# Analytical Model of Visibility of a Landmark 

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#### Abstract

Landmarks in an urban area, such as castles, isolated towers, tall buildings, great hills and tall trees, are accents of turning point in a landscape. On the other hand, steel construction and elevators have pushed buildings higher, so the views of the landmarks have been obstructed by the buildings. This paper develops a simple analytical model to examine how buildings height and density affect the area from which the landmark is visible. The functional form of the probability of visibility from viewpoints will be derived where the distance between the successive buildings is represented by a renewal process.


Keyword: landmark; visibility; regulation; renewal process; landscape

JEL Classification: H71; H73; R51

## 1. Introduction

Landmarks in an urban area, such as castles, isolated towers, tall buildings, great hills and tall trees, are accents of turning point in a landscape. Typically, they are seen from many angles and distances, over the tops of short buildings. They are identified by their contrast in size, form, color, texture, function, or content of symbolism of sentiment with their surroundings. Researches on both psychology and geography, for example, Cohen and Schuepfer (1980), Allen, Kirasic, Siegel, and Herman (1979) have demonstrated that landmarks are visual configurations for course-maintaining aids. In addition, there is a tendency for those who are more familiar with an urban area to rely increasingly on systems of landmarks for their guidance. So, the landmarks can be thought as features of community or regional landscapes, as discussed in the famous book by Lynch(1960).

As pointed out by Felleman(1986), Landscape Institute et al.(2002), visibility studies play an important role in landscape analyses. In particular, Felleman(1986), Aguiló and Iglesias(1995) asserted that the landscape has to be evaluated based on the visibility from observers on the surface of the terrain rather than form above. In order to identify the area from which the landmark is visible, we have to specify the objects which block the sightlines to the landmark through cross-section maps. Aguiló and Iglesias(1995) discussed systematical calculation methods to detect the area visible from a fixed viewpoint, taking account of topographical conditions such as an uneven plane. Recent remarkable progress in computer graphics techniques enables us to define the area more easily by using standard GIS packages: see Jones(1997), Hanna(1999), Landscape Institute et al.(2002).

On the other hand, steel construction and elevators have pushed buildings higher. Modern architects have frequently ignored the relationship between the buildings and the existing landmarks. This is because the building height and building density usually are determined upon the necessity to preserve adequate interior space. Tall buildings and high building density block the views to landmarks. Thus, the tremendous burden of building bulk occupying the urban area has contributed to not only the congestion of people and traffic but also the disappearances of landmarks. Controlling the building height and density will be necessary for aesthetic considerations. Zoning regulates the land use to be administered in the public interest by protecting the interests of each individual who invests in the urban community. The zoning regulates building setbacks from property lines, volume envelope, sloping planes from street lines, maximum height, lot coverage and so on. Thus the zoning has a profound
effect on building height and density. To preserve the views to the landmark, an appropriate zoning ordinance on building height and density must be established.

Of course, computer mappings of visibility are one of helpful means, but generated results directly depend on individual study areas. Hence, it seems to be difficult to capture general characterization finding from the results. Contrary to these studies, we set up an analytical model in order to get some of the central features of the interaction between building controls and the visibility of a landmark. This paper formulates a probability model in a crosssection to investigate how location of viewpoints, building height and density and building distribution affect the visibility from surrounding area. From physical considerations, it may be reasonable to consider the location of buildings as random variables. We utilize renewal process to express these locations. To restrict the number of possibilities that need to be considered, we assume that all buildings within a zone have the same building height, and they are distributed according to the same density on a flat region.

First, we demonstrate how the visibility from viewpoints changes according to their distance from the landmark. Although, of course, the perceptible size of the landmark becomes smaller farther from the landmark, we concentrate on the simple question whether the landmark is visible or not. Second, we examine the impacts of the building height and density on the visibility. Information about the sensitivity of building height and building density are useful because relaxing a restriction of the building height and density may not be critical if the change in the visibility is minor. Alternatively, it may be beneficial to impose the existing requirements of the building height and density if the visibility will decrease significantly. Finally, most real-world building-distributions look as though they have been produced by a process which has either deterministic or probabilistic components. We examine the visibility of random building-distribution and that of regular one to recognize the impact of building-distributions.

The rest of this paper is organized as follows. The next section sets up our models under renewal process, and then carries out the sensitivity analyses against the building height and density. Section 3 considers the visibility under random distribution of buildings and that under their regular distribution. Conclusions are drawn in Section 4.

