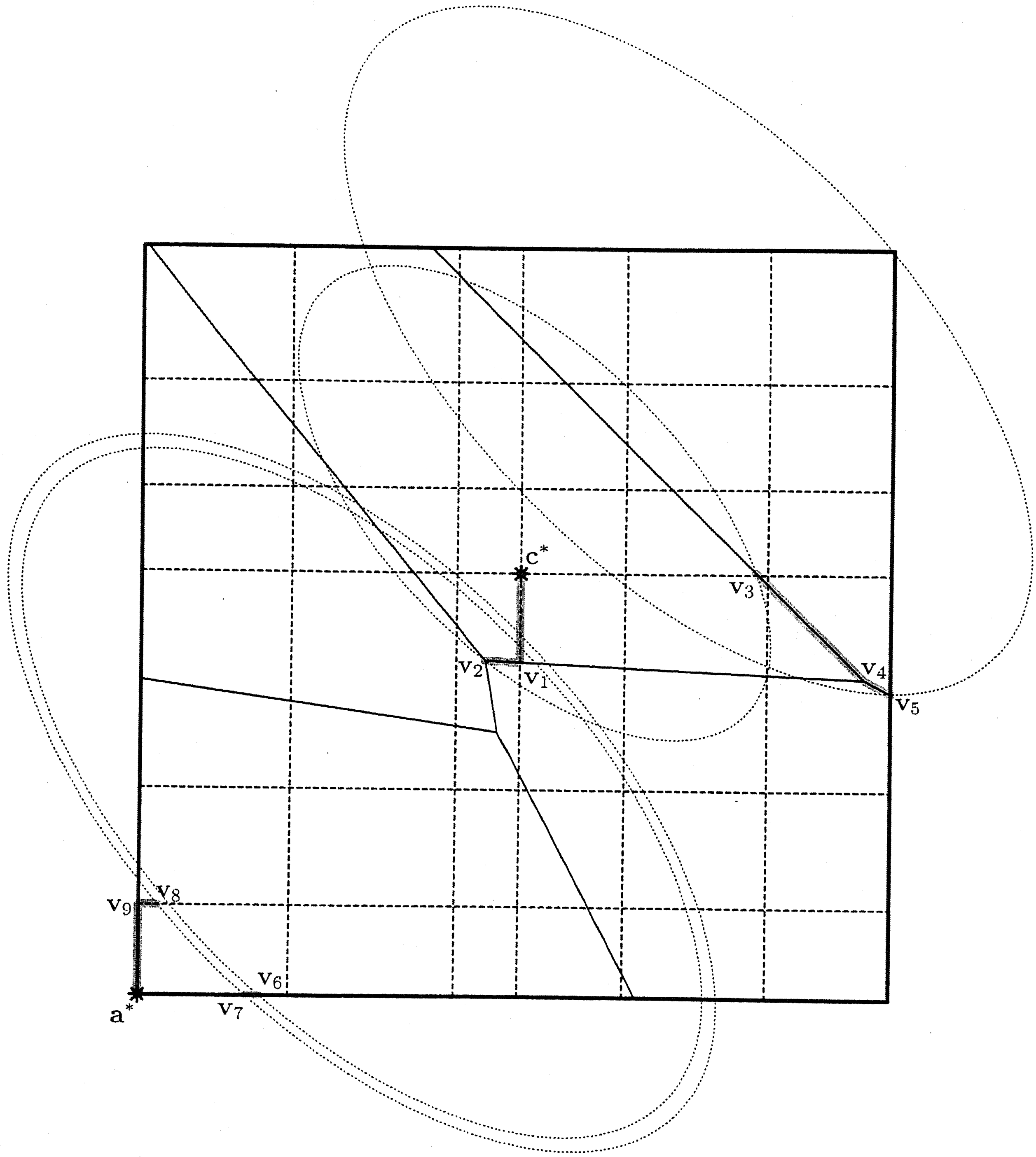

Figure 4: Efficient set of elliptic maximin and rectangular minisum criteria

