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An Analysis of Japanese Automobile Market in Market Equilibrium

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Abstract

Analyzing consumers' product preferences has become more powerful tool as information technology progresses and marketing methods improves. Using POS data, we can analyze consumers characteristics and their diversity. In many cases, however, we may not be able to get hold of the individually-based micro data. In this paper, we empirically analyze Japanese automobile market using market-level aggregate data under market equilibrium. We use the random coefficient logit model to allow flexibility in substitution patterns. We found that consumers, on average, seem to prefer vehicles belonging to mini vehicle category. However, there are no significant heterogeneity in almost all the variables.

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1 Introduction

Japan is a highly mortorized society. As such, each year nearly two hundred new models are introduced by the domestic and foreign manufacturers, and close to 5 million passenger cars and light trucks are being sold. There is a wide range of makes and models, and people make their choices based on their own preferences and needs. In Japanese new vehicles market, the share of SUVs, minivans and station wagons and mini vehicles (less than 660cc in displacement) increased rapidly in 1990's, from 5.1% and 3.4% in 1989 to 30.2% and 17.1% in 1999 respectively as seen in Figure 1.

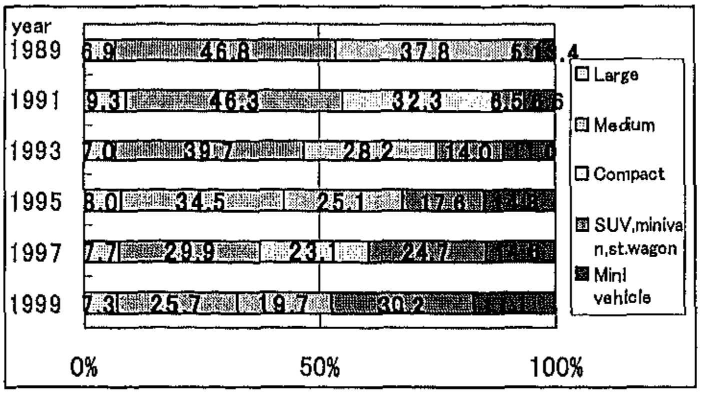

Figure 1: Share of vehice segment household use in Japan

Why do Japanese consumers increasingly prefer these vehicles? What determines their preferences for and choice of a certain kind of automobiles? What characteristics do the consumers have in common who drive the same type of vehicles? We wrote this article to answer these questions for 2002 using the model-by-model new vehicle sales data that are available publicly. Incidentally 2002 is the latest year in which these data are available to us.

We follow in a tradition of applied economic and industrial organization literature that tries to reveal basic parameters of demand and supply. In this framework, products are recognized as collections of characteristics. Each consumer chooses the product that maximizes the utility derived from product characteristics. Each manufacturer, on the other hand, is assumed to employ pricing policies that maximize the joint profits of the firm across all the products it produces. Their costs of producing such products, however, are assumed to depend upon the product characteristics as well as the economy of scale. Specifically we employ as the demand framework a class of differentiated products demand models called the random coefficient model of discrete choice of demand proposed in Berry (1994). Subsequently it was applied to the U.S. automobile market in Berry, Levinsohn and Pakes (1995) (henceforth: BLP), Sudhir (2001), and Petrin (2002). The model can account for heterogeneous preferences of the utility maximizing consumers and is able to realize more reasonable substitution patterns between similar products.

Our analysis indicates that in 2002 mini vehicles as a vehicle category may have increased the utility of all consumers slightly, while minivans did so for some but on the average their effects are negative, suggesting that the potential market size of the latter be much smaller than previously believed. It appears that this article is the first one to analyze the Japanese automobile market using the random coefficient model in market equilibrium and obtain the Japanese consumers' heterogeneous automobile preferences that the widely-used logit model of demand cannot uncover.

Previous studies

The evolution of discrete choice models originated from McFadden (1973) where he combined logit model with utility maximization. Then it diverged into two main approaches. The first one makes use of individual consumer-level data, and the second one utilizes aggregate market-level data.