## 2. Model

### 2.1. Probability of visibility

Consider a cross-section with a perfectly even surface, which is divided into $n$ zones, as illustrated in Figure 1. We place a toll structure with height $l$ at an origin, and measure the distance from the structure along the horizontal axis to the right. The vertical axis measures the height from the surface. We assume that the landmark is idealized as a point and it is located at the top of the structure. This is indicated by a circle in the figure. Let $b_{i}$ be the position of the boundary between the $i$-th and $(i+1)$-th zones. For notational purposes, $b_{0}=0$. Different building height and density may be imposed on each zone. Within the $i$-th zone, the building height and density are fixed at $h_{i}(\geq 0)$ and $\lambda_{i}(\geq 0)$.

We are interested in how the probability of visibility from viewpoints with height $v$, to the landmark varies with their positions, when buildings are probabilistically distributed over the study area. Whether or not the landmark from each viewpoint is visible depends on the height and position of buildings, the height and position of the viewpoint, and the landmark height. However, the basic concept is that the landmark is visible from the viewpoint if and only if the straight line from the viewpoint towards the landmark does not intersect any building, as pointed out by Alonso et al.(1986), Felleman(1986), Jones(1997).

The analysis will be limited to the case of $v \leq h_{i} \leq l$ for any $i$, as shown in Figure 2. Define the ratio of the two heights $\alpha_{i}$ by $\alpha_{i} \equiv \frac{l-h_{i}}{l-v}$, so we have $0 \leq \alpha_{i} \leq 1$. When a building exists between $\alpha_{i} r$ and $r$, it blocks view from the site at $r$ to the landmark. Thus, if the building location is confined to the $i$-th zone, the landmark is visible from the site at $r$ if and only if there exists no buildings along the interval between $\max \left\{b_{i-1}, \alpha_{i} r\right\}$ and $\min \left\{b_{i}, r\right\}$. This is illustrated in Figure 2, where both axes measure the same with Figure 1. The length of the interval, denoted as $I_{i}(r)$, can be expressed as

$$
I_{i}(r) \equiv \begin{cases}\min \left\{b_{i}, r\right\}-\max \left\{b_{i-1}, \alpha_{i} r\right\} & \text { for } b_{i-1} \leq r<\frac{b_{i}}{\alpha_{i}}  \tag{1}\\ 0 & \text { otherwise }\end{cases}
$$

Figure 3 shows the graph of $\min \left\{b_{i}, r\right\}$ and that of $\max \left\{b_{i-1}, \alpha_{i} r\right\}$ by the broken and dot-dash lines, respectively. As is evident on referring to this figure, $I_{i}(r)$ is concave, piecewise linear and continuous with respect to $r$. Also, $I_{i}(r)$ with respect to $r$ is maximized at $r=b_{i}$.

Let $X_{j}$ be the random variable which indicates the position of the $j$-th building. Hence, its density is given by $\lambda_{i}$. We assume that these buildings are identically and independently distributed each other. For convenience, we assume that $X_{j} \leq X_{j+1},(j=1, \cdots)$. The random variable $Y_{j}$ is defined by $Y_{j} \equiv X_{j+1}-X_{j}$, so $\mathrm{E}[Y] \equiv \frac{1}{\lambda_{i}}$. Let $f_{Y}(y)$ be its probability
density function. Since $X_{j}$ 's are identically distributed and are mutually independent, $Y_{j}$ 's are according to renewal process. The random variable $Z$ is defined by the distance between the random point and its nearest building in the direction of the origin. By invoking the renewal process, the probability density function of $Z$ can be represented as follows:

$$
g_{Z}(z)=\lambda_{i}\left(1-F_{Y}(z)\right),
$$

where $F_{Y}(x)$ is the cumulative distribution of $Y$. The development of $g_{Z}(z)$ is given in Larson and $\operatorname{Odoni}(1981)$, $\operatorname{Ross}(1985)$. It should be noted that the building density $\lambda_{i}$ is a scale parameter. Accordingly, $\frac{X_{j}}{\lambda_{i}}$ is independent of $\lambda_{i}$, so do $\frac{Y_{j}}{\lambda_{i}}$ and $\frac{Z}{\lambda_{i}}$.

Let $p(r)$ be such probability from viewpoint located at $r$. In order to obtain $p(r)$, we begin to derive the probability of visibility cutted off only by the buildings along the $i$-th zone, denoted as $p_{i}(r)$. Note that if either $\lambda_{j}=0$ or $h_{j}=v$ for all $j$ with $j \neq i$, then $p(r)=p_{i}(r)$. Hence $p_{i}(r)$ can be regarded as the probability of visibility for a simplest city with only one zone where the building is higher than the viewpoint. Since the landmark is visible from $r$ is identical that no buildings exist along the interval with its length $I_{i}(r), p_{i}(r)$ can be expressed as follows:

$$
\begin{equation*}
p_{i}(r)=1-\lambda_{i} \int_{0}^{I_{i}(r)} 1-F_{Y}(u) d u \tag{2}
\end{equation*}
$$

This expression is intuitively reasonable because only the open space in front of the viewpoint towards the landmark matters for its visibility. Since $p_{i}(r)$ is a decreasing function with respect to $I_{i}(r), p_{i}(r)$ is also concave and continuous with respect to $r$. In addition, since $I_{i}(r)$ is maximized at $r=b_{i}$, the visibility from $r=b_{i}$ is the worst. Furthermore, the buildings within a zone cut off the visibility from its outer neighbors. Accordingly, we can characterize the probability of visibility $p_{i}(r)$ in terms of viewers' position as follows:

Property 1 Starting from $r=b_{i-1}, p_{i}(r)$ gradually decreases to a minimum at $r=b_{i}$, and then gradually increases until $r=\frac{b_{i}}{\alpha_{i}}$ with $r$.

This property means that the visibility due to the $i$-th zone $p_{i}(r)$ takes smaller values near the outer boundary of the $i$-th zone.

Then we extend to the plural number of zones. Since $p_{i}(r)$ and $p_{j}(r)$ with $i \neq j$ are probabilistically independent, we have

$$
\begin{equation*}
p(r)=\Pi_{i=1}^{n} p_{i}(r) . \tag{3}
\end{equation*}
$$

### 2.2. Building Regulation and Visibility

We have assumed that both the building height $h_{i}$ and the building density $\lambda_{i}$ are given. However, for many regulations which may be a balance between building bulk and exterior space required for the evolution of an urban landscape, $h_{i}$ and $\lambda_{i}$ vary over time, according to social values in the corresponding era. We examine whose views will be affected by raising either $h_{i}$ or $\lambda_{i}$, as in Blair(1986). Different from Blair(1986), we pursue the analytical finding. We evaluate the partial derivatives of $p_{i}(r)$ with respect to $h_{i}$ and $\lambda_{i}$, which provide useful quantitative information about its sensitivity against the building height and density, all other things being equal. Their numerically larger absolute values imply that $p_{i}(r)$ is more responsive to changes in the building height and density.

First, let us examine the effect of the building height $h_{i}$ on $p_{i}(r)$. Taking the partial derivative of $p_{i}(r)$ with respect to $h_{i}$ gives

$$
\frac{\partial p_{i}(r)}{\partial h_{i}}=-\lambda_{i}\left(1-F_{Y}\left(I_{i}(r)\right)\right) \frac{\partial I_{i}(r)}{\partial h_{i}} .
$$

It follows from (1) that

$$
\frac{\partial I_{i}(r)}{\partial h_{i}}= \begin{cases}\frac{1}{l-v} r & \text { for } \frac{b_{i-1}}{\alpha_{i}} \leq r<\frac{b_{i}}{\alpha_{i}} ; \\ 0 & \text { otherwise. }\end{cases}
$$

Hence, we have

$$
\frac{\partial p_{i}(r)}{\partial h_{i}}= \begin{cases}\frac{\gamma_{i}}{l-v} r\left(1-F_{Y}\left(I_{i}(r)\right)\right) & \text { for } \frac{b_{i-1}}{\alpha_{i}} \leq r<\frac{b_{i}}{\alpha_{i}} ; \\ 0 & \text { otherwise. }\end{cases}
$$

Since $1-F_{Y}\left(I_{i}(r)\right)$ is non-decreasing for $b_{i} \leq r \leq \frac{b_{i}}{\alpha_{i}}$, the minimum location of $\frac{\partial p_{i}(r)}{\partial h_{i}}$ subject to $b_{i} \leq r<\frac{b_{i}}{\alpha_{i}}$ is given by $r=\frac{b_{i}}{\alpha_{i}}$. The value of $\frac{\partial p_{i}(r)}{\partial h_{i}}$ evaluated at $r=\frac{b_{i}}{\alpha_{i}}$ is given by $-\frac{\lambda_{i}}{l-v} \frac{b_{i}}{\alpha_{i}}$. On the other hand, for $\frac{b_{i-1}}{\alpha_{i}} \leq r \leq b_{i}, \frac{\partial p_{i}(r)}{\partial h_{i}} \geq-\frac{\lambda_{i}}{l-v} r \geq-\frac{\lambda_{i}}{l-v} b_{i}$, where the first inequality holds since $1-F_{Y}\left(I_{i}(r)\right) \leq 1$ and the last inequality holds since $r \leq b_{i}$. Comparing these two results indicates that $\frac{\partial p_{i}(r)}{\partial h_{i}}$ is minimized at $r=\frac{b_{i}}{\alpha_{i}}$. Thus, we have

Property 2 1) $\frac{\partial p_{i}(r)}{\partial h_{i}}<0$ for $\frac{b_{i-1}}{\alpha_{i}} \leq r \leq \frac{b_{i}}{\alpha_{i}}$; 2) $\frac{\partial p_{i}(r)}{\partial h_{i}}$ is minimized at $r=\frac{b_{i}}{\alpha_{i}}$.
The first claim of this proposition states that raising building height of a zone cannot harm the visibility from the interval $\left[b_{i-1}, \frac{b_{i}}{\alpha_{i}}\right]$, i.e., the left-hand part of its zone anymore. An intuitive explanation for this result is that the visibility there depends on whether or not at least one building is located in front of them, no matter how high these buildings are.

The second claim means that the most sensitive viewpoint against the building height is $\frac{b_{i}}{\alpha_{i}}$. An underlying mechanism of this result is as follows. Raising building height makes some buildings newly block the view from a fixed viewpoints. For a sufficiently small increment $\epsilon$, the length of the interval where such buildings are located is given by $\frac{\epsilon}{l-v} r$, that is, the length is in perfectly proportion to the distance of the viewpoint from the landmark. Therefore, the most sensitive viewpoint is situated at the outer boundary.

Next, let us inspect the effect of the building density $\lambda_{i}$ on $p_{i}(r)$. Taking the partial derivative of $p_{i}(r)$ with respect to $\lambda_{i}$, while using that $\lambda_{i}$ is a scale parameter, gives

$$
\frac{\partial p_{i}(r)}{\partial \lambda_{i}}=-I_{i}(r)\left(1-F_{Y}\left(I_{i}(r)\right)\right)
$$

This indicates that $\frac{\partial p_{i}(r)}{\partial \lambda_{i}}$ can be regarded as a function with respect to $I_{i}(r)$. Since $I_{i}(r)$ is continuous and convex, $\frac{\partial p_{i}(r)}{\partial \lambda_{i}}$ has either a unique or two global minimum location(s). Note that $I_{i}(r)$ is independent of $\lambda_{i}$. If $\lambda_{i} \approx 0$, then $1-F_{Y}\left(I_{i}(r)\right) \approx 1$ for any $r$, so the minimum location of $\frac{\partial p_{i}(r)}{\partial \lambda_{i}}$ will coincide with the maximum location of $I_{i}(r)$, i.e., $r=b_{i}$. As $\lambda_{i}$ is increased, $1-F_{Y}\left(I_{i}(r)\right)$ will more contribute to the derivative $\frac{\partial p_{i}(r)}{\partial \lambda_{i}}$ than $I_{i}(r)$. This indicates that two minimum locations of $\frac{\partial p_{i}(r)}{\partial \lambda_{i}}$ originating from $b_{i}$, will move towards $b_{i-1}$ and $\frac{b_{i}}{\alpha}$ which both maximize $1-F_{Y}\left(I_{i}(r)\right)$, respectively.

Property 3 1) $\frac{\partial p_{i}(r)}{\partial \lambda_{i}}<0$ for $b_{i-1} \leq r \leq \frac{b_{i}}{\alpha_{i}}$; 2) for small $\lambda_{i}, \frac{\partial p_{i}(r)}{\partial \lambda_{i}}$ is minimized at $r=b_{i}$. As $\lambda_{i}$ is increased, its two global minima move from $r=b_{i}$ to $r=b_{i-1}$ and $r=\frac{b_{i}}{\alpha_{i}}$, respectively. The first claim of this proposition implies that raising the building density will harm the visibility from the whole of the zone. The second claim states that the most sensitive viewpoints move from $b_{i}$ towards the opposite directions. To interpret the second claim, we consider two extreme cases. When $\lambda_{i}$ is very small such that buildings rarely exists, the number of building in front of any viewer are almost zero. As a result, the impact of a small increment in the building density on its visibility is approximately proportional to the length of the open space necessary for visibility in the direction of the landmark, i.e., $I_{i}(r)$. On the other hand, when $\lambda_{i}$ is very large such that buildings are crowed, the landmark is invisible from almost all viewpoints. So constructing a new building does not affect visibility, except for the viewpoints where have smallest open space necessary for visibility, i.e., the inner end of the zone $r=b_{i-1}$ and the farthest affected position $r=\frac{b_{i}}{\alpha_{i}}$. Combining these two extreme cases indicates that as $\lambda_{i}$ is increased, the most sensitive viewpoints against the building density move from the location maximizing $I_{i}(r)$ towards the one minimizing $I_{i}(r)$.

Comparing Properties 2 with 3 indicates that an increment in building density and height causes different influence on the visibility of the landmark, even though increasing the building density and height both harm such visibility. This finding does not seem to be in accord with intuition. In addition, the visibility from the vicinity of $\frac{b_{i}}{\alpha_{i}}$ is most sensitive to building height. This indicates that loosening a limitation on building height may be critical in the neighborhood of $\frac{b_{i}}{\alpha_{i}}$. Moreover, for small building density, raising the building density has the greatest influence on $b_{i}$, so loosening a limitation on building density may be critical near $b_{i}$. On the other hand, for large building density, two most sensitive locations against building density exist such that $b_{i}$ is between them. As the building density is increased, both locations move farther away from $b_{i}$.

## 3. Examples

### 3.1. Random Distribution

As long as the renewal process of building distribution is maintained, no clear conclusions can be derived. In fact, it is very difficult to get analytical expressions for $p(r)$. Hence, we consider two extreme cases: (1) random building-distribution; (2) regular building-distribution. In the former, there are buildings scattered at random in each zone with its density $\lambda_{i}$. In the latter, the buildings in each zone are distributed perfect regularly having rate $\lambda_{i}$. Thus, although the former is produced by a purely probabilistic process, the latter is perfectly ordered in a way such that the building locations are completely dependent upon one another.

For the random distribution, all buildings in the $i$-th zone are distributed in accordance with a Poisson process having rate $\lambda_{i}(>0)$. Accordingly, the distance between successive buildings is independent exponential random variable with its mean $\frac{1}{\lambda_{i}}$ : see $\operatorname{Ross}(1985)$. Hence,

$$
F_{Y}^{R A N}(y)=1-\exp \left(-\lambda_{i} y\right), \quad(0 \leq y)
$$

Going now back to (2) and (3) yields the probability in the random distribution $p^{R A N}(r)$

$$
\begin{align*}
p^{R A N}(r) & =\Pi_{i=1}^{n}\left(1-\lambda_{i} \int_{0}^{I_{i}(r)} \exp \left(-\lambda_{i} u\right) d u\right), \\
& =\Pi_{i=1}^{n} \exp \left(-\lambda_{i} I_{i}(r)\right), \\
& =\exp \left(-\sum_{i=1}^{n} \lambda_{i} I_{i}(r)\right), \quad(0 \leq r) . \tag{4}
\end{align*}
$$

Since $I_{i}(r)$ is continuous for $r \geq 0, p^{R A N}(r)$ is also continuous for $r \geq 0$.

### 3.2. Regular Distribution

For the regular distribution, the distance between successive buildings in the $i$-th zone is always $\frac{1}{\lambda_{i}}$ apart. As a result, the cumulative distribution function of the distance between successive buildings, dented as $F_{Y}^{R E G}(y)$, is given by

$$
F_{Y}^{R E G}(y)= \begin{cases}0, & \text { for } 0 \leq y \leq \frac{1}{\lambda_{i}} \\ 1, & \text { for } \frac{1}{\lambda_{i}}<y .\end{cases}
$$

Using (2) and (3), the probability in the regular distribution $p^{R E G}(r)$ is expressed as

$$
\begin{align*}
p^{R E G}(r) & =\Pi_{i=1}^{n}\left(1-\lambda_{i} \int_{0}^{\min \left\{I_{i}(r), \frac{1}{\lambda_{i}}\right\}} d u\right), \\
& =\Pi_{i=1}^{n} \max \left\{1-\lambda_{i} I_{i}(r), 0\right\}, \quad(0 \leq r) \tag{5}
\end{align*}
$$

Since $I_{i}(r)$ is continuous and piecewise linear for $r \geq 0, p^{R E G}(r)$ is also continuous and piecewise linear for $r \geq 0$. Since $1-\lambda_{i} I_{i}(r) \leq \exp \left(-\lambda_{i} I_{i}(r)\right)$ for any $r(\geq 0)$, making a comparison (4) with (5) yields

Property $4 p^{R E G}(r) \leq p^{R A N}(r)$ for any location $r(\geq 0)$.
Thus, for fixed building height $h_{i}$ 's and densities $\lambda_{i}$ 's, the visibility in the regular distribution is always worse than that in the random one from any viewpoint. This result is hardly surprising. Since the average distance from a viewpoint to its nearest building in the direction of the landmark in the regular distribution is always smaller than that in the random one.

### 3.3. Monocentric City

In order to understand the extent to which visibility is affected by the layout of building regulation, we consider a monocentric city which is the simplest theoretical model in urban economics. The city is divided into three zones by their boundaries $b_{1}=100(m), b_{2}=200(\mathrm{~m})$ and $b_{3}=300(m)$. Thus, the city is envisaged as a central business district of radius $b_{1}$ where is surrounded with a circular high-rise residential area of radius $b_{2}$, and a circular low-rise residential area. The height and density of buildings of these three zones are restricted as follows: $h_{1}=15(m), h_{2}=10(m), h_{3}=5(m), \lambda_{1}=0.02(1 / m), \lambda_{2}=0.01(1 / m)$ and $\lambda_{3}=0.005(1 / m)$. Hence, the regulation on building height and density of monocentric city is more tighten with the distance from the CBD, as most real-world cities. A landmark with height $l=30(m)$ is located on the center of the CBD. We evaluate the probability of visibility from the viewpoint with its height $v=1.5(m)$ and location $r$, when all buildings are distributed either randomly or regularly.

The probabilities of visibility $p^{R A N}(r)$ and $p^{R E G}(r)$ can be represented by the solid and dotted curves in Figure 4, respectively. On the basis of the examples, we can show some distinct characteristics of the visibility of a landmark located at the CBD. First, roughly speaking, in the monocentric city, both probabilities $p^{R A N}(r)$ and $p^{R E G}(r)$ have U-shaped structures. Second, these probabilities have local minima near $b_{i}$ 's and local maxima in the neighborhood of $\frac{b_{i}}{\alpha_{i}}$ 's, . Thus, we see that the visibility is the worst near $b_{i}$ 's. Of course, this is consistent with Property 1. Finally, comparing the solid curve with dotted curve indicates that the graph of $p^{R A N}(r)$ is always above that of $p^{R E G}(r)$, which coincides with Property 4.

In order to confirm Properties 1 and 2, the probabilities $p_{2}(r)$ 's for the random distribution are graphically depicted in Figure 5. Five types of graphs are constructed, corresponding to five values of the building height $h_{2}\left(h_{2}=5,7.5,10,12.5,15(m)\right)$. Similarly, in order to confirm Properties 1 and $3, p_{2}(r)$ 's for the random distribution corresponding to five building densities $\lambda_{2}\left(\lambda_{2}=0.005,0.075,0.01,0.015,0.02,(1 / m)\right)$ are graphically depicted in Figure 6. Clearly, these numerical results agree with Properties 1, 2 and 3.

## 4. Concluding Remarks

In this paper, visibility is defined with respect to the landmark where stands out from its surroundings visually. Although our discussion was presented in the context of an analytical framework, our findings seem to be useful in characterizing the interaction between the visibility of a the landmark and building regulations at the conceptual level. First, we showed that the probability of visibility has U-shaped structures, and the visibility of the landmark is the worst when the observer is located at the outer end of zones. Second, we demonstrated that the impact of building height on the visibility is rather different from that of building density. Finally, we proved that the visibility under the random building-distribution is better than that under regular building-distribution. Note that the landmark is visible from a viewpoint if and only if the viewpoint also is visible from the landmark. Therefore, our findings are applicable to the visibility from observation platforms to cities.

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Figure 1: Study Area


Figure 2: Height of Landmark, Buildings and Viewpoint


Figure 3: Length of Open Interval for Visibility $I_{i}(r)$


Figure 4: Visibility in a Monocentric City


Figure 5: Visibility versus Building Height


Figure 6: Visibility versus Building Density