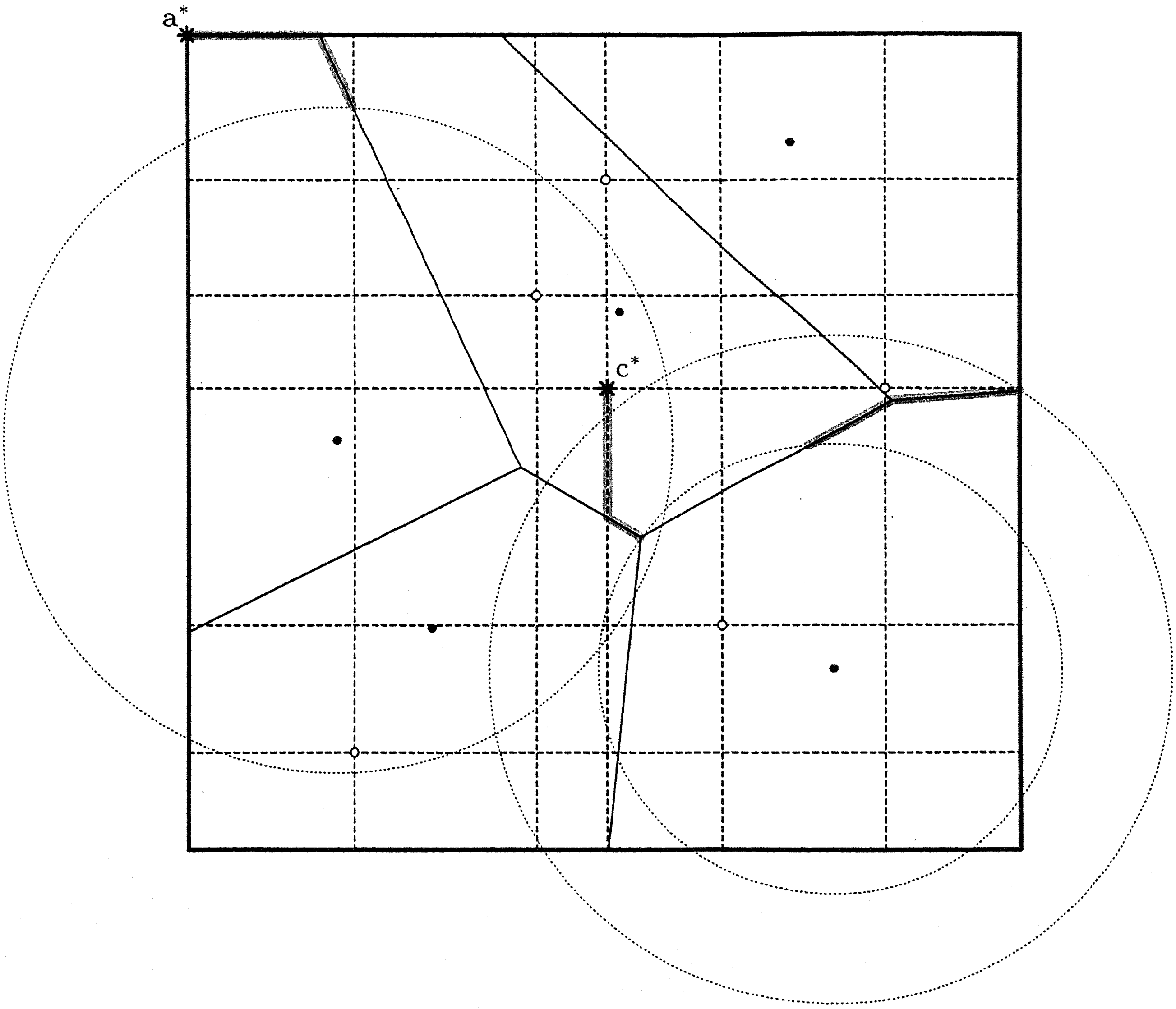

Figure 5: Efficient set of Euclidean maximin and rectangular minisum criteria