The former is mainly based on logit models that estimate demand at an individual level either directly (Train, 1986) or through nested versions of logit model assuming an apriori ordering (Berkovec and Rust, 1985), (Mannering, 2002). Brownstone and Train (1999) analyze individual vehicle choices with random coefficient model to realize consumers' substitution pattern. In recent Japanese literature, using nested logit model and random coefficient model, Yamamoto et al. (2001) analyze car ownership including vehicle choice (new/old, a vehicle size combination) and allocation in household with questionnaire (individual) data. Hibiki and Arimura (2001) also analyze new vehicle ownership but could not get consumer-level data. They substitute used car transaction data including who was selling what car with what characteristics. Their logit model focuses on running cost of choice vehicle. These papers require product characteristics to match consumer characteristics, thereby allowing both for a high degree of product differentiation and for consumer heterogeneity, but it pays the price of neglecting the supply side and the market equlibrium considerations.

Although formally classified as the latter, many classical Japanese literature using aggregate data ties with market share but often neglects the supply side as well as the rival products prices or characteristics and dealing the vehicle price with an exogeneous variable (Katahira (1977), Ohta (1980)). Katahira (1977) uses five passenger car sales volume data in the same kind of vehicle segment and estimates market share with logit model. If the product substitution patterns were incorporated, however, the large number of products would make too many parameters to be estimated. In the U.S., the earliest applications of the random coefficient model were apparently the automobile demand models of Boyd and Melman (1980) and Cardell and Dunbar (1980). They used aggregate, market-share data rather than customer-choice data.

The stream of aggregate industry literature directry addresses demand and supply. Under the assumption of the existence of a Nash equilibrium, BLP (1995) proposes estimation of consumers' indirect utility with the random coefficient model in the U.S. automobile market and analyzes substitution patterns. Sudhir (2000) follows BLP, takes a theory-driven empirical approach to gain a deeper understanding of the competitive pricing behavior of firms in the U.S. auto market, but he does not assume the Bertrand equilibrium. Petrin (2002) also estimates with the random coefficient model, but incorporates available data that relate the average characteristics of consumer to the characteristics of products they purchase. In Japan, Tanishita et al. (2002) analyzes impact of car-related taxes on fuel consumption using logit model on consumption side and a first-order profit maximization condition on supply side.

Why we follow BLP?

In this paper, we would explore the utility behind consumers' vehicle type choice and safety considerations for new vehicle purchase in Japan. We specially focus on minivans and mini vehicles, and new safety features.

There are reasons why we follow BLP for estimating demand for differentiated products. The method is superior to other prior models because (1) the model can be estimated using only market-level price and quantity data, (2) it deals with the endogeneity of prices, and (3) it allows interaction between product characteristics and consumers' preference, so it produces demand elasticities that are more realistic.

In section 3, we specify the consumers' utility and derive the market share from a general class of discrete choice models.

2 Japanese Automobile Market

2.1 Domestic Sales of New vehicles

Figure 2 shows the total sales of new passenger vehicles in Japan for the last nine years. Passenger vehicle sales grew for the fourth consecutive year, rising by 3.5% to 4,441,354 units in 2002. [1]

The small (661cc-2000cc) and mini vehicles (660cc and under) sectors have a dominant presence in the Japanese market. In 2002, these two sectors combined accounted for 84.8% of the market, of which small cars accounted for 55.4% and mini vehicles took a 29.4% share. By way of comparison, standard cars (2000cc+) peaked in 1995 with a 20% share of the market but have been on a steady downward trend since then.

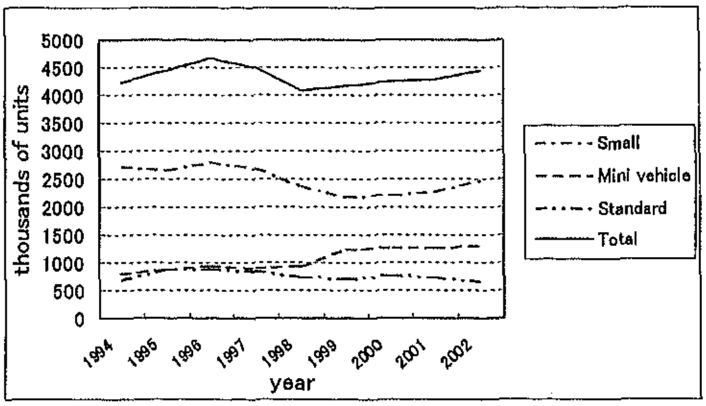

Figure 2: The total sales of new passenger vehicles in Japan (1994-2002)

[1] With all vehicles included, registrations of new vehicles in 2002 declined for the second straight year, resulting in a year-on-year decrease of 1.9% to total 5,792,093 units. Sales of trucks declined for the seventh consecutive year, dropping by 16.6% to 1,334,380 units, while sales of buses were up 2.7%, the first rise in two years. Japanese Automobile Manufacturers Association (JAMA) says that the drop in vehicle sales was attributed to the weak market for trucks in the wake of Japan's prolonged economic slowdown.

Figure 3 shows the shares within the Japanese new vehicle market that each manufacturer claimed in the years between 1994 and 2002. As evidenced by the figure, Japanese automobile market is oligopolistic market.

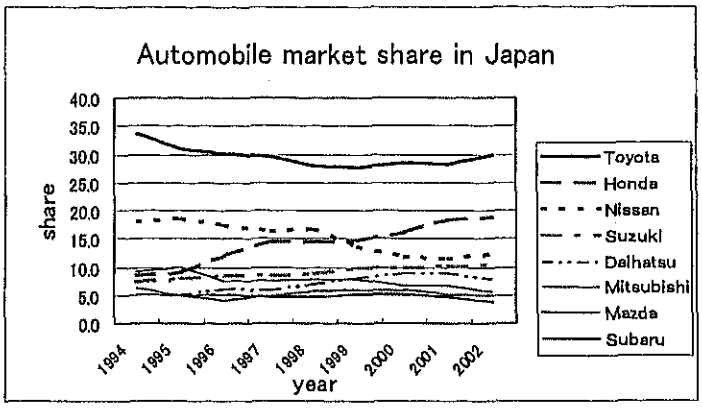

Figure 3: Firm-by-firm Market shares in Japan (1994-2002)

2.2 The vehicle segments and characteristics of choice

When we discuss Japanese automobile market, two vehicle segments—minivans and mini vehicles, and two vechicle characteristics or features—mileage and safety are important.

In terms of the number of models as well as the sales figures, minivans with the third row seats have been very popular in Japan with thirty one models and approximately twenty percent of the total passenger vehicles sales in 2002. The Japanese consumers seem to be enamored with the idea of carrying up-to-seven people despite their consistently shrinking average family size.

For the obvious 80% reduction in vehicle taxes and close to 30% liability insurance advantage over vehicles with 2000cc displacement, the Japanese consumers are interested in purchasing mini vehicles at least as a second vehicle. Established in 1949, the mini vehicles category is a distinctive sector in Japan. After 1998 changes in regulation addressing the safety concern, the category is currently expanded to vehicles whose lengths, widths, and heights are respectively less than 3.4m, 1.48m, and 2.0m and with an engine displacement of 660cc or lower. As these size and displacement figures indicate, mini vehicles additionally offer excellent fuel economy and the ability to maneuver in Japan's narrow streets. In 2002, mini vehicles sales figure is at 1.3 million units, just about growing for every consecutive year.

Since Japan is one of those industrial countries to keep gas prices artificially high to encourage fuel conservation, high mileage is obviously very important to the Japanese consumers. Also rapidly growing environmental awareness in Japan forces the vehicle manufacturers to develop "green" technology to meet the needs of society and the consumer. However these developments are likely to put an upward pressure on the production costs.[2]

Driver and passenger side air bags and anti-lock braking systems (ABS) have been the standard equipment of recent vehicles. According to Japan Automobile Manufacturers' Association (JAMA) report No.83 (Life style and vehicle type choice), consumers were found to pay more attention to the safety equipment when buying new vehicle. In the report, consumers inter-

[2]For example, some of the Japanese manufacturers are moving aggressively towards introducing hybrid vehicles to the market. These vehicles combine the traditional combustion engine with electric motor technology. They have low levels of emissions and high levels of fuel economy, often exceeding 60 mpg. For example, Toyota has sold more than 120,000 hybrid vehicles since the introduction of its Prius and forecasts selling 300,000 a year by 2005. Honda has been aggressively selling its two hybrids, the Civic and the Insight. It is widely believed, however, that these campanies are selling hybrids at or below cost.

ested in the safety equipment grew considerably from 29% in 1995 to 58% in 1999. Furthermore, 83% of them were found willing to buy safer vehicles even if its price is higher. Therefore, we assume that safety features and equipments increase the utility of the vehicles, though they are definitely cost shifters for the manufacturers as well.

3 The model and distributional assumptions

In the following, we discuss why the random coefficient models of discrete choice is a more realistic framework to use. This also implies that all the previous studies on Japanese automobile market based on logit or nested logit models of discrete choice are unsatisfactory.

3.1 McFadden's (1973) utility specification

Assume that the utility $u_{ij} = u(\boldsymbol{x}_j, \xi_j, p_j, \theta)$ of consumer i, $i = 1, \ldots, n$, from consuming product j, $j = 0, \ldots, J$, where product $j = 0$ is the outside good, depends on observed and unobserved (by the researcher) product characteristics $\boldsymbol{x}_j$ and ξ_j, price p_j, and unknown parameters θ, respectively.

McFadden (1973) utilized a linear version of the utility,

$$u_{ij} = \delta_j + \epsilon_{ij}, \qquad i = 1, \ldots, n, \quad j = 0, \ldots, J, \tag{1}$$

where ϵ_{ij} is a mean-zero stochastic variation in consumer tastes. The variation in consumer tastes enters in (1) only through the additive term ϵ_{ij}, which is assumed to be independently and identically distributed across consumers and products. He defined the mean utility which is common to all consumers as

$$\delta_j \equiv \boldsymbol{x}_j \boldsymbol{\beta} - \alpha p_j + \xi_j, \tag{2}$$

where $\theta = (\alpha, \beta)$ are parameters to be estimated. Consumers are assumed to purchase one unit of the product that gives the highest utility. So consumer i purchases one unit of product j if and only if,

$$u_{ij} > u_{ik}, \qquad 0 \leq k \leq J, \quad k \neq j.$$

The probability s_{ij} of consumer i purchasing product j is

$$\begin{aligned} s_{ij} &= \Pr\{\delta_j + \epsilon_{ij} > \delta_k + \epsilon_{ik}, j \neq k\} \\ &= \Pr\{\epsilon_{ik} < \epsilon_{ij} + \delta_j - \delta_k, j \neq k\} \\ &= \int_{-\infty}^{\infty} F_j(\epsilon_{ij} + \delta_j - \delta_0, \cdots, \epsilon_{ij}, \cdots \epsilon_{ij} + \delta_j - \delta_J) d\epsilon_{ij}. \end{aligned} \tag{3}$$

where F_j denotes the partial derivatives of the joint cumulative distribution function F of the stochastic error terms $(\epsilon_{i0}, \ldots, \epsilon_{iJ})$ with respect to its jth argument.

3.2 McFadden's logit model

In equation (3), after integration over ϵ_{ij}, which is assumed to have a type 1 extreme value (Gumbel) distribution, the probability of consumer i purchasing product j is given by

$$s_{ij} = \frac{\exp(\delta_j)}{\sum_{k=0}^{J} \exp(\delta_k)}, \tag{4}$$

according to equation (12) in McFadden (1973, p.110). This is called the logit model of discrete choice. Deviation of equation (4) is in Appendix A. Since δ_j does not vary with consumers, s_{ij} is the same for all consumers and so this equals the market share s_j of product j

$$s_j = \frac{\exp(\delta_j)}{\sum_{k=0}^{J} \exp(\delta_k)}. \tag{5}$$

By substituting (2) for (5), we obtain

$$s_j = \frac{\exp(x_j\beta - \alpha p_j + \xi_j)}{\sum_{k=0}^{J} \exp(x_k\beta - \alpha p_k + \xi_k)}. \tag{6}$$

The specification of demand system is completed with the introduction of an outside good. With the mean utility of the outside good normalized to zero ($\delta_0 = 0$), we obtain the demand equation for product j to be

$$\ln(s_j) - \ln(s_0) = \delta_j = x_j\beta - \alpha p_j + \xi_j. \tag{7}$$

One possible estimation strategy is to choose parameters that minimize the distance between the market shares predicted by equation (7) and the observed shares. This estimation strategy will yield estimates of parameters that determine the distribution of individual attributes, but it does not account for the correlation between the prices and the unobserved product characteristics, which leads to inconsistent estimation of β and α.

The standard procedure for consistently estimating β and α is the method of two-stage least squares (2SLS). The procedure consists of running two regressions. First the regress the explanatory variables on the instrument variables to obtain its fitted values. Then regress the response on the fitted values of the explanatory variables to obtain the parameter estimates of β and α.

Three weaknesses of logit model of discrete choice of demand

Although this model is appealing due to its tractability, it has three serious shortcomings, namely, the appropriateness of independence of irrelevant alternatives, of own price elasticity, and of cross product price elasticity. We will elaborate these issues below.

Independence of irrelevant alternatives

In logit model, the ratio of purchasing product j relative to product l can be expressed from equation (4) as

$$\frac{s_j}{s_l} = \frac{\exp(\delta_j)/\sum_{k=0}^{J}\exp(\delta_k)}{\exp(\delta_l)/\sum_{k=0}^{J}\exp(\delta_k)} = \frac{\exp(\delta_j)}{\exp(\delta_l)} = \exp(\delta_j - \delta_l), \tag{8}$$

which means that the odds is not influenced by other alternatives. This is called independence of irrelevant alternatives (I.I.A).

While the I.I.A axiom is realistic in some choice situations, it causes inplausible decisions, as first point out by Debreu (1960).

For example, suppose a group of individual has a choice of either traveling by their own vehicle or by a public transportation such as by bus, and two-thirds of them choose to travel by their own vehicle. Now assume that a second mode of public transportation is introduced such as the subway which gives the the same utility to the group of individuals. Under I.I.A axiom implied in logit model, the odds of traveling by their own vehicles to traveling by bus is unaffected. Intuitively, however, a half of those who chose public transportation before will opt for traveling by subway.

Own price elasticity

The own price elasticity $E_{s_j|p_j}$ of the market share s_j of product j is defined as

$$E_{s_j|p_j} = \frac{\partial s_j/s_j}{\partial p_j/p_j} = \frac{\partial s_j}{\partial p_j} \times \frac{p_j}{s_j}. \tag{9}$$

Substituing equation (4) for $\partial s_j/\partial p_j$ in (9), we obtain

$$\begin{aligned}
\frac{\partial s_j}{\partial p_j} &= \frac{\partial[\exp(\delta_j)\left(\sum\exp(\delta_k)\right)^{-1}]}{\partial p_j} \\
&= \frac{\partial \delta_j}{\partial p_j} \cdot \frac{\partial[\exp(\delta_j)\left(\sum\exp(\delta_k)\right)^{-1}]}{\partial \delta_j}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\partial \delta_j}{\partial p_j}\exp(\delta_j)\left(\sum \exp(\delta_k)\right)^{-1} + \frac{\partial \delta_j}{\partial p_j}\exp(\delta_j)\left(-\sum \exp(\delta_k)\right)^{-2}\cdot \exp(\delta_j) \\
&= \frac{\partial \delta_j}{\partial p_j}\left[\exp(\delta_j)\left(\sum \exp(\delta_k)\right)^{-1} - (\exp(\delta_j))^2\left(\sum \exp(\delta_k)\right)^{-2}\right] \\
&= \frac{\partial \delta_j}{\partial p_j}(s_j - s_j^2) = \frac{\partial \delta_j}{\partial p_j}s_j(1-s_j). \qquad (10)
\end{aligned}
$$

Therefore, (9) becomes

$$E_{s_j|p_j} \;=\; \frac{\partial \delta_j}{\partial p_j}\cdot s_j(1-s_j)\times\frac{p_j}{s_j} = -\alpha p_j(1-s_j). \qquad (11)$$

If product j has a small market share s_j, $E_{s_j|p_j} \approx -\alpha_j \cdot p_j$, which means that its own price elasticity is almost proportional to own price. This implies that the lower the price of product j, the lower its price elasticity, which further implies that firms could obtain higher markups from the low priced products.

Cross price elasticity

The cross price elasticity $E_{s_j|p_m}$ of the market share s_j of product j with respect to the price p_m of product m is defined as

$$E_{s_j|p_m} = \frac{\partial s_j/s_j}{\partial p_m/p_m} = \frac{\partial s_j}{\partial p_m}\times\frac{p_m}{s_j}. \qquad (12)$$

Substituing equation (4) for $\partial s_j/\partial p_m$ in (12), we obtain

$$
\begin{aligned}
\frac{\partial s_j}{\partial p_m} &= \frac{\partial \delta_m}{\partial p_m}\cdot\frac{\partial s_j}{\partial \delta_m} \\
&= \frac{\partial \delta_m}{\partial p_m}\cdot\frac{\partial[\exp(\delta_j)\left(\sum \exp(\delta_k)\right)^{-1}]}{\partial \delta_m} \\
&= \frac{\partial \delta_m}{\partial p_m}\cdot\exp(\delta_j)\left(\sum \exp(\delta_k)\right)^{-2}\cdot\exp(\delta_m)\cdot(-1) \\
&= \frac{\partial \delta_m}{\partial p_m}\cdot\exp(\delta_j)\left(\sum \exp(\delta_k)\right)^{-1}\cdot\left(\sum \exp(\delta_k)\right)^{-1}\cdot\exp(\delta_m)\cdot(-1) \\
&= -\frac{\partial \delta_m}{\partial p_m}\cdot s_j\cdot s_m. \qquad (13)
\end{aligned}
$$

Therefore, (12) becomes

$$E_{s_j|p_m} = -\frac{\partial \delta_m}{\partial p_m}\cdot s_j\cdot s_m\times\frac{p_m}{s_j} = \alpha\cdot s_m\cdot p_m. \qquad (14)$$

The equation (14) shows that the cross price elasticity of product j does not depend on its share nor its price, but only on the share and the price of the product m from which the substitution occurs. The logit model restricts consumers to substitute towards other products in proportion to market shares, regardless of brand categories.

3.3 Nested logit model (Generalized extreme value model)

Here, all ϵ_{ij} within the group B^r of similar products are correlated with each other, but ϵ_{ij} between products belonging to different groups are not. Then, the market share of product j within group B^r, $(0 \leq r \leq T)$ is given by

$$s_{ij} = \frac{\exp(\delta_j/\lambda_r)\left(\sum_{k \in B^r} \exp(\delta_k/\lambda_r)\right)^{\lambda_r - 1}}{\sum_{t=0}^{T}\left(\sum_{k \in B^t} \exp(\delta_k/\lambda_g)\right)^{\lambda_g}}, \qquad (0 \leq k \leq J) \tag{15}$$

where λ_r is correlation indicators within a group. See Appendix C for the derivation. Using (15), we can derive the following demand equation

$$\ln(s_j) - \ln(s_0) = x_j\beta - \alpha p_j + (1 - \lambda_r)\ln(\frac{s_j}{\bar{s}_r}) + \xi_j, \tag{16}$$

so that estimates of α, β, and λ_r can be obtained again by way of a linear instrumental variables regression of differences in log market shares on prices, product characteristics, log of within group share. See Appendix B for the derivation. This model allows consumer tastes to be correlated within a group of similar products but restricts substitution to occur only within a group.

3.4 Random coefficient model

The utility function for the random coefficient model of discrete choice of demand can be modeled as

$$u_{ij} \;=\; \alpha \log(y_i - p_j) + \sum_{k=1}^{K} x_{jk}(\beta_k + \sigma_k \nu_{ik}) + \xi_j + \epsilon_{ij}, \tag{17}$$

where y_i is the income over five years of consumer i, $\boldsymbol{x}_j = (x_{j1}, \ldots, x_{jK})$ is the K observed product characteristics, ξ_j is the unobserved product characteristics, and p_j is the price of product j, ϵ_{ij} is the random utility across products and consumers and is assumed to be i.i.d. with extreme value distribution. Implicit in the specification given by the equation above is the acknowledgment that a quasilinear utility function often used for modeling small-ticket items is not reasonable for some products such as automobiles. BLP builds on a Cobb-Douglas utility function, generating the term $\log(y_i - p_j)$ of disposable income and we are following their specification.

In this model, $\beta_k x_{jk}$ represents the average utiliity of the all consumers for characteristics k, ν_{ik} is the standardized random coefficient representing consumer i's preference for the characterictics k, so that $\sigma_k \nu_{ik} x_{jk}$ represents the consumer i's deviation from the average preference among the consumers for the characterictics k. We also define the utility for the outside good as

$$u_{i0} \quad = \quad \alpha \log(y_i) + \epsilon_{i0}. \tag{18}$$

By allowing the possiblitiy of consumers choosing not to buy in the model, we can capture utility from products other than the new vehicles.

Let us redefine the utility as the diference $U_{ij} = u_{ij} - u_{i0}$ from that of outside goods. Then, the newly defined utlity U_{ij} can be decomposed as

$$U_{ij} \quad = \quad \delta_j(\boldsymbol{x}_j, \xi_j; \boldsymbol{\beta}) + \mu_{ij}(\boldsymbol{x}_j, p_j, \boldsymbol{\nu}_{2i}; \boldsymbol{\theta}_2) + \epsilon_{ij} - \epsilon_{i0}, \tag{19}$$

where

$$\begin{aligned} \delta_j(\boldsymbol{x}_j, \xi_j; \boldsymbol{\beta}) &= \boldsymbol{x}_j \boldsymbol{\beta} + \xi_j, \\ \mu_{ij}(\boldsymbol{x}_j, p_j, \boldsymbol{\nu}_{2i}; \boldsymbol{\theta}_2) &= \alpha \log(1 - p_j/y_i) + \sum_{k=1}^{K} \sigma_k x_{jk} \nu_{ik}, \end{aligned}$$

and δ_j is the product-specific term independent of the individual consumer characteristics, and μ_{ij} is a function of both consumer as well as product-specific characteristics. The parameters $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_K)'$ are associated with average preferences of consumers. Let us define $(\alpha, \sigma_1, \ldots, \sigma_K)$ as $\boldsymbol{\theta_2}$, then the parameters $\boldsymbol{\theta_2}$ are dependent both on consumer and product characteristics.

For this model, the market share function s_j can be obtained in two stages. First, the probability that consumer i purchases product j is given by the logit formula:

$$s_{ij} = \frac{\exp(\delta_j + \mu_{ij})}{\sum_{j=0}^{J} \exp(\delta_j + \mu_{ij})} = \frac{\exp(\delta_j + \mu_{ij})}{1 + \sum_{j=1}^{J} \exp(\delta_j + \mu_{ij})}. \tag{20}$$

Integrating out this s_{ij} over the distribution of ν_{2i} gives the market share s_j:

$$s_j = \int_{\nu_{2i}} \frac{\exp(\delta_j + \mu_{ij})}{1 + \sum_{j=1}^{J} \exp(\delta_j + \mu_{ij})} f(\nu_{2i}) d\nu_{2i}, \tag{21}$$

where $f(\boldsymbol{\nu_{2i}})$ is the joint probability density function of consumer characteristics $\boldsymbol{\nu_{2i}} = (y_i, \nu_{i1}, \ldots, \nu_{iK})$.

Note that though the random coefficient model's specification allows for more realistic cross price elasticities, it introduces the problem of computing burden in the integral in (21), even if we know the exact distