INSTITUTE OF POCICY AND PLANNING SCIENCES

Discussion Paper Series

No. 1112

Development of Parametric Simulation Models for Structural Analysis of Voting Behaviors in Public Referendum

by
Mizue Ohe, Nobuko Igaki, Ushio Sumita

March 2005

UNIVERSITY OF TSUKUBA
Tsukuba, Ibaraki 305-8573
JAPAN

Development of Parametric Simulation Models for Structural Analysis of Voting Behaviors in Public Referendum

Mizue Ohe Nobuko Igaki* Ushio Sumita†

January 27, 2005

Abstract

Parametric simulation models are developed for structural analysis of voting behaviors in public referendum. By decomposing the residents into eight groups, a mechanism is established to construct transition probability matrices defined on three states (0: undecided; 1: YES; 2: NO), thereby capturing behavioral patterns of the residents in forming their individual opinions toward the voting date. This approach enables one to devise a strategy concerning how to transform the formation of the eight residential groups so as to achieve a target voting result. The validity is tested through eight real cases of Japan and a mock public referendum.

Key Words: Voting Behavior, Public Referendum, Parametric Simulation Models, Transition Probability Matrices, Eight Real Cases, Mock Public Referendum, Strategic Implications

1 Introduction

In Japan, as the municipal governments are forced, more and more, to be independent of the central government in managing their local matters and financial needs, the importance of public referendums has been rapidly increasing. When the local legislation for public referendum was established in four municipalities (Kubokawa, Kochi in 1982, Yonago, Tottori in 1988, Minamijima, Mie in 1993, and Kushima, Miyazaki in 1993) in Japan, the legislation requires the approval of the local assembly for implementing any public referendum. Such requests were often denied by the local assemblies and no public referendum actually took place until 1996, when the issue of whether or not a nuclear power plant should be built in a town of Maki was subjected to a public referendum. Maki's public referendum was approved by the town assembly because of the pressure from people of Japan and the mass media. Prior to this public referendum, the mayor strongly supporting to build the power plant was recalled and a new mayor was elected from the opposition side. The voting result of the public referendum was NO with voting ratio of 88.29 % and pro-vote ratio of 34.04 %. Since then, public referendums

*School of Policy Studies, Kwansei Gakuin University

†Institute of Policy and Planning Sciences, University of Tsukuba

have spread all over Japan, with the total of 142 cases between 1996 and 2004 involving 142 municipalities over a variety of issues such as whether or not to build a nuclear power plant, a waste site, a heliport for the U.S. army and a barrage near the exit of the Yoshino river, to shrink the U.S. army camp, and to merge into a city among others.

Voting behavior, in general, has been studied extensively in the literature, analyzing the structural properties from many different angles and validating such results with actual data, e.g. a game theoretic models of McKelevey (1976, 1979), Schofield (1978) and Browne and James (1991), statistical models of Caputo (1977), Kaplan and Venezky (1994) and Singh, el. al (1995), and Hashimoto (1997), Spatial probabilistic models of Fiorina (1976), Calvert (1985), Enelow and Hinich (1994), Ball (1999) and Lin, et. al (1999), and models based on rational choice theory by Brunk (1980), Austen-Smith (1987), Hansson (1992), Aldrich (1993), to name only a few. However, the literature specifically addressing to voting behavior of public referendums and the like is rather limited. Karaham and William (2004) analyzes a special election held in 2001 in the State of Mississippi over the issue of whether or not the current state flag is identified using regression models. Thalman (2004) deals with a public referendum held in Switzerland to vote on three proposals for taxes for fossil fuels. Using individual data of post referendum survey, some determinants are evaluated based on probit models of YES -NO type. To the authors' best knowledge, there have been no explicit models available in the literature which give an insight into the structure of voting behaviors in public referendum. The purposes of this paper is to fill this gap by developing parametric simulation models in a systematic manner.

The voting residents are first decomposed into eight groups. A mechanism is then proposed to construct transition probability matrices for the eight groups defined on three states (0: Undecided; 1: YES; 2: NO). These matrices may change as time goes by or as the ratio of approvers or disapprovers change, thereby capturing behavioral patterns of the residents in forming their individual opinions toward the voting date. The underlying parameters are plenty and it is difficult to estimate them accurately. However one set of the values of such parameters is identified, which enables one to reconstruct, via simulation, the voting results of eight real cases of public referendums that actually took place in Japan. Based on the set of the parameter values, our approach is to establish three basic models (Urban Basic model, Rural Basic model, Max-Vote Basic model) by altering the formation of the eight residential groups as well as the voting population, where approvers and disapprovers are almost balanced with Voting Ratio $\approx$ 2 x Pro-Vote Ratio.

The next step is to establish an algorithm for vertical expansion, resulting in 9 basic models from Urban Basic model to Max-Vote Basic model. Given one of such basic models, a mechanism for horizontal expansion is developed by transforming those who have a tendency to vote for YES to those who have a tendency to vote for NO while keeping the voting population intact. The horizontal expansion is represented by 11 models with Con-Type on the left edge, Basic-Type on the center, and Pro-Type on the right edge. Consequently, we have (Urban, Rural, Max-Vote) x (Con-Type, Basic-Type, Pro-Type) = 9 representative models and 9 x 11 = 99 detailed models via vertical and horizontal expansions.

The parametric simulation approach turns out to be quite robust in changes of the robust in changes of the voting population. If the formation of the eight residential groups is kept intact, the voting results of (Pro-Vote Ratio, Voting Ratio) remains almost the same regardless of the voting population. Consequently, given a voting result of (Pro-Vote Ratio, Voting Ratio), it is possible to estimate the underlying formation of the eight residential groups. This in turn enables one to device a strategy concerning how to achieve a target voting result starting from an estimated current situation by transforming the formation of the eight residential groups appropriately via campaign efforts and the like.

The parametric simulation approach is validated via a mock public referendum which took place at School of Policy Studies, Kwansei Gakuin University in the spring term involving 149 students. The issue was whether the final grade of a course entitled “ Introduction to Human Ecology” should be determined by a term report or by an ordinary final exam. Based on the set of the parameter values and the formation of the eight groups constructed from a survey, the simulation produced the voting result of (Pro-Vote Ratio, Voting Ratio) = (53.15 %, 86.88 %) in comparison with the actual voting result of (Pro-Vote Ratio, Voting Ratio) = (53.69 %, 88.59 %), yielding satisfactory accuracy.

The set of the values of the underlying parameters is identified in such a way that the voting results of the eight real cases that took place in Japan can be reconstructed via simulation. In this sense, the simulation results and their implications presented in this paper are peculiar to Japan. However, it should be noted that the parametric simulation approach itself may be valid in other countries where the set of the values of the underlying parameters ought to be specified differently by reflecting the peculiarities of the country under considerlations.

The structure of this paper is as follows. Parametric simulation models are formally introduced in Section 2, with a mechanism to construct transition probability matrices where details are summarized in Appendix A. Section 3 addresses itself to establish a methodological approach for structural analysis of voting behaviors in public referendum. A set of the values of the underlying parameters is identified in such a way that the voting results of eight real cases in Japan can be reconstructed via simulation with satisfactory accuracy. Also discussed are the robustness of the parametric simulation approach in changes of the voting population, and how to estimate the formation of the eight groups given a voting result. In Section 4, the validity of the parametric simulation approach is tested against a mock public referendum, yielding quite satisfactory results. Section 5 is devoted to the discussion of impact of formation of the eight residential groups on public referendum and its strategic implications. Finally in Section 6, some concluding remarks are given.

2 Model Description

We consider a population of residents who are to make a collective decision of YES-or-NO type over an issue through public referendum. The voting is to take place K days later. It is assumed that the residents are classified into eight groups, characterizing behavioral patterns of the residents in forming their individual opinions toward the voting date. The influential relationships among the eight groups are depicted in Figure 2.1, where group l is denoted by $G(l)$ and $G(l) \rightarrow G(l')$ means that $G(l)$ directly influences $G(l')$. Verbal descriptions of the eight groups are given below.

$G(1)$ (Convinced Approvers): those who are determined to vote for YES from the very beginning and to make serious efforts to convince others in line with them.

$G(2)$ (Adaptable Approvers): those who have not formed their opinions in the beginning but have a tendency to vote for YES when the ratio of approvers in $G(1)$ and $G(2)$ increases.

$G(3)$ (Independent Approvers): those who have not formed their opinions in the beginning, are not influenced by others, and independently have an inclination to vote for YES.

$G(4)$ (Convinced Disapprovers): those who are determined to vote for NO from the very beginning and to make serious efforts to convince others in line with them.

$G(5)$ (Adaptable Disapprovers): those who have not formed their opinions in the beginning but have a tendency to vote for NO when the ratio of disapprovers in $G(4)$ and $G(5)$ increases.

$G(6)$ (Independent Disapprovers): those who have not formed their opinions in the beginning, are not influenced by others, and independently have an inclination to vote for NO.

$G(7)$ (Local Opportunists): those who have not formed their opinions in the beginning and have a tendency to be influenced toward voting for YES by the ratio of approvers among $G(2)$, $G(3)$, $G(5)$, $G(6)$, and $G(7)$.

$G(8)$ (Global Opportunists): those who have not formed their opinions in the beginning and have a tendency to be influenced toward voting for YES by the ratio of approvers among the entire population.

Let $\mathscr{N} = \{1, \ldots, N\}$ be the set of the whole population. The cardinality of $G(l)$ is denoted by $N(l) = |G(l)|$, $1 \leq l \leq 8$. We assume that $G(l)$s are mutually exclusive and exhaustive so that

(2.1) $$\mathscr{N} = \bigcup_{l=1}^{8} G(l); \quad G(l) \bigcap G(l') = \emptyset \text{ for } l \neq l'$$

and

(2.2) $$N = \sum_{l=1}^{8} N(l).$$

Figure 2.1: Influential Relationships among Eight Residential Groups

A formation $\underline{g}$ of the eight residential groups is then defined as

$$\underline{g} = (g(1),\ \dots,\ g(8));\ \ g(l) = \frac{N(l)}{N},\ \ 1 \leq l \leq 8. \tag{2.3}$$

At time k, individual residents are in one of the three states: (0)Undecided; (1)YES; (2)NO. More specifically let $\{S_i(k) : 0 \leq k \leq K\}$ be a discrete time stochastic process describing the state of resident $i \in \mathscr{N}$ at time k where , for $0 \leq k \leq K$,

$$S_i(k) = \begin{cases} 0 & \text{if resident } i \text{ is undecided at time } k, \\ 1 & \text{if resident } i \text{ would vote for YES at time } k, \\ 2 & \text{if resident } i \text{ would vote for NO at time } k. \end{cases} \tag{2.4}$$

For notational convenience, we define vector stochastic processes $\boldsymbol{S}_{G(l)}(k)$ for $1 \leq l \leq 8$ and $0 \leq k \leq K$ where

$$\boldsymbol{S}_{G(l)}(k) = [S_i(k)]_{i \in G(l).} \tag{2.5}$$

Given $\boldsymbol{S}_{G(l)}(k)$ at time k for $1 \leq l \leq 8$, individual residents independently choose their state at time $k+1$ based on time-dependent transition probability matrices. Such time-dependent transition probability matrices are assumed to be common within one group, but differ from each other across different groups significantly so that their behavioral patterns can be captured.