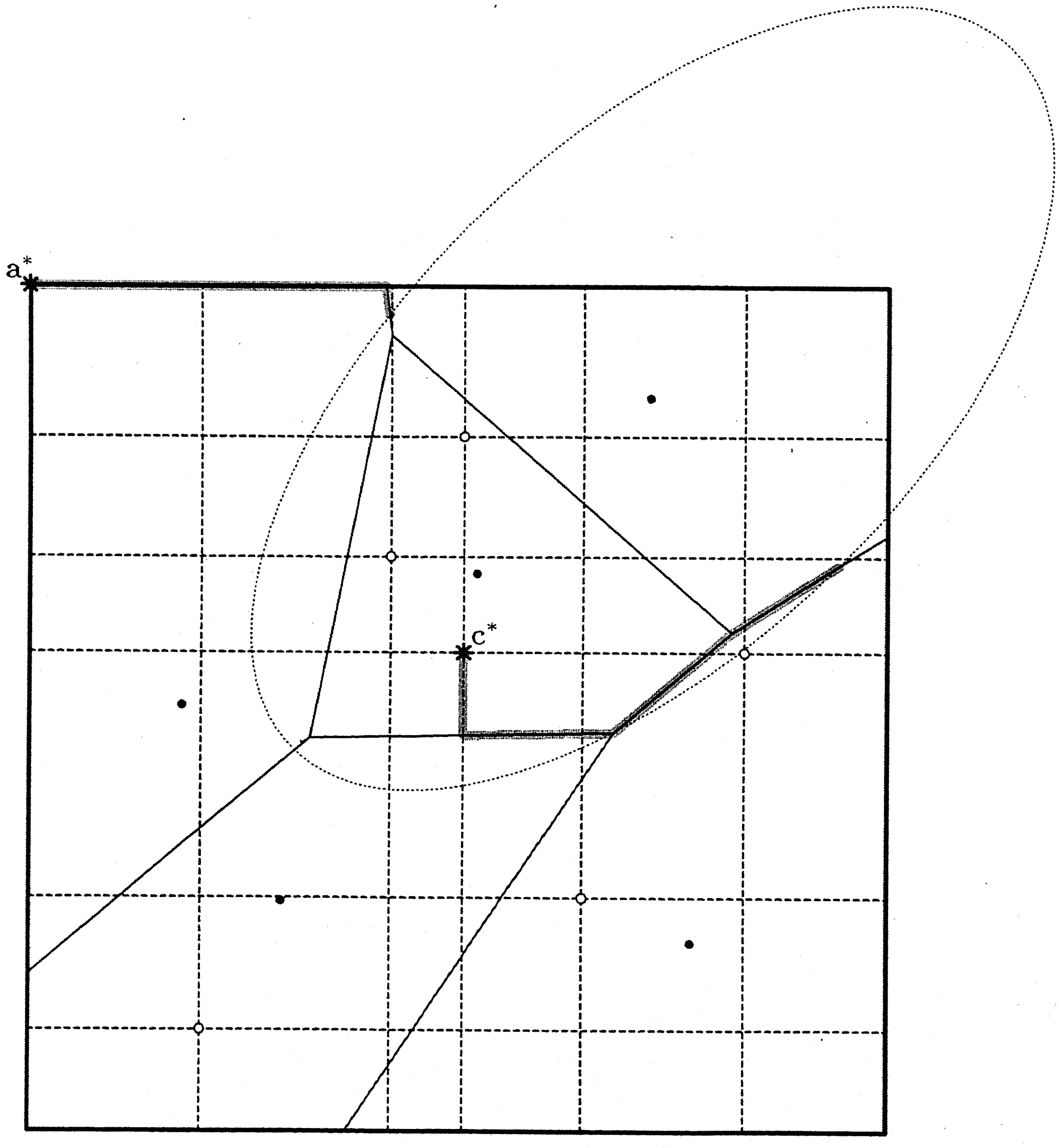

Figure 6: Efficient set of another elliptic maximin and rectangular minisum criteria

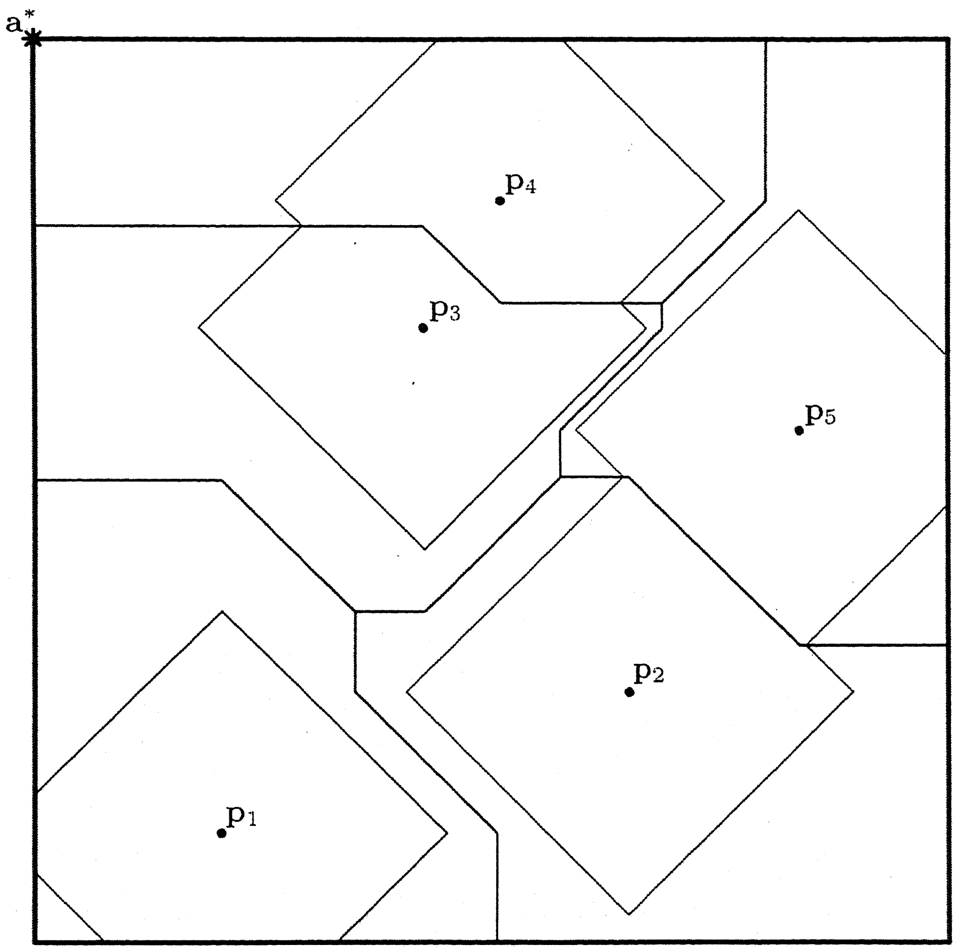

Figure 7: Rectangular Voronoi diagram, and contour of maximin criterion

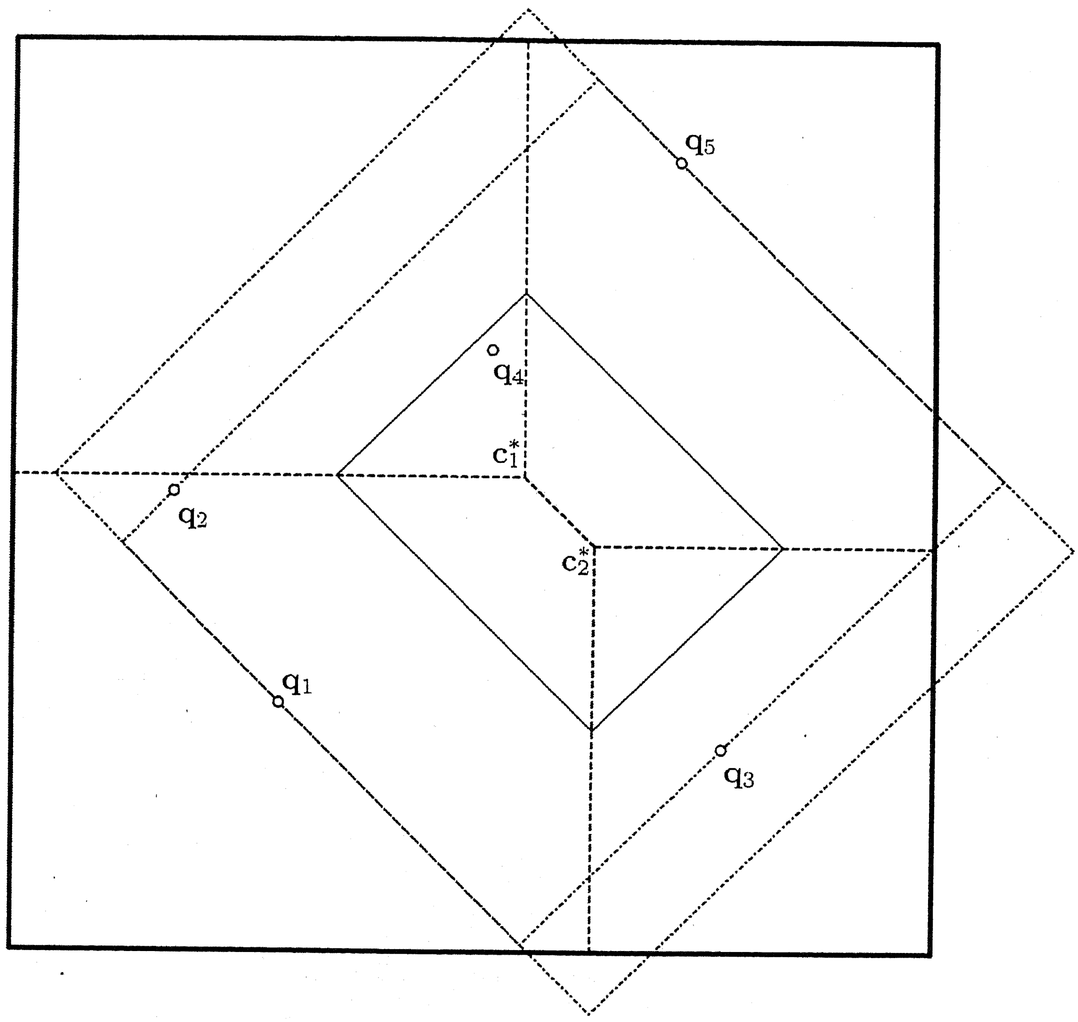

Figure 8: Rectangular farthest-point Voronoi diagram, and contour of minimax criterion

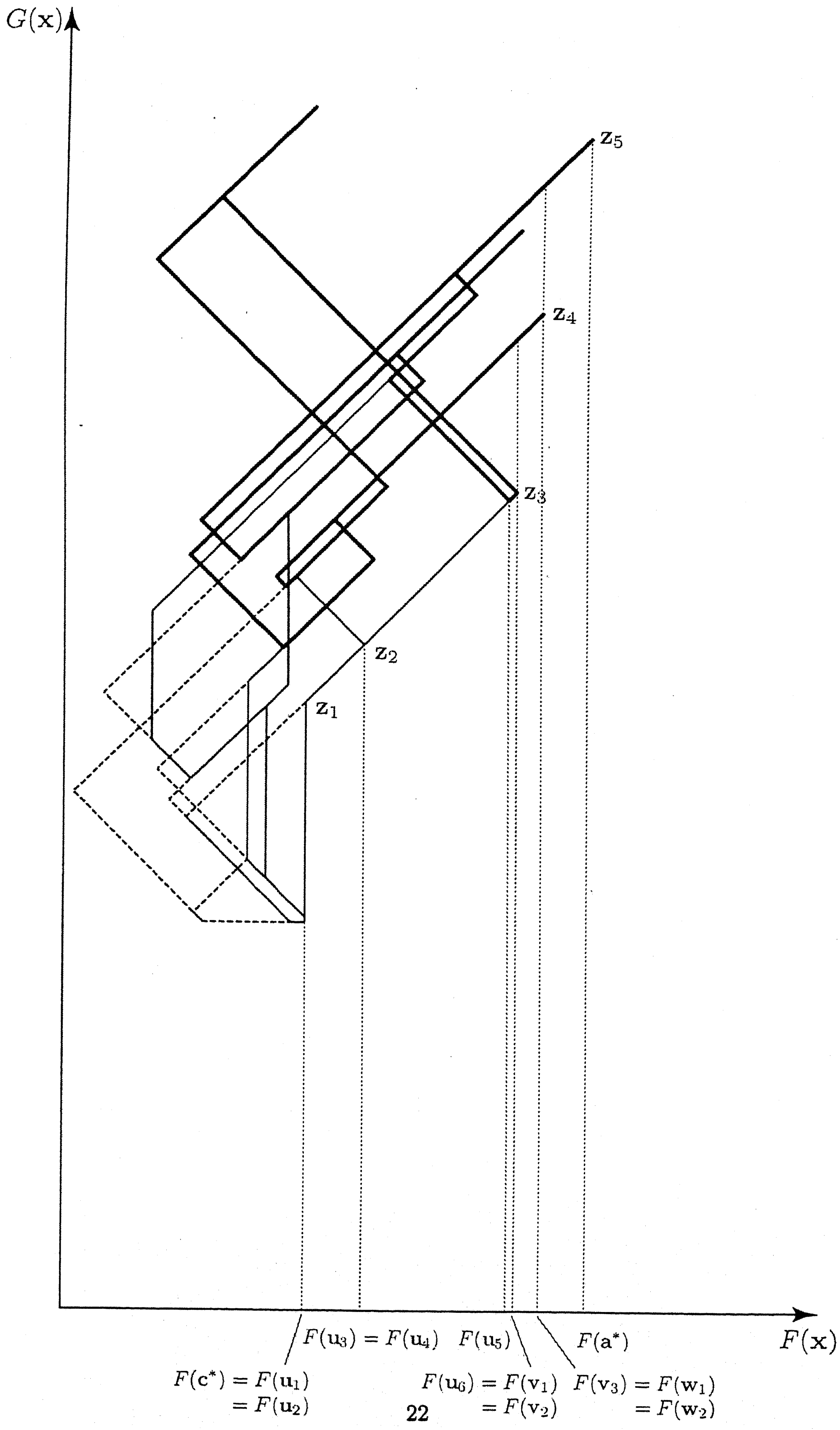

Figure 9: Tradeoff curve of rectangular maximin and mininax criteria

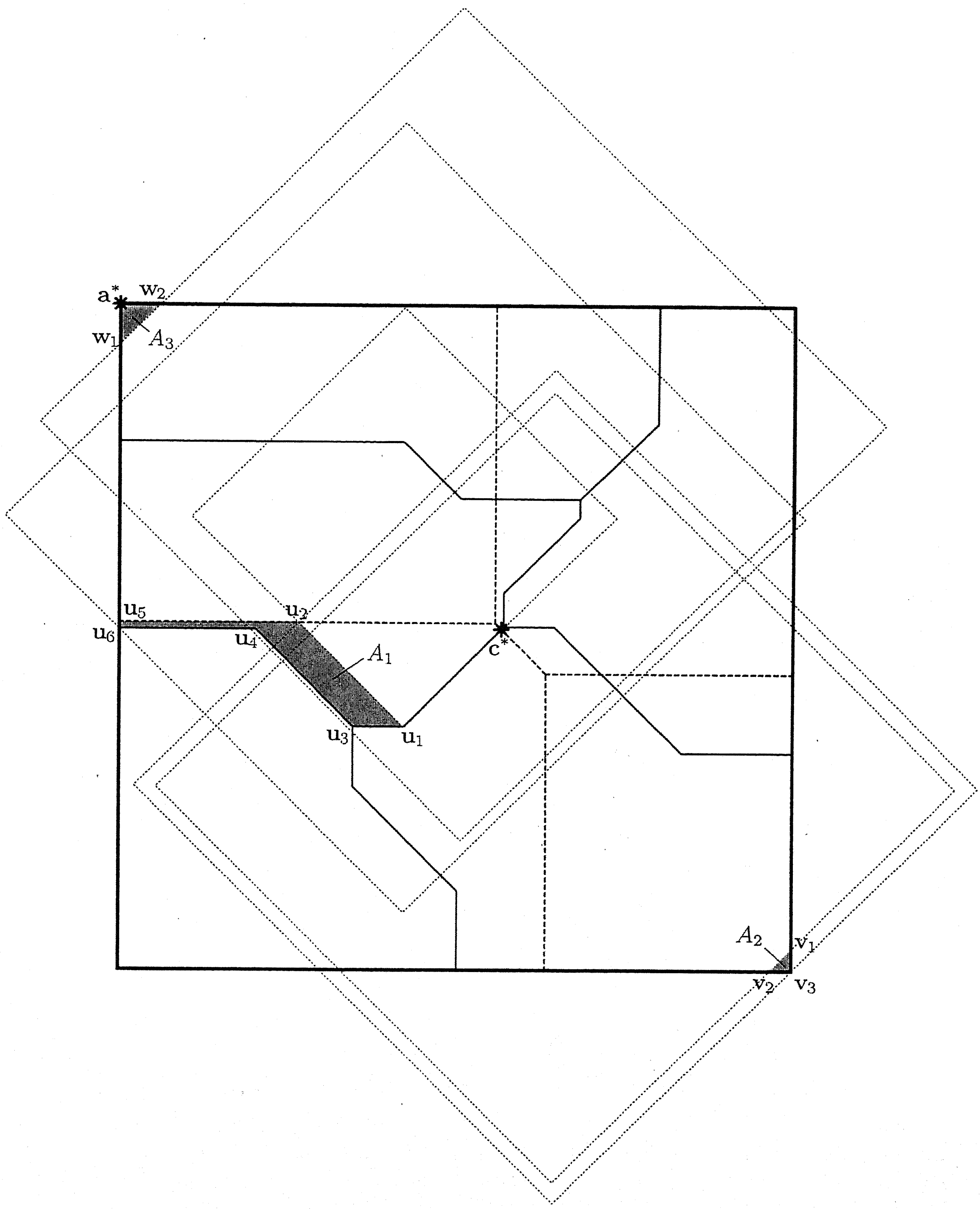

Figure 10: Efficient set of rectangular maximin and mininax criteria