In order to characterize how $i \in G(l)$ forms his/her decision toward the voting date, we assume that governing transition probability matrices would change over three time periods: the initial stage, the middle stage, and the final stage. More specifically, let $K^-(l)$ and $K^+(l)$ be such that $0 \le K^-(l) \le K^+(l) \le K$ and define

$$U(l,k) = \begin{cases} 1 & \text{if} \quad 0 \le k < K^-(l) \\ 2 & \text{if} \quad K^-(l) \le k < K^+(l) \\ 3 & \text{if} \quad K^+(l) \le k \le K. \end{cases} \tag{2.6}$$

We note that the three stages may be perceived differently across different groups.

For certain groups, governing transition probability matrices also depend on the ratio of approvers or disapprovers in their own and other groups. Let $I(l)$ be the set of groups influencing $G(l)$, i.e., from Figure 2.1, one has

$$\begin{cases} I(1) = I(3) = I(4) = I(6) = \phi \\ I(2) = \{1,2\}, \;\; I(5) = \{4,5\}, \\ I(7) = \{2,3,5,6,7\}, \;\; I(8) = \{1,2,3,4,5,6,7,8\}. \end{cases} \tag{2.7}$$

Corresponding to $K^-(l)$, $K^+(l)$ and $U(l,k)$, we introduce $0 \le Y^-(l) \le Y^+(l) \le 1$ and

$$V(l,k) = \begin{cases} 1 & \text{if} \quad 0 \le d(l,k) < Y^-(l) \\ 2 & \text{if} \quad Y^-(l) \le d(l,k) < Y^+(l) \\ 3 & \text{if} \quad Y^+(l) \le d(l,k) \le 1, \end{cases} \tag{2.8}$$

where $d(l,k)$ denotes the ratio of approvers among groups $\{G(l')\}_{l' \in I(l)}$ at time k for $l = 2,7,8$. For $l = 5$, $d(l,k)$ denotes the ratio of disapprovers among groups $\{G(l')\}_{l' \in I(l)}$ at time k.

Given $U(l,k) = u$ and $V(l,k) = v$ ($u,v \in \{1,2,3\}$), the corresponding transition probability matrix for individual residents in $G(l)$ is denoted by

$$\boldsymbol{\alpha}(l,u,v) = \begin{bmatrix} \alpha_{00}(l,u,v) & \alpha_{01}(l,u,v) & \alpha_{02}(l,u,v) \\ \alpha_{10}(l,u,v) & \alpha_{11}(l,u,v) & \alpha_{12}(l,u,v) \\ \alpha_{20}(l,u,v) & \alpha_{21}(l,u,v) & \alpha_{22}(l,u,v) \end{bmatrix} \tag{2.9}$$

where, for $m,n, \in \{0,1,2\}$,

$$\alpha_{mn}(l,u,v) = P[S_i(k+1) = n \mid S_i(k) = m, \; U(l,k) = u, \; V(l,k) = v] \text{ for all } i \in G(l). \tag{2.10}$$

In general, there are 3 x 3 = 9 transition probability matrices describing the behavioral pattern of $G(l)$, except that $G(1)$ and $G(4)$ have only one matrix, and $G(3)$ and $G(6)$ have 3 matrices that are dependent on u but independent of v. For $G(2)$, for example, one has

$$(2.11)\qquad \boldsymbol{\alpha}(2,1,1) = \begin{bmatrix} \alpha_{00}(2) & p(2)\{1-\alpha_{00}(2)\} & \{1-p(2)\}\{1-\alpha_{00}(2)\} \\ 0 & 1 & 0 \\ 1-\tilde{\alpha}_{00}(2) & q(2)\tilde{\alpha}_{00}(2) & \{1-q(2)\}\tilde{\alpha}_{00}(2) \end{bmatrix}.$$

When a resident in $G(2)$ is in state 0 at time k with $u = v = 1$, the resident remains undecided at time $k+1$ with probability $\alpha_{00}(2)$, moves to state 1 (YES) with probability $p(2)\{1-\alpha_{00}(2)\}$, and moves to state 2 (NO) with probability $\{1-p(2)\}\{1-\alpha_{00}(2)\}$. If his/her voting position at time k is state 2, the resident becomes undecided with probability $1-\tilde{\alpha}_{00}(2)$, switches to state 1 with probability $q(2)\hat{\alpha}_{00}(2)$, and remains in state 2 with probability $\{1-q(2)\}\tilde{\alpha}_{00}(2)$. Reflecting a rather strong tendency of those in $G(2)$ to vote for YES, it is assumed that once a resident in $G(2)$ decides to vote for YES, then the position will never be changed. In addition, one should have $\frac{1}{2} \leq p(2), q(2) \leq 1$ so that the probability of moving from state 0 to state 1 (YES) would be higher than that of moving from state 0 to state 2 (NO). Further details about how to construct $\boldsymbol{\alpha}(l,u,v)$ in (2.9) for capturing the group characteristics of $G(1)$ through $G(8)$ are given in Appendix A.

3 Methodological Approach for Structural Analysis of Public Referendum

The purpose of this section is to establish a methodological approach for structural analysis of public voting based on the parametric simulation approach discussed in Section 2 and Appendix A. In order to define the values of the underlying parameters in a realistic setting, 8 cases of public referendums that actually took place in Japan are plotted in Figure 3.1, where the horizontal axis represents Pro-Vote Ratio (i.e. the ratio of YES votes among the entire voting population) and the vertical axis corresponds to the Voting Ratio(i.e. the ratio of YES or NO votes among the entire voting population). Further details of these cases are summarized in Table 3.1. As can be seen in Table 3.1, five of these cases have a 10 day campaign period, whereas three of them have a 5 day campaign period.

It should be noted that the voting results of the eight cases are all negative with Pro-Vote Ratio $< \frac{\text{Voting Ratio}}{2}$. If the issues are questioned in a reversed manner (i.e. YES becomes NO and NO becomes YES), one should observe the symmetric reflection of the results of the eight real cases along the line Voting Ratio $= 2$ x Pro-Vote Ratio. In this case, the formation of the eight residential groups for each real case also ought to be reversed in a similar manner, that is, $(g(2), g(3))$ and $(g(5), g(6))$ are interchanged while $g(1), g(4), g(7)$ and $g(8)$ remain intact. In consistent with this observation, our methodological approach is to establish a systematic way to develop simulation models which cover the results of the eight real cases and their symmetric reflections by altering the formation of the eight residential groups. For this purpose, we first establish three basic models along the vertical line Voting Ratio$= 2$ x Pro-Vote Ratio, followed by their vertical and horizontal expansions.

Table 3.1: Details of Eight Real Cases

	Municipality	Issue	Campaign Period	Voting Population	Pro-Vote Ratio(%)	Voting Ratio(%)	Final Approval
1	Maki, Niigata	Nuclear plant	10	23,222	34.04	88.29	No
2	Mitake, Gifu	Waste disposal site	10	14,884	16.41	87.50	No
3	Kobayasi, Miyazaki	Waste disposal site	10	31,575	30.43	75.86	No
4	Kariwa, Niigata	Nuclear plant	10	4,092	37.46	88.14	No
5	Higashisonogi, Nagasaki	Merger into a city	10	7,665	37.04	83.26	No
6	Kamio, Saitama	Merger into a city	5	168,297	26.56	64.48	No
7	Miyama, Mie	Nuclear plant	5	8,705	28.86	88.64	No
8	Iwaki, Akita	Merger into a city	5	5,427	29.96	81.24	No

3.1 Establishment of Basic Models and Specification of Underlyning Paramenter Values

Let a public voting model be described by a triplet (Population, Pro-Vote Ratio, Voting Ratio) = (P, PV-R, V-R). A model (P, PV-R, V-R) is called Basic if the model has approvers and disapprovers almost equal, i.e. PV-R $\approx \frac{\text{V-R}}{2}$. We first aim at developing two basic models representing a typical rural community and a typical urban community. Considering the fact that rural areas in Japan typically achieve higher voting ratios than urban areas, we assume that the

Figure 3.1: Pro-Vote Ratio vs. Voting Ratio for Eight Real Cases and Their Symmetric Reflections

former is located near (10,000, 40%, 80%) while the latter is approximately at (100,000, 25%, 50%). Keeping characteristics of rural and urban communities in mind, we roughly estimate the formations of the eight residential groups for the two basic models as below.

	G(1)	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)
Rural Basic Model	5%	25%	5%	5%	25%	5%	10%	20%
Urban Basic Model	2%	3%	10%	2%	3%	10%	20%	50%

Here, those in $G(1), G(2), G(3)$ with tendency to vote for YES and those in $G(4), G(5)$, $G(6)$ with tendency to vote for NO are balanced. In rural communities in Japan, people are tied closely through various human relationships. Accordingly, $G(7)$ and $G(8)$ are minority groups, estimated as 10% and 20% of the voting population respectively. In contrast, $G(7)$ and $G(8)$ clearly constitute the majority in urban communities, estimated as 20% and 50% of the voting population. For $G(1), G(2)$ and $G(3)$, rural communities are likely to have more people in $G(1)$ and $G(2)$ than urban communities. However the ratio of independent people in $G(3)$ ought to be higher in urban communities than that in rural communities.

Given the formations of the residential groups $G(1)$ through $G(8)$ as above, the next task is to set the underlying parameter values so as to achive about (10,000, 40%, 80%) for the Rural Basic model, and (100,000, 25%, 50%) for the Urban Basic model. For the parameter values specified in Table 3.2 and the threshold values given in Table 3.3, the two basic models are simulated and the first difference of the cumulative average of the first *n*-runs for PV-R and that for V-R are depicted in Figure 3.2 as a function of *n*. One sees that the cumulative averages converge to a range within $\pm 0.1\%$ after 10 simulation runs. As we see in Table 3.4, with campaign periods of 5 and 10 days, Rural Basic model results in (10,000, 38.58%, 76.03%)

and (10,000, 44.85%, 83.76%), while Urban Basic model has (100,000, 25.63%, 51.15%) and (100,000, 26.46%, 52.86%) respectively. Based on these observations, it is decided that we adopt the parameter values and the threshold values given in Tables 3.2 and 3.3, and use 10 simulation runs throughout the paper unless specified otherwise.

Table 3.2: Parameter Values

	$\alpha_{00}(l)$	$\tilde{\alpha}_{00}(l)$	$a(l)$	$b(l)$	$\tilde{b}(l)$	$p(l)$	$q(l)$	$\tilde{q}(l)$	$r(l)$	$s(l)$	$\tilde{s}(l)$	$t(l)$	$w(l)$
G(1): Convinced Approvers	-	-	-	-	-	-	-	-	-	-	-	-	-
G(2): Adaptable Approvers	0.45	0.8	0.95	0.8	1	0.55	0.1	-	1.1	0.01	-	0.9	0.95
G(3): Independent Approvers	0.45	0.8	0.95	-	-	0.55	0.1	-	1.1	0.01	-	-	-
G(4): Convinced Dispprovers	-	-	-	-	-	-	-	-	-	-	-	-	-
G(5): Adaptable Disapprovers	0.45	0.8	0.95	0.8	1	0.45	0.9	-	1.1	0.01	-	0.9	0.95
G(6): Independent Disapprovers	0.45	0.8	0.95	-	-	0.45	0.9	-	1.1	0.01	-	-	-
G(7): Local Opportunists	0.8	0.6	-	0.8	1	0.5	0.9	0.1	-	0.01	0.01	0.9	0.95
G(8): Global Opportunists	0.8	0.6	-	0.8	1	0.5	0.9	0.1	-	0.01	0.01	0.9	0.95

Table 3.3: Threshold Values $(1 \leq l \leq 8)$

	$K^-(l)$	$K^+(l)$	$Y^-(l)$	$Y^+(l)$
10 day Model	4	8	0.4	0.8
5 day Model	3	4	0.4	0.8

Figure 3.2: First Differences of Cumulative Averages of Pro-Vote Ratio and Voting Ratio

Table 3.4: Rural and Urban Basic Models

	Population	Campaign Period			
		10 days		5 days	
		PV-R (%)	V-R (%)	PV-R (%)	V-R (%)
Rural Basic Model	10,000	44.85	83.76	38.58	76.03
Urban Basic Model	100,000	26.46	52.86	25.63	51.15

3.2 Vertical and Horizontal Expansion of Basic Models

In order to cover the major area of the spread of the point $(\circ, \bullet)$ in Figure 3.1 with simulation results, we consider a vertical expansion of the two basic models by developing 4 basic models between Urban Basic model and Rural Basic model in the following manner. The j-th Basic model has a total population of $100,000 - 18,000 * j$ with the population of $G(l)$, denoted by $N_j(l)$, specified by

$$N_j(l) = N_0(l) - \frac{j}{5}(N_0(l) - N_5(l)), \quad 0 \le j \le 5, \tag{3.1}$$

where $j = 0$ and $j = 5$ correspond to Urban Basic model and Rural Basic model respectively. As can be seen in Figure 3.1, some real cases of public referendum experienced V-R beyond 80%. For covering this area, we develop Basic models beyond Rural Basic model by exterpolating (3.1) for $j > 5$. In this case, however, $N_j(l)$ may become negative. In order to avoid this situation, we define

$$j^* = min_{1\le l\le 8}\{j_l^*\}; \quad j_l^* = max\{j : N_j(l) \ge 0\}, \quad 1 \le l \le 8 \tag{3.2}$$

and the exterpolation stops at j^*. The resulting Basic model is called Max-Vote Basic model. The expansion of the two basic models is vertical in that the entire voting population decreases and V-R increases as j increases while PV-R remains within a limited range.

We next consider a horizontal expansion of a Basic model, for which the entire voting population is kept constant and V-R remains within a limited range while PV-R varies over a wide range from Con-Type (low) to Pro-Type (high) through Basic model at the middle point. Given a Basic model, we develop 5 models in each side of Basic model by altering the formation of the eight residential groups. Let $N_B(l)$ and $N_{Hi}(l)$ be the population of $G(l)$ for Basic model under consideration and the i-th horizontal model respectively, $-5 \le i \le 5$, where $i = 0$ corresponds to Basic model. So as to keep the V-R within a narrow range, the group population of $G(1), G(4), G(7)$, and $G(8)$ are fixed, that is,

$$N_{Hi}(l) = N_B(l) \quad \text{for } l = 1,4,7,8 \quad \text{and} \quad -5 \le i \le 5. \tag{3.3}$$

For $G(2), G(3), G(5)$, and $G(6)$, members move from $G(2)$ to $G(5)$ and from $G(3)$ to $G(6)$ as i changes from 0 to -5. These shifts are reversed when i increases from 0 to 5. More specifically, we define:

$$\begin{cases} N_{Hi}(2) = N_B(2)(1+\frac{i}{5}), & N_{Hi}(3) = N_B(3)(1+\frac{i}{5}) \\ N_{Hi}(5) = N_B(5)(1-\frac{i}{5}), & N_{Hi}(6) = N_B(6)(1-\frac{i}{5}). \end{cases} \tag{3.4}$$

It should be noted that, for Basic model, one has $N_B(2) = N_B(5)$ and $N_B(3) = N_B(6)$. Consequently the entire voting population remains constant over all horizontal models. We call the horizontal model with $i = -5$ Con-Type, with $i = 0$ Basic-Type, and with $i = 5$ Pro-Type.

In Table 3.5, simulation results of PV-R and V-R are summarized for the nine representative models (Urban, Rural, Max-Vote) x (Con-Type, Basic-Type, Pro-Type) with a campaign period of 10 days. Similar results are summarized in Table 3.6 for a campaign period of 5 days. It should be noted that one has PV-R $\approx \frac{\text{V-R}}{2}$ for the three basic models, while PV-R $< \frac{\text{V-R}}{2}$ for Con-Type models and PV-R $> \frac{\text{V-R}}{2}$ for Pro-Type models. Corresponding to Tables 3.5 and 3.6, Figures 3.3 and 3.4 exhibit these models graphically, together with other models generated by vertical-horizontal expansions of the three basic models, the eight real cases and their symmetric reflections. One sees that the major area of the spread of the real cases and their symmetric reflections are well covered by our simulation models.

Table 3.5: Simulation Results of Nine Representative Models with 10 day campaign period

		Population	PV-R (%)			V-R (%)		
Max-Vote Model	Con-Type	10,000	12.06			94.03		
	Basic-Type			48.62			95.30	
	Pro-Type				84.03			95.91
Rural Model	Con-Type	10,000	13.77			78.64		
	Basic-Type			44.85			83.76	
	Pro-Type				71.49			84.56
Urban Model	Con-Type	100,000	15.71			52.86		
	Basic-Type			26.46			52.86	
	Pro-Type				47.16			62.09

Table 3.6: Simulation Results of Nine Representative Models with 5 day campaign period

		Population	PV-R (%)			V-R (%)		
Max-Vote Model	Con-Type	10,000	20.73			89.36		
	Basic-Type			45.86			90.60	
	Pro-Type				69.44			90.38
Rural Model	Con-Type	10,000	20.01			74.84		
	Basic-Type			38.58			76.03	
	Pro-Type				60.29			79.92
Urban Model	Con-Type	100,000	18.60			51.21		
	Basic-Type			25.63			51.19	
	Pro-type				34.98			53.42

Figure 3.3: Pro-Vote Ratio vs. Voting Ratio: Simulation Results vs. Real Cases and Their Symmetric Reflections with 10 Day Campaign Period

Figure 3.4: Pro-Vote Ratio vs. Voting Ratio: Simulation Results vs. Real Cases and Their Symmetric Reflections with 5 Day Campaign Period

3.3 Robustness of the Parametric Simulation Approach and Estimation of Group Formation for Real Cases

The parametric simulation approach discussed in the previous sections turns out to be quite robust in changes of the voting population. Figure 3.5 depicts simulation results of the nine representative models (Urban, Rural, Max-Vote) x (Con-Type, Basic-Type, Pro-Type) where the voting populations are scaled by factors of 0.1, 0.5, 1.0, and 2.0, while keeping the formations of the eight residential groups intact. It can be seen that both PV-R and V-R are hardly affected by changes of the voting population. This robustness enables one to estimate the group formation of real cases as we discuss next.

Through the vertical-horizontal expansions, we have generated 9 x 11 = 99 models. Based on the robustness of the simulation model described above, we ignore the voting population and let a real case be represented by (PV-R*, V-R*), which is known, with the group formation $\underline{g}^* = (g^*(1), \dots, g^*(8))$ unknown. One sees that there exist three models (PV-R$_m$, V-R$_m$), m=1, 2, 3, constituting the smallest triangle which contains (PV-R*, V-R*), with the respective group formations $\underline{g}_m = (g_m(1), \dots, g_m(8))$, m=1, 2, 3. It then follows that (PV-R*, V-R*) can be expressed as a convex combination of (PV-R$_m$, V-R$_m$), m=1, 2, 3. The associated weights $(w(m))_{m=1}^3$ can be found by solving the following simultaneous linear equations.

$$(3.5) \qquad \begin{cases} \sum_{m=1}^3 w(m)\text{PV-R}(m) = \text{PV-R}^* \\ \sum_{m=1}^3 w(m)\text{V-R}(m) = \text{V-R}^* \\ \sum_{m=1}^3 w(m) = 1 \end{cases} .$$

The group formation $\underline{g}^*$ of the real case is then estimated by

$$(3.6) \qquad \underline{g}^* = \sum_{m=1}^{3} w(m)\, \underline{g}_m .$$

The soundness of the above approach can be confirmed by testing it through the eight real cases described in Section 2. Table 3.7 exhibits the group formations estimated via (3.6) for the eight real cases. Table 3.8 summarizes simulation results obtained by reconstructing the eight real cases via simulation using the real voting populations and the estimated group formations. Comparison of the results with the actual PV-Rs and V-Rs reveal that the differences are within 1.5%.

Figure 3.5: Robustness of the Simulation Model in Changes of Voting Population

Table 3.7: Estimated Formations of Eight Residential Groups for Real Cases

Real Case	Group Formation (%)								
	G(1)	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)	Total
1	6.1	27.0	2.6	6.1	39.3	3.7	6.3	8.9	100.0
2	6.0	6.5	0.6	6.0	58.8	5.9	6.5	9.6	100.0
3	4.4	20.9	6.2	4.4	20.0	5.9	12.1	26.2	100.0
4	5.9	28.3	3.3	5.9	34.4	3.8	7.1	11.3	100.0
5	4.9	33.7	7.2	4.9	14.8	3.1	10.3	21.0	100.0
6	3.4	21.8	13.2	3.4	4.4	2.3	15.4	36.3	100.0
7	6.7	18.1	1.1	6.7	57.0	3.2	4.3	2.9	100.0
8	5.7	24.5	3.0	5.7	36.3	4.5	7.5	12.6	100.0

Table 3.8: Pro-Vote Ratio and Voting Ratio by Reconstruction of Real Cases

Case	Campaign Period	Voting Population	PV-R (%)			V-R (%)		
			Real Case (a)	Simulation (b)	(a) - (b)	Real Case (a)	Simulation (b)	(a) - (b)
1	10	23,222	38.55	38.26	0.29	88.29	88.23	0.06
2	10	14,884	18.75	18.60	0.15	87.50	87.69	-0.19
3	10	31,575	40.18	39.60	0.58	75.86	75.50	0.36
4	10	4,092	42.52	43.91	-1.39	88.14	89.36	-1.22
5	10	7,665	54.81	55.43	-0.62	83.26	83.45	-0.19
6	5	168,297	41.56	40.89	0.67	64.48	64.06	0.42
7	5	8,705	32.40	32.50	-0.10	88.64	88.54	0.10
8	5	5,427	36.88	36.88	0.00	81.24	80.98	0.26

4 Validation of the Parametric Simulation Approach via a Mock Public Referendum

In general, it is awfully difficult to examine how the parametric simulation model described in the previous sections may (or may not) realistically capture the behavioral characteristics of the voters in public voting. In an effort to respond to this difficulty, a mock public referendum was conducted involving 149 students who took a course entitled "Introduction to Human Ecology" in the spring term of 2004 at School of Policy Studies, Kwansei Gakuin University of Japan. The issue here was whether or not the final grade of the course should be determined by a term repot to be written and submitted over a five day period. If the students disapprove the proposal, then an ordinary in-class closed-book final exam would take place. The students were also informed that the difficulty level of the term report would be higher than that of the final exam.

In organizing a mock public referendum concerning this issue, the following steps were taken over a four week period in the middle of the course. For notational convenience, the four weeks are called Week $i, 1 \leq i \leq 4$. In reality, an action taken in Week i means that the action was taken during the class hour of the week.

1. Before Week 1

By directly communicating with the students, five students who strongly supported the proposal, and another five students who opposed to the proposal and preferred the final examination were identified. The former five students were considered to be convinced approvers belonging to $G(1)$, while the latter five students were understood to be convinced disapprovers belonging to $G(4)$.

2. Week 1

1) In the beginning of the class, a questionnaire survey was conducted to find out the current voting position of each student, as well as to figure out later which group each student would belong to among the eight groups $G(1)$ through $G(8)$. The actual questionnaire is given in Appendix B.

2) In the end of the class, the voting result at this point in time (i.e. the numbers of students who approved, disapproved, and were still undecided) was announced.

3) After 2), some of the students identified as $G(1)$ or $G(4)$ made a short presentation to approve or disapprove the proposal respectively, followed by a ten minute discussion session involving the whole class.

3. Week 2 and Week 3

1) In the beginning of the class, each student was asked to inform of whether he/she approved, disapproved or was still undecided at this point in time by marking an answer sheet.

2) In the end of the class, the voting result at this point in time was announced.

3) After 2), some of the students identified as $G(1)$ or $G(4)$ in Week 1 made a short presen-

tation to approve or disapprove the proposal respectively, followed by a ten minute discussion session involving the whole class.

4. Week 4

1) The final vote took place in the beginning of the class.

For examining the validity of the parametric simulation approach, the next task would be to classify the 149 students into the eight groups $G(1)$ through $G(8)$ based on their responses to the questionnaire as well as their temporal voting positions from Week 1 until Week 3. For this purpose, the questionnaire was designed, as shown in Appendix B, to figure out the degrees of independence, sympathy and sensitivity of individual students to local atmosphere by asking how they chose the university, how they conducted information search for academic purposes, what type of club activities they were engaged in, and what role they played in such club activities. Based on the survey results, the following five indicators are introduced using scores of relevant questions as specified for each indicator. Here scores are in scale of 1 through 10, and the associated average for each indicator is defined as the value of the indicator.

IP1: Independence Point 1

Q1.1-1), Q1.2-1), Q1.2-5)

IP2: Independence Point 2

Q1.4-1), Q1.4-2), Q1.5-1), Q1.5-2)

Remark: For Q1.4-1) and Q1.4-2), the point is 10 if the answer is 1 or 2, and is 0 else.

SP1: Sympathy Point 1

Q1.1-2), Q1.1-3), Q1.2-2), Q1.2-3), Q1.2-4)

SP2: Sympathy Point 2

Q1.5-3), Q1.5-4)

SL: Sensitivity to Local Atmosphere

Q1.5-5), Q1.5-6)

For classification purposes, the threshold values of the five indicators are defined. For notational convenience, let $W(i)$ be the score of the i-th student for indicator W, and we write TH(W) to denote the threshold value of W where W=IP1, IP2, ... etc.

(4.1) $$\mu(W) = \frac{1}{149}\sum_{i=1}^{149} W(i); \quad \sigma(W) = \sqrt{\frac{1}{148}\sum_{i=1}^{149}(W(i)-\mu(i))^2}$$

(4.2) $$\text{For } W = \text{IP1, IP2 or SL}, \quad \text{TH}(W) = \mu(W) + \sigma(W).$$

(4.3) $$\text{For } W = \text{SP1 or SP2}, \quad \text{TH}(W) = \mu(W) - \sigma(W).$$

The classification criterion for (4.2) is " greater than or equal to ", and that for (4.3) is "less than or equal to". Applying those indicator values of the individual students to the corresponding threshold values together with self-claims by the students concerning which group they thought they would belong to, an algorithm is developed for classifying the students into the eight groups $G(1)$ through $G(8)$. This algorithm is summarized in Appendix C in a form of a flow-chart. The resulting group formation is exhibited in Table 4.1.

Table 4.1: Group Formation Based on Survey

	Population	%
G(1): Convinced Approvers	11	7.4
G(2): Adaptable Approvers	66	44.3
G(3): Independent Approvers	13	8.7
G(4): Convinced Disapprovers	5	3.4
G(5): Adaptable Disapprovers	36	24.2
G(6): Independent Disapprovers	6	4.0
G(7): Local Opportunists	6	4.0
G(8): Global Opportunists	6	4.0
Total	149	100.0

With the group formation in Table 4.1, the parameter values in Table 3.2 and the voting population of 149, we are now in a position to run the simulation model. A few remarks are worth noting.

1) While the mock public referendum was conducted over a period of 4 weeks, it is highly unlikely for the students to discuss the issue outside class hours. Because of this reason, the unit of time is taken to be one week, and we set $K = 4$.

2) Since $K = 4$ is rather small, the distinction of the initial, middle and final stage is discarded, while $Y^{-}(l)$ and $Y^{+}(l)$ are employed as in Table 3.3.

3) The voting population of 149 is much smaller than 10,000 or more in the previous discussions. Accordingly, it is necessary to redetermine the number of simulation runs needed to obtain reliable resluts. As before, the first difference of the cumulative average of the first n-runs for PV-R and that for V-R are depicted in Figure 4.1 as a function of n. One finds that the relative error is contained within $\pm 0.2\%$ for $n = 50$ or more. Based on this observation, 50 simulation runs were made.

Figure 4.1: First Difference of the Cumulative Average of Pro-Vote Ratio and Voting Ratio

In Table 4.2, the actual voting resluts of PV-R* $= 53.69$ % and $V - R^* = 88.59$ % are summarized in the second column. The third column exhibits the silumation results besed on the group formation given in Table 4.1, with PV-R = 53.15 % and V-R = 86.88 %. One may conclude that the reality of the mock public referendum was well captured by the parametric simulation approach.

Table 4.2: Comparison of Simulation Results with Actucal Result

	Actual	Group Formation Based on Survey	Group Formation Based on Estimation
Pro-Vote Ratio (%)	53.69	53.15	51.65
Voting Ratio (%)	88.59	86.88	88.10

The experiment of the mock public referendum can also provide an opportunity to examine how reliable the estimation procedure of Subsection 3.3 may be, where the group formation is estimated given the voting outcome of (PV-R*, V-R*). In Table 4.3, the group formation obtained by the algorithm in Appendix C using the survey resluts is compared with the group formation constructed by the estimation procedure. While the maximum estimation error amounts to 7.3%, or the over-estimation of 11 students for $G(2)$, this error seems to result from misinterpretation of $G(3)$ as $G(2)$, and to be contained within Pro-Groups. Similar observations can be made for other estimation errors. Indeed, the estimation errors of the sub-totals of $G(1), G(2)$ and $G(3)$ for Pro-Groups, $G(4), G(5)$ and $G(6)$ for Con-Groups and $G(7)$ and $G(8)$ for Neutral Groups are contained within 1.0% of surprising accuracy. Simulation results using this estimational formation are given in the last column of Table 4.2, demonstrating consistency strongly.

Table 4.3: Group Formation: Survey vs. Estimation

	Survey (%) (a)	Estimation (%) (b)	(a) - (b)
G(1): Convinced Approvers	7.4	6.7	0.7
G(2): Adaptable Approvers	44.3	51.6	-7.3
G(3): Independent Approvers	8.7	3.1	5.7
Pro-Total	60.4	61.4	-1.0
G(4): Convinced Disapprovers	3.4	6.7	-3.3
G(5): Adaptable Disapprovers	24.2	22.6	1.6
G(6): Independent Disapprovers	4.0	1.3	2.7
Con-Total	31.6	30.6	1.0
G(7): Local Opportunists	4.0	4.5	-0.5
G(8): Global Opportunists	4.0	3.5	0.5
Neutral-Total	8.0	8.0	0.0

5 Impact of Formation of Residential Groups on Public Referendum and Strategic Implications

In this section, the parametric simulation approach discussed in the previous sections is employed to examine how the formation of the eight residential groups would affect the outcome of public voting.

In Figure 5.1, the group formations of the nine replesentative models (Urban, Rural, Max-Vote) x (Con-Type, Basic-Type, Pro-Type) are plotted. We note that those with independent mind or less concern decrease as the model moves vertically from the Urban model to the Max-Vote model. More specifically, one has $G(1)\uparrow$, $G(2)\uparrow$, $G(3)\downarrow$, $G(4)\uparrow$, $G(5)\uparrow$, $G(6)\downarrow$, $G(7)\downarrow$ and $G(8)\downarrow$ from the bottom to the top. As the model moves horizontally from Con-Type to Pro-Type, the transfer from $G(5)$ to $G(2)$ and that from $G(6)$ to $G(3)$ occur increasingly.

Simulation results with 10 day campaign period are discussed here since those with 5 day capmiagn results have the same trends. Simulation results of (PV-R, V-R) are depicted in Figure 5.2 for 9 x 11 = 99 models constructed from the three basic models via the horizontal and vertical expansions as discussed in Subsection 3.2. It should be noted that, for each horizontal expansion, the opposing party wins in the public referendum for the models on the left-side of the basic model since one has PV-R $< \frac{V-R}{2}$. We call these models L-models of a horizontal expansion. In contrast, for the basic model and the models on the right-side of it, which we call BR-models of a horizontal expansion, one sees that PV-R $> \frac{V-R}{2}$ and the supporting party wins.

For each horizontal expansion, L-models and BR-models may be approximated by two separate lines via the least-square method. Figure 5.3 illustrates these lines and the leading coefficients are plotted in Figure 5.4. It can be seen that the leading coefficients for L-models are almost zero for all the models, while those for BR-models are monotonically decreasing as the model moves vertically from Urban model to Max-Vote model, and are almost zero beyond Rural model. Leading coefficients and constants are summarized in Table 5.1. These results combined with the robustness of the parametric simulation approach in changes of the voting

Figure 5.1: Group Formations of Nine Representative Models

Figure 5.2: L-Models vs. BR-Models: Simulation Results with 10 Day Campaign Period

population then lead to the following observations:

Observation 5.1

As discussed in 3.3, given a voting reslut of (PV-R, V-R), the underliying formation of the eight residential group can be estimated. Consequently, it is possible to device a strategy concerning how to achieve a target voting result starting from an estimated current situation by transforming the formation of the eight residential groups appropriately via campaign efforts and the like.

Observation 5.2

When the supporting party is perceived to be in minority, it may be possible to increase PV-R without necessarily increasing V-R. In this case, the focus should be on transforming less-convinced disapprovers into approvers.

Observation 5.3

When V-R is expected to be low, say 60% or less, and the supporting party is perceived to be close to a tie, it is difficult to increase PV-R without increasing V-R. In addition to the efforts to convince people, general efforts to increase V-R would help. Using the coefficients in Table 5.1, one may estimate the desired V-R level so as to achieve a prespecified PV-R level.

Figure 5.3: Liner Approximation of L-Models and BR-Models: Simulation Results with 10 Day Campaign Period

Figure 5.4: Leading Coefficients of L-Models vs. BR-Models: Simulation Results with 10 Day Campaign Period

Table 5.1: Leading Coefficients and Constants of L-Models and BR-Models: Simulation Results with 10 Day Campaign Period

Model	L-Model		BR-Model	
	leading coeff.	constant	leading coeff.	constant
8: Max-Vote Model	0.0186	93.76	0.0137	94.50
7	0.0019	86.84	0.0178	90.38
6	0.0077	83.54	0.0223	86.36
5: Rural Model	0.0006	78.59	0.0236	82.58
4	0.0096	73.26	0.1425	71.62
3	0.0105	68.10	0.1965	65.08
2	0.0105	62.98	0.3705	52.18
1	0.0089	57.86	0.4407	45.22
0: Urban Model	0.0005	52.86	0.4348	40.76

6 Conculding Remarks

Major findings of this paper can be summarized as follows:

1) The parametric simulation approach developed in this paper is quite robust in changes of the voting population. If the formation of the eight residential groups is kept intact, the voting results of (Pro-Vote Ratio, Voting Ratio) remains almost the same regardless of the voting population.

2) The parametric simulation approach is validated via a mock public referendum which took

place at School of Policy Studies, Kwansei Gakuin University in the spring term involving 149 students. The issue was whether the final grade of a course entitled “ Introduction to Human Ecology” should be determined by a term report or by an ordinary final exam. In comparison with the actual voting result of (Pro-Vote Ratio, Voting Ratio) = (53.69 %, 88.59 %), based on the set of the parameter values and the formation of the eight groups constructed from a survey, the simulation produced the voting result of (Pro-Vote Ratio, Voting Ratio) = (53.15 %, 86.88 %), yielding satisfactory accuracy.

3) Given a voting result of (Pro-Vote Ratio, Voting Ratio), the parametric simulation approach enables one to estimate the underlying formation of the eight residential groups.

4) Consequently, it is possible to device a strategy concerning how to achieve a target voting result starting from an estimated current situation by transforming the formation of the eight residential groups appropriately via campaign efforts and the like.

5) When the supporting party is perceived to be in minority, it may be possible to increase Pro-Vote Ratio without necessarily increasing Voting Ratio. In this case, the focus should be on transforming less-convinced disapprovers into approvers.

6) When Voting Ratio is expected to be low, say 60% or less, and the supporting party is perceived to be close to a tie, it is difficult to increase Pro-Vote Ratio without increasing Voting Ratio. In addition to the efforts to convince people, general efforts to increase Voting Ratio would help. Using the coefficients in Table 5.1, one may estimate the desired Voting Ratio level so as to achieve a prespecified Pro-Vote Ratio level.

Appendix

A Construction of Governing Transition Probability Matrices

Using the basic model described in Section 2, we now provide detailed specifics about how to construct governing transition probability matrices $\boldsymbol{\alpha}(l,u,v)$ in (2.9) for each $G(l)$, which collectively describe the behavioral characteristics of the group toward the voting date. We call $\boldsymbol{\alpha}(l,1,1)$ the basic transition probability matrix for $G(l)$. In order to describe 9 matrices succinctly, we introduce the following notation.

$$\begin{aligned} &\boldsymbol{\alpha}(l,1,1)=\boldsymbol{\alpha}(l), \quad \boldsymbol{\alpha}(l,1,2)=\hat{\boldsymbol{\alpha}}(l), \quad \boldsymbol{\alpha}(l,1,3)=\hat{\hat{\boldsymbol{\alpha}}}(l) \\ &\boldsymbol{\alpha}(l,2,1)=\boldsymbol{\beta}(l), \quad \boldsymbol{\alpha}(l,2,2)=\hat{\boldsymbol{\beta}}(l), \quad \boldsymbol{\alpha}(l,2,3)=\hat{\hat{\boldsymbol{\beta}}}(l) \\ &\boldsymbol{\alpha}(l,3,1)=\boldsymbol{\gamma}(l), \quad \boldsymbol{\alpha}(l,3,2)=\hat{\boldsymbol{\gamma}}(l), \quad \boldsymbol{\alpha}(l,3,3)=\hat{\hat{\boldsymbol{\gamma}}}(l) \end{aligned} \tag{A.1}$$

It should be noted that the changes of the matrices in u over the three stages along the time axis are denoted by Greek letters $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$, and the changes of the matrices in v over the states $[\boldsymbol{S}_{G(l)}(k)]_{l=1}^{8}$ are distinguished by $\boldsymbol{\alpha}$, $\hat{\boldsymbol{\alpha}}$ and $\hat{\hat{\boldsymbol{\alpha}}}$. Throughout the paper, we assume that all the residents, except those in $G(1)$ and $G(4)$, have not decided their voting position and are in state 0 at time $k=0$.

$G(1)$: Convinced Approvers

The residents in this group are determined to vote for YES and the corresponding diagram is depicted in Figure A.1(1). Accordingly, the group has only one transition probability matrix $\boldsymbol{\alpha}(1)=\boldsymbol{\alpha}(1,u,v)$, independent of u and v, given by

$$\boldsymbol{\alpha}(1)=[\alpha_{mn}(1)]=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \tag{A.2}$$

$G(2)$: Adaptable Approvers

Figure A.1(2) illustrates the transition structure of individual residents in $G(2)$. When the voting position is not decided (i.e. one is in state 0) at time k, one may remain in state 0, move to state 1 deciding to vote for YES, or move to state 2 deciding to vote for NO at time $k+1$. If the voting position at time k is in state 2, one may remain in state 2 or switch to state 1 deciding to vote for YES at time $k+1$. However, once a resident in $G(2)$ decides to vote for YES, this voting position will never be changed.

Reflecting the above transition structure and considering the fact that $u=v=1$ at time $k=0$ since everyone except those in $G(1)$ and $G(4)$ are in state 0 at time 0, the basic transition probability matrix $\boldsymbol{\alpha}(2,1,1)$ may be defined as

Figure A.1: Transition Structures of Residential Groups

(A.3) $$\boldsymbol{\alpha}(2) = [\alpha_{mn}(2)] = \begin{bmatrix} \alpha_{00}(2) & p(2)\{1-\alpha_{00}(2)\} & \{1-p(2)\}\{1-\alpha_{00}(2)\} \\ 0 & 1 & 0 \\ 1-\tilde{\alpha}_{00}(2) & q(2)\tilde{\alpha}_{00}(2) & \{1-q(2)\}\tilde{\alpha}_{00}(2) \end{bmatrix}.$$

Naturally, one has $0 \leq \alpha_{00}(2), \tilde{\alpha}_{00}(2), p(2), q(2) \leq 1$. In addition, we impose the condition $\frac{1}{2} \leq p(2), q(2) \leq 1$ so that the probability of moving from state 0 to state 1 (YES) would be higher than that of moving from state 0 to state 2 (NO).

For capturing the changes over $u = 1,2,3$ along the time axis, three additional parameters $a(2), r(2), s(2)$ are introduced where one has $0 \leq a(2),\ s(2) \leq 1$ and $1 \leq r(2)$. With notation in (A.1), one sees that

(A.4)
$$\boldsymbol{\beta}(2) = [\beta_{mn}(2)] = \begin{bmatrix} a(2)\alpha_{00}(2) & r(2)\alpha_{01}(2) & 1-\beta_{00}(2)-\beta_{01}(2) \\ 0 & 1 & 0 \\ \alpha_{20}(2)-s(2)\alpha_{21}(2) & 1-\beta_{20}(2)-\beta_{22}(2) & \alpha_{22}(2)-s(2)\alpha_{21}(2) \end{bmatrix}$$

and

(A.5) $$\boldsymbol{\gamma}(2) = [\gamma_{mn}(2)] = \begin{bmatrix} a(2)\beta_{00}(2) & r(2)\beta_{01}(2) & 1-\gamma_{00}(2)-\gamma_{01}(2) \\ 0 & 1 & 0 \\ \beta_{20}(2)-s(2)\beta_{21}(2) & 1-\gamma_{20}(2)-\gamma_{22}(2) & \beta_{22}(2)-s(2)\beta_{21}(2) \end{bmatrix}.$$

When a resident in $G(2)$ is either in state 0 or state 2, the transition probability to state 1 increases over the three stages along the time axis. Because of the nature of $G(2)$, one also expects that, for $u,v \in \{1,2,3\}$, $\boldsymbol{\alpha}(2,u,v)$ are stochastic matrices satisfying

$$\alpha_{m1}(2,u,v) \geq \alpha_{m2}(2,u,v), \quad m \in \{0,1,2\}, \tag{A.6}$$

reflecting the tendency to vote for YES with higher probabbility. After a little algebra, the following necessary and sufficient condition can be found for securing (A.6).

(A.7)

$$\begin{cases} max\{\frac{1-a(2)\alpha_{00}(2)}{2p(2)\{1-\alpha_{00}(2)\}}, \sqrt{\frac{1-a^2(2)\alpha_{00}(2)}{2p(2)\{1-\alpha_{00}(2)\}}}, 1\} < r(2) \leq min\{\frac{1-a(2)\alpha_{00}(2)}{p(2)\{1-\alpha_{00}(2)\}}, \sqrt{\frac{1-a^2(2)\alpha_{00}(2)}{p(2)\{1-\alpha_{00}(2)\}}}\}, \\ 0 \leq s(2) \leq \frac{-1+\sqrt{\frac{1+3q(2)\alpha_{00}(2)}{q(2)\alpha_{00}(2)}}}{2}. \end{cases}$$

Stochastic matrices $\hat{\boldsymbol{\alpha}}(2)$ and $\hat{\hat{\boldsymbol{\alpha}}}(2)$ are treated in a similar manner as u changes with the constraints in (A.7) modified accordingly.

The residents of $G(2)$ tend to move to state 1 as the ratio of the number of approvers among $\bigcup_{l\in I(2)} G(l)$ increases. In order to reflect this tendency, stochastic matrices $\boldsymbol{\alpha}(2,u,v)$ are changed as v increases from 1 to 3. We recall that $\boldsymbol{\alpha}(2) = \boldsymbol{\alpha}(2,1,1), \hat{\boldsymbol{\alpha}}(2) = \boldsymbol{\alpha}(2,1,2)$ and $\hat{\hat{\boldsymbol{\alpha}}}(2) = \boldsymbol{\alpha}(2,1,3)$ as introduced in (A.1).

(A.8)

$$\hat{\boldsymbol{\alpha}}(2) = [\hat{\alpha}_{mn}(2)] = \begin{bmatrix} b(2)\alpha_{00}(2) & 1-\hat{\alpha}_{00}(2)-\hat{\alpha}_{02}(2) & \tilde{b}(2)\alpha_{02}(2) \\ 0 & 1 & 0 \\ \alpha_{20}(2)+t(2)\{1-w(2)\}\alpha_{22}(2) & \alpha_{21}(2)+\{1-t(2)\}\{1-w(2)\}\alpha_{22}(2) & w(2)\alpha_{22}(2) \end{bmatrix}$$

and

(A.9)

$$\hat{\hat{\boldsymbol{\alpha}}}(2) = [\hat{\hat{\alpha}}_{mn}(2)] = \begin{bmatrix} b(2)\hat{\alpha}_{00}(2) & 1-\hat{\hat{\alpha}}_{00}(2)-\hat{\hat{\alpha}}_{02}(2) & \tilde{b}(2)\hat{\alpha}_{02}(2) \\ 0 & 1 & 0 \\ \hat{\alpha}_{20}(2)+t(2)\{1-w(2)\}\hat{\alpha}_{22}(2) & \hat{\alpha}_{21}(2)+\{1-t(2)\}\{1-w(2)\}\hat{\alpha}_{22}(2) & w(2)\hat{\alpha}_{22}(2) \end{bmatrix}.$$

Here when a resident in $G(2)$ is in state 0, the transition probabilities to state 0 or state 2 for $v=2$ are reduced by a factor of $b(2)$, $0 \leq b(2) \leq 1$, in comparison with those for $v=1$. The reduced probabilities are added to the transition probability to state 1. When the current voting position is NO in state 2, the self- transition probability is reduced by a factor of $w(2), 0 \leq w(2) \leq 1$, as v changes from 1 to 2. The reduced probability is split between state 0 with weight $t(2)$ and state 1 with weight $1-t(2)$. The transformation mechanism from $\hat{\boldsymbol{\alpha}}(2)$ to $\hat{\hat{\boldsymbol{\alpha}}}(2)$ as v changes from 2 to 3 is identical. Stochastic matrices $\boldsymbol{\beta}(2)$ and $\boldsymbol{\gamma}(2)$ are treated in a similar manner. Table A.1 exhibits the nine stochastic matrices for $G(2)$, summarizing these transformation mechanisms.

$G(3)$: Independent Approvers

Table A.1: Stochastic Matrices for G(2)

		v=1			v=2			v=3		
		0	1	2	0	1	2	0	1	2
u=1	0	α_{00}	α_{01} $= p(1-\alpha_{00})$	α_{02} $= (1-p)(1-\alpha_{00})$	$\hat{\alpha}_{00}$ $= b\alpha_{00}$	$\hat{\alpha}_{01}$ $= 1-\hat{\alpha}_{00}-\hat{\alpha}_{02}$	$\hat{\alpha}_{02}$ $= \tilde{b}\alpha_{02}$	$\hat{\hat{\alpha}}_{00}$ $= b\hat{\alpha}_{00}$	$\hat{\hat{\alpha}}_{01}$ $= 1-\hat{\hat{\alpha}}_{00}-\hat{\hat{\alpha}}_{02}$	$\hat{\hat{\alpha}}_{02}$ $= \tilde{b}\hat{\alpha}_{02}$
	1	0	1	0	0	1	0	0	1	0
	2	α_{20} $= 1-\tilde{\alpha}_{00}$	α_{21} $= q\tilde{\alpha}_{00}$	α_{22} $= (1-q)\tilde{\alpha}_{00}$	$\hat{\alpha}_{20}$ $= \alpha_{20}+t(1-w)\alpha_{22}$	$\hat{\alpha}_{21}$ $= \alpha_{21}+(1-t)(1-w)\alpha_{22}$	$\hat{\alpha}_{22}$ $= w\alpha_{22}$	$\hat{\hat{\alpha}}_{20}$ $= \hat{\alpha}_{20}+t(1-w)\hat{\alpha}_{22}$	$\hat{\hat{\alpha}}_{21}$ $= \hat{\alpha}_{21}+(1-t)(1-w)\hat{\alpha}_{22}$	$\hat{\hat{\alpha}}_{22}$ $= w\hat{\alpha}_{22}$
u=2	0	β_{00} $= a\alpha_{00}$	β_{01} $= r\alpha_{01}$	β_{02} $= 1-\beta_{00}-\beta_{01}$	$\hat{\beta}_{00}$ $= b\beta_{00}$	$\hat{\beta}_{01}$ $= 1-\hat{\beta}_{00}-\hat{\beta}_{02}$	$\hat{\beta}_{02}$ $= \tilde{b}\beta_{02}$	$\hat{\hat{\beta}}_{00}$ $= b\hat{\beta}_{00}$	$\hat{\hat{\beta}}_{01}$ $= 1-\hat{\hat{\beta}}_{00}-\hat{\hat{\beta}}_{02}$	$\hat{\hat{\beta}}_{02}$ $= \tilde{b}\hat{\beta}_{02}$
	1	0	1	0	0	1	0	0	1	0
	2	β_{20} $= \alpha_{20}-s\alpha_{21}$	β_{21} $= 1-\beta_{20}-\beta_{22}$	β_{22} $= \alpha_{22}-s\alpha_{21}$	α_{20} $= \beta_{20}+t(1-w)\beta_{22}$	$\hat{\beta}_{21}$ $= \beta_{21}+(1-t)(1-w)\beta_{22}$	$\hat{\beta}_{22}$ $= w\beta_{22}$	$\hat{\hat{\beta}}_{20}$ $= \hat{\beta}_{20}+t(1-w)\hat{\beta}_{22}$	$\hat{\hat{\beta}}_{21}$ $= \hat{\beta}_{21}+(1-t)(1-w)\hat{\beta}_{22}$	$\hat{\hat{\beta}}_{22}$ $= w\hat{\beta}_{22}$
u=3	0	γ_{00} $= a\beta_{00}$	γ_{01} $= ra\beta_{01}$	γ_{02} $= 1-\gamma_{00}-\gamma_{01}$	$\hat{\gamma}_{00}$ $= b\gamma_{00}$	$\hat{\gamma}_{01}$ $= 1-\hat{\gamma}_{00}-\hat{\gamma}_{02}$	$\hat{\gamma}_{02}$ $= \tilde{b}\gamma 02$	$\hat{\hat{\gamma}}_{00}$ $= b\hat{\gamma}_{00}$	$\hat{\hat{\gamma}}_{01}$ $= 1-\hat{\hat{\gamma}}_{00}-\hat{\hat{\gamma}}_{02}$	$\hat{\hat{\gamma}}_{02}$ $= \tilde{b}\hat{\gamma}_{02}$
	1	0	1	0	0	1	0	0	1	0
	2	γ_{20} $= \beta_{20}-s\beta_{21}$	γ_{21} $= 1-\gamma_{20}-\gamma_{22}$	γ_{22} $= \beta_{22}-s\beta_{21}$	γ_{20} $= \gamma_{20}+t(1-w)\gamma_{22}$	$\hat{\gamma}_{21}$ $= \gamma_{21}+(1-t)(1-w)\gamma_{22}$	$\hat{\gamma}_{22}$ $= w\gamma_{22}$	γ_{20} $= \hat{\gamma}_{20}+t(1-w)\hat{\gamma}_{22}$	$\hat{\hat{\gamma}}_{21}$ $= \hat{\gamma}_{21}+(1-t)(1-w)\hat{\gamma}_{22}$	$\hat{\hat{\gamma}}_{22}$ $= w\hat{\gamma}_{22}$

As shown in Figure A.1(3), the transition structure for $G(3)$ is the same as that for $G(2)$. Accordingly, we set the structure of the basic transition matrix of $G(3)$ identical to that of $G(2)$. In addition, the residents of $G(3)$ share the same tendency as those of $G(2)$ to move to state 1 with higher probability as time goes by , and hence, the transformation mechanism as u changes from 1 to 3 is also similar. One has:

(A.10) $$\boldsymbol{\alpha}(3) = [\alpha_{mn}(3)] = \begin{bmatrix} \alpha_{00}(3) & p(3)\{1-\alpha_{00}(3)\} & \{1-p(3)\}\{1-\alpha_{00}(3)\} \\ 0 & 1 & 0 \\ 1-\tilde{\alpha}_{00}(3) & q(3)\tilde{\alpha}_{00}(3) & \{1-q(3)\}\tilde{\alpha}_{00}(3) \end{bmatrix},$$

(A.11)
$$\boldsymbol{\beta}(3) = [\beta_{mn}(3)] = \begin{bmatrix} a(3)\alpha_{00}(3) & r(3)\alpha_{01}(3)) & 1-\beta_{00}(3)-\beta_{01}(3) \\ 0 & 1 & 0 \\ \alpha_{20}(3)-s(3)\alpha_{21}(3) & 1-\beta_{20}(3)-\beta_{22}(3) & \alpha_{22}(3)-s(3)\alpha_{21}(3) \end{bmatrix}$$

and
(A.12)
$$\boldsymbol{\gamma}(3) = [\gamma_{mn}(3)] = \begin{bmatrix} a(3)\beta_{00}(3) & r(3)\beta_{01}(3)) & 1-\gamma_{00}(3)-\gamma_{01}(3) \\ 0 & 1 & 0 \\ \beta_{20}(3)-s(3)\beta_{21}(3) & 1-\gamma_{20}(3)-\gamma_{22}(3) & \beta_{22}(3)-s(2)\beta_{21}(3) \end{bmatrix}.$$

The conditions in (A.7) can be rewritten accordingly.

The residents of $G(3)$ differ from those of $G(2)$ in that they are not influenced by others in deciding their voting positions. Consequently, $\boldsymbol{\alpha}(3,u,v)$ is independent of v so that

(A.13) $$\begin{cases} \boldsymbol{\alpha}(3) = \hat{\boldsymbol{\alpha}}(3) = \hat{\hat{\boldsymbol{\alpha}}}(3) \\ \boldsymbol{\beta}(3) = \hat{\boldsymbol{\beta}}(3) = \hat{\hat{\boldsymbol{\beta}}}(3) \\ \boldsymbol{\gamma}(3) = \hat{\boldsymbol{\gamma}}(3) = \hat{\hat{\boldsymbol{\gamma}}}(3). \end{cases}$$

As can be seen from Figures A.1(4) through A.1(6), the groups $G(4), G(5)$ and $G(6)$ are similar to $G(1), G(2)$ and $G(3)$ respectively in their transition structure, except that the transition structure from/to state 0 is interchanged with the transition structure from/to state 1. Accordingly, we construct the transition probability matrices for these groups in the following manner.

$G(4)$: Convinced Disapprovers

This group has only one transition probability matrix $\boldsymbol{\alpha}(4) = \boldsymbol{\alpha}(4,u,v)$, independent of u and v, given by

$$\boldsymbol{\alpha}(4) = [\alpha_{mn}(4)] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}. \tag{A.14}$$

$G(5)$: Adaptable Disapprovers

The basic transition probability matrix $\boldsymbol{\alpha}(5,1,1) = \boldsymbol{\alpha}(5)$ may be defined as

$$\boldsymbol{\alpha}(5) = [\alpha_{mn}(5)] = \begin{bmatrix} \alpha_{00}(5) & p(5)(1-\alpha_{00}(5)) & \{1-p(5)\}\{1-\alpha_{00}(5)\} \\ 1-\tilde{\alpha}_{00}(5) & q(5)\tilde{\alpha}_{00}(5) & \{1-q(5)\}\tilde{\alpha}_{00}(5) \\ 0 & 0 & 1 \end{bmatrix}. \tag{A.15}$$

Naturally, one has $0 \le \alpha_{00}(5), \tilde{\alpha}_{00}(5), p(5), q(5) \le 1$. In addition, we impose the condition $0 \le p(5), q(5) \le \frac{1}{2}$ so that the probability of moving from state 0 to state 2 (NO) would be higher than that of moving from state 0 to state 1 (YES).

For capturing the changes over $u = 1,2,3$ along the time axis, three additional parameters $a(5), r(5), s(5)$ are introduced where, for $0 \le a(5), s(5) \le 1$ and $1 \le r(5)$, we define:

(A.16)

$$\boldsymbol{\beta}(5) = [\beta_{mn}(5)] = \begin{bmatrix} a(5)\alpha_{00}(5) & 1-\beta_{00}(5)-\beta_{01}(5) & r(5)\alpha_{02}(5)) \\ \alpha_{10}(5)-s(5)\alpha_{12}(5) & \alpha_{11}(5)-s(5)\alpha_{12}(5) & 1-\beta_{10}(5)-\beta_{11}(5) \\ 0 & 0 & 1 \end{bmatrix}$$

and

(A.17)

$$\boldsymbol{\gamma}(5) = [\gamma_{mn}(5)] = \begin{bmatrix} a(5)\beta_{00}(5) & r(5)\beta_{01}(5)) & 1-\gamma_{00}(5)-\gamma_{02}(5) \\ \beta_{10}(5)-s(5)\beta_{12}(5) & \beta_{11}(5)-s(5)\beta_{12}(5) & 1-\gamma_{10}(5)-\gamma_{11}(5) \\ 0 & 0 & 1 \end{bmatrix}.$$

Because of the nature of $G(5)$, in contrast with (A.6), one expects that, for $u, v \in \{1,2,3\}$, $\boldsymbol{\alpha}(5,u,v)$ are stochastic matrices satisfying

$$\alpha_{m1}(5,u,v) \le \alpha_{m2}(5,u,v), \quad m \in \{0,1,2\}. \tag{A.18}$$

After a little algebra, the following necessary and sufficient condition can be found for securing (A.18).

$$\text{(A.19)} \quad \begin{cases} max\{\frac{1-a(5)\alpha_{00}(5)}{2\{1-p(5)\}\{1-\alpha_{00}(5)\}}, \sqrt{\frac{1-a^2(5)\alpha_{00}(5)}{2\{1-p(5)\}\{1-\alpha_{00}(5)\}}}, 1\} < r(5) \leq \\ \quad min\{\frac{1-a(5)\alpha_{00}(5)}{\{1-p(5)\}\{1-\alpha_{00}(5)\}}, \sqrt{\frac{1-a^2(5)\alpha_{00}(5)}{\{1-p(5)\}\{1-\alpha_{00}(5)\}}}\}, \\ \\ \quad 0 \leq s(5) \leq \frac{-1+\sqrt{\frac{1+3\{1-q(5)\}\alpha_{00}(5)}{\{1-q(5)\}\alpha_{00}(5)}}}{2}. \end{cases}$$

Stochastic matrices $\hat{\boldsymbol{\alpha}}(5)$ and $\hat{\hat{\boldsymbol{\alpha}}}(5)$ are treated in a similar manner as u changes with the constraints in (A.19) modified appropriately.

The residents of $G(5)$ tend to move to state 2 as the ratio of the number of disapprovers among $\bigcup_{l \in I(5)} G(l)$ increases. In order to reflect this tendency, stochastic matrices $\boldsymbol{\alpha}(5,u,v)$ are changed as v increases from 1 to 3 in parallel with (A.8) and (A.9). Keeping this in mind, we define:

(A.20)

$$\hat{\boldsymbol{\alpha}}(5) = [\hat{\alpha}_{mn}(5)] = \begin{bmatrix} b(5)\alpha_{00}(5) & \tilde{b}(5)\alpha_{01}(5) & 1-\hat{\alpha}_{00}(5)-\hat{\alpha}_{01}(5) \\ \alpha_{10}(5)+t(5)(1-w(5))\alpha_{11}(5) & w(5)\alpha_{11}(5) & \alpha_{12}(5)+\{1-t(5)\}\{1-w(5)\}\alpha_{11}(5) \\ 0 & 0 & 1 \end{bmatrix}$$

and

(A.21)

$$\hat{\hat{\boldsymbol{\alpha}}}(5) = [\hat{\hat{\alpha}}_{mn}(5)] = \begin{bmatrix} b(5)\hat{\alpha}_{00}(5) & \tilde{b}(5)\hat{\alpha}_{01}(5) & 1-\hat{\hat{\alpha}}_{00}(5)-\hat{\hat{\alpha}}_{01}(5) \\ \hat{\alpha}_{10}(5)+t(5)\{1-w(5)\}\hat{\alpha}_{11}(5) & w(5)\hat{\alpha}_{11}(5) & \hat{\alpha}_{12}(5)+\{1-t(5)\}\{1-w(5)\}\hat{\alpha}_{11}(5) \\ 0 & 0 & 1 \end{bmatrix}.$$

$\boldsymbol{\beta}(5,u,v)$ and $\boldsymbol{\gamma}(5,u,v)$ are treated similarly.

$G(6)$: Independent Disapprovers

We set the structure of the basic transition matrix of $G(6)$ identical to that of $G(5)$. The transformation mechanism as u changes from 1 to 3 is also similar. One has:

$$\text{(A.22)} \quad \boldsymbol{\alpha}(6) = [\alpha_{mn}(6)] = \begin{bmatrix} \alpha_{00}(6) & p(6)\{1-\alpha_{00}(6)\} & \{1-p(6)\}\{1-\alpha_{00}(6)\} \\ 1-\tilde{\alpha}_{00}(6) & q(6)\tilde{\alpha}_{00}(6) & \{1-q(6)\}\tilde{\alpha}_{00}(6) \\ 0 & 0 & 1 \end{bmatrix},$$

(A.23)

$$\boldsymbol{\beta}(6) = [\beta_{mn}(6)] = \begin{bmatrix} a(6)\alpha_{00}(6) & 1-\beta_{00}(6)-\beta_{02}(6) & r(6)\alpha_{02}(6)) \\ \alpha_{10}(6)-s(6)\alpha_{12}(6) & \alpha_{11}(6)-s(6)\alpha_{12}(6) & 1-\beta_{10}(6)-\beta_{12}(6) \\ 0 & 0 & 1 \end{bmatrix}$$

and
(A.24)

$$\boldsymbol{\gamma}(6)=[\gamma_{mn}(6)]=\begin{bmatrix} a(6)\beta_{00}(6) & 1-\gamma_{00}(6)-\gamma_{02}(6) & r(6)\beta_{02}(6)) \\ \beta_{10}(6)-s(6)\beta_{12}(6) & \beta_{11}(6)-s(6)\beta_{12}(6) & 1-\gamma_{10}(6)-\gamma_{12}(6) \\ 0 & 0 & 1 \end{bmatrix}.$$

The conditions in (A.19) can be rewritten accordingly. Because of the underlying independence, one has $\boldsymbol{\alpha}(6,u,v)$ being independent of v so that

(A.25)
$$\begin{cases} \boldsymbol{\alpha}(6)=\hat{\boldsymbol{\alpha}}(6)=\hat{\hat{\boldsymbol{\alpha}}}(6) \\ \boldsymbol{\beta}(6)=\hat{\boldsymbol{\beta}}(6)=\hat{\hat{\boldsymbol{\beta}}}(6) \\ \boldsymbol{\gamma}(6)=\hat{\boldsymbol{\gamma}}(6)=\hat{\hat{\boldsymbol{\gamma}}}(6). \end{cases}$$

$G(7)$: Local Opportunists

The residents in this group make their voting decisions by observing the ratio of approvers in their own group as well as in $G(2)$, $G(3)$, $G(5)$ and $G(6)$. The structure of the basic matrix is identical with other groups except $G(1)$ and $G(4)$.

(A.26)
$$\boldsymbol{\alpha}(7)=[\alpha_{mn}(7)]=\begin{bmatrix} \alpha_{00}(7) & p(7)\{1-\alpha_{00}(7)\} & \{1-p(7)\}\{1-\alpha_{00}(7)\} \\ 1-\tilde{\alpha}_{00}(7) & q(7)\tilde{\alpha}_{00}(7) & \{1-q(7)\}\tilde{\alpha}_{00}(7) \\ 1-\tilde{\alpha}_{00}(7) & \tilde{q}(7)\tilde{\alpha}_{00}(7) & \{1-\tilde{q}(7)\}\tilde{\alpha}_{00}(7) \end{bmatrix},$$

However, the parameter $p(7)$ affecting the transition probability from state 0 (Undecided) to state 1 (YES) takes a value different from $p(2)$ for $G(2)$ and $p(5)$ for $G(5)$, satisfying $p(5)<p(7)<p(2)$. This reflects the fact that, when a resident is in state 0 without any decision yet, those in $G(7)$ are more undecided than those in $G(2)$ having inclination to state 1 (YES) or those in $G(5)$ with inclination to state 2 (NO).

In constructing $\boldsymbol{\beta}(7)=\boldsymbol{\alpha}(7,2,1)$ and $\boldsymbol{\gamma}(7)=\boldsymbol{\alpha}(7,3,1)$, the transition probability structure in state 0 is identical to the basic matrix $\boldsymbol{\alpha}(7)=\boldsymbol{\alpha}(7,1,1)$, while the transition probability structure in state 1 is identical to that of $G(5)$ and that in state 2 is identical to that of $G(2)$. One has:
(A.27)

$$\boldsymbol{\beta}(7)=[\beta_{mn}(7)]=\begin{bmatrix} \alpha_{00}(7) & \alpha_{01}(7) & \alpha_{02}(7) \\ \alpha_{10}(7)-s(7)\alpha_{11}(7) & 1-\beta_{10}(7)1-\beta_{12}(7) & \alpha_{12}(7)-s(7)\alpha_{11}(7) \\ \alpha_{20}(7)-\tilde{s}(7)\alpha_{22}(7) & \alpha_{21}(7)-\tilde{s}(7)\alpha_{22}(7) & 1-\beta_{20}(7)1-\beta_{21}(7) \end{bmatrix},$$

and
(A.28)

$$\boldsymbol{\gamma}(7)=[\gamma_{mn}(7)]=\begin{bmatrix} \beta_{00}(7) & \beta_{01}(7) & \beta_{02}(7) \\ \beta_{10}(7)-s(7)\beta_{11}(7) & 1-\gamma_{10}(7)1-\gamma_{12}(7) & \beta_{12}(7)-s(7)\beta_{11}(7) \\ \beta_{20}(7)-\tilde{s}(7)\beta_{22}(7) & \beta_{21}(7)-\tilde{s}(7)\beta_{22}(7) & 1-\gamma_{20}(7)1-\gamma_{21}(7) \end{bmatrix}.$$

Because of the nature of $G(7)$, one expects that for $1 \leq u, v, \leq 3$,

$$
\text{(A.29)} \qquad \begin{cases} \alpha_{01}(7,u,v) = \alpha_{02}(7,u,v) \\ \alpha_{11}(7,u,v) \geq \alpha_{12}(7,u,v) \\ \alpha_{21}(7,u,v) \leq \alpha_{22}(7,u,v). \end{cases}
$$

The conditions (3.7) hold true if and only if

$$
\text{(A.30)} \qquad \begin{cases} p(7) = \frac{1}{2}, \\ 0 \leq s(7) \leq \frac{-1+\sqrt{\frac{1+3\{1-q(7)\}\alpha_{00}(7)}{\{1-q(7)\}\alpha_{00}(7)}}}{2}, \\ 0 \leq \tilde{s}(7) \leq \frac{-1+\sqrt{\frac{1+3\{1-q(7)\}\alpha_{00}(7)}{\{1-q(7)\}\alpha_{00}(7)}}}{2}. \end{cases}
$$

For $\hat{\boldsymbol{\alpha}}(7) = \boldsymbol{\alpha}(7,1,2)$ and $\hat{\hat{\boldsymbol{\alpha}}}(7) = \boldsymbol{\alpha}(7,1,3)$, the transition probability from state 0 to state 0 or state 2 is decreased by a factor $b(7)$, with the resulting decreased probabilies added to the transition probability from state 0 to state 1. In state 1, the transition probability structure is similar except that $b(7)$ is replaced by $\tilde{b}(7)$ with $\tilde{b}(7) \geq b(7)$. For state 2, the transition probability structure is similar to that of $\hat{\boldsymbol{\alpha}}(2)$ and $\hat{\hat{\boldsymbol{\alpha}}}(2)$ for $G(2)$. We define that:

(A.31)

$$
\hat{\boldsymbol{\alpha}}(7) = [\hat{\alpha}_{mn}(7)] = \begin{bmatrix} b(7)\alpha_{00}(7) & 1-\hat{\alpha}_{00}(7)-\hat{\alpha}_{02}(7) & b(7)\alpha_{02}(7)) \\ \tilde{b}(7)\alpha_{10}(7) & 1-\hat{\alpha}_{10}(7)-\hat{\alpha}_{12}(7) & \tilde{b}(7)\alpha_{02}(7)) \\ \alpha_{20}(7)+t(7)\{1-w(7)\}\alpha_{22}(7) & \alpha_{21}(7)+\{1-t(7)\}\{1-w(7)\}\alpha_{22}(7) & w(7)\alpha_{22}(7) \end{bmatrix},
$$

(A.32)

$$
\hat{\hat{\boldsymbol{\alpha}}}(7) = [\hat{\hat{\alpha}}_{mn}(7)] = \begin{bmatrix} b(7)\widehat{\alpha_{00}}(7) & 1-\hat{\hat{\alpha}}_{00}(7)-\hat{\hat{\alpha}}_{02}(7) & b(7)\widehat{\alpha_{02}}(7)) \\ \tilde{b}(7)\widehat{\alpha_{10}}(7) & 1-\hat{\hat{\alpha}}_{10}(7)-\hat{\hat{\alpha}}_{12}(7) & \tilde{b}(7)\hat{\alpha}_{02}(7)) \\ \hat{\alpha}_{20}(7)+t(7)\{1-w(7)\}\hat{\alpha}_{22}(7) & \hat{\alpha}_{21}(7)+\{1-t(7)\}\{1-w(7)\}\hat{\alpha}_{22}(7) & w(7)\hat{\alpha}_{22}(7) \end{bmatrix}.
$$

$G(8)$: Global Opportunists

The probabilistic structure of $G(8)$ in forming their voting positions is identical to that of $G(7)$, except that the likelihood of voting for YES would be increased by the ratio of approvers among the entire population rather than the subset $\bigcup_{l \in I(7)} G(l)$ in case of $G(7)$. The relevant parameters for $G(8)$ may take values different from those for $G(7)$.

B Questionnaire

Questionnaire: Week 1

Q1.1 Why did you choose School of Policy Studies at Kwansei Gakuin University to enter? For each factor below, please indicate the level of importance in your decision by marking one number on the right.

not important at all — very important

1) Because the curriculum matched with my career plan. 1–2–3–4–5–6–7–8–9–10
2) Because my friend recommended. 1–2–3–4–5–6–7–8–9–10
3) Because my parents or teachers recommended. 1–2–3–4–5–6–7–8–9–10

Q1.2 In writing class essays and term papers, what would be your typical information sources? For each factor below, please indicate the level of importance by marking one number on the right.

1) Library 1–2–3–4–5–6–7–8–9–10
2) Professors 1–2–3–4–5–6–7–8–9–10
3) Senior students 1–2–3–4–5–6–7–8–9–10
4) Classmates 1–2–3–4–5–6–7–8–9–10
5) Massmedia such as TV and newspapers 1–2–3–4–5–6–7–8–9–10

Q1.3 Do you participate in any club activities?

1) Yes
2) No

Q1.4 If you answered YES in Q1.3, what is your role in the club? Please choose one.

1) A leader
2) A sub-leader
3) A member

Q1.5 Why did you join the club? For each factor below, please indicate the level of importance in your decision by marking one number on the right.

1) Because I organized it. 1–2–3–4–5–6–7–8–9–10
2) Because the club matched with my interest. 1–2–3–4–5–6–7–8–9–10
3) Because my friend recommended. 1–2–3–4–5–6–7–8–9–10

4) Because my parents or professors recommended. 1–2–3–4–5–6–7–8–9–10
5) Because the club seemed interesting. 1–2–3–4–5–6–7–8–9–10
6) Because I had nothing to do else. 1–2–3–4–5–6–7–8–9–10

Q1.6 Concerning how to determine the final grade of this course, please choose one below that suits you best at this point in time.

1) An ordinary in-class closed-book final exam. The difficulty level of the final exam would be equal to that of last year.
2) A term report to be written and submitted over a five day period. The difficulty level of the term report would be higher than that of the final exam.
3) I haven't decided yet.

Q1.7 Through Week 1 to Week 4, which group below do you think you would belong to?

1) Those who strongly support the term report and will try to convince others.
2) Those who may be convinced to choose the term report.
3) Those who support the term report without influencing others or being influenced by others.
4) Those who strongly support the in-class closed-book final exam and will try to convince others.
5) Those who may be convinced to choose the in-class closed-book final exam.
6) Those who oppose the in-class closed-book final exam without influencing others or being influenced by others.
7) Those who are undecided, and may have a tendency to be influenced by close friends.
8) Those who are undecided, and may have a tendency to be influenced by the whole class atmosphere.

Questionnaire: Week 2 through Week 4

Q2.1 / Q3.1 / Q4.1
Concerning how to determine the final grade of this course, please choose one below that suits you best at this point in time.

1) An ordinary in-class closed-book final exam. The difficulty level of the final exam would be equal to that of last year.
2) A term report to be written and submitted over a five day period. The difficulty level of the term report would be higher than that of the final exam.
3) I haven't decided yet.

C Flow Chart of Algorithm for Classifying the 149 students into 8 Groups

References

[1] Aldrich, John H. (1993). Rational Choice and Turnout. *American Journal of Political Science* 37(1): 246-278.

[2] Austen-Smith, David (1987). Interest Groups, Campaign Contributions, and Probabilistic Voting. *Public Choice* 54(2): 123-139.

[3] Ball, Richard (1999). Discontinuity and Non-Existence of Equilibrium in the Probabilistic Spatial Voting Model. *Social Choice and Welfare* 16(4): 533-556.

[4] Browne, Eric C. and James, Peggy A. (1991). Simulation of Global and Local Intransitivities in a Simple Voting Game under Majority Rule. *Behavioral Science* 36(2): 148-156.

[5] Brunk, Gregory G. (1980). The Impact of Rational Participation Models on Voting Attitudes. *Public Choice* 35(5): 549-564.

[6] Calvert, Randall L. (1985). Robustness of the Multidimentional Voting Model: Candidate Motivations, Uncertainty, and Convergence. *American Journal of Political Science* 29(1): 69-95.

[7] Caputo, David A. (1977). Community Size, Public Attitudes, and Population-Policy Preferences. *Urban Affairs Quarterly* 13(2): 207-222.

[8] Enelow, J. M. and Hinich, M. J. (1994). A Test of the Predictive Dimensions Model in Spatial Voting Theory. *Public Choice* 78(2): 155-169.

[9] Fiorina, Morris P. (1976). The Voting Decision: Instrumental and Expressive Aspects. *Journal of Politics* 38(2): 390-413.

[10] Hansson, Sven Ove (1992). A Procedural Model of Voting. *Theory and Decision* 32: 269-301.

[11] Hashimoto, Akihiro (1997). A Ranked Voting System Using a DEA/AR Exclusion Model: A Note. *European Journal of Operational Research* 97: 600-604.

[12] Kaplan, David and Venezky, Richard L. (1994). Literacy and Voting Behavior: A Bivariate Probit Model with Sample Selection. *Social Science Research* 23: 350-367.

[13] Karahan, Gokhan R. and William F. Shughart (2004). Under Two Flags: Symbolic Voting the State of Mississippi. *Public Choice* 118: 105-124.

[14] Lin, Tse-Min, Enelow, James M. and Dorussen, Han (1976). Equilibrium in Multicandidate Probabilistic Spatial Voting. *Public Choice* 98(102): 59-82.

[15] McKelevey, G. H. (1976). Intransitivities in Multidimensional Voting Models and Some Implications for Agenda Control. *Journal of Economic Theory* 12: 472-482.

[16] McKelevey, G. H. (1979). General Conditions for Global Intransitivies in Formal Voting Models. *Econometrica* 47: 1085-1112.

[17] Schofield, N. (1978). Instability of Simple Dynamic Games. *Review of Economic Studies* XLV: 575-594.

[18] Singh, Kulwant, Siew Meng Leong, Chi Tiong Tan, and Kwei Cheong Wong (1995). A Theory of Reasoned Action Perspective of Voting Behavior: Model and Empirical Test. *Psychology and Marketing* 12(1): 37-51.

[19] Thalmann, Philippe (2004). The Public Acceptance of Green Taxes: 2 Million Voters Express Their Opinion. *Public Choice* 119: 179-217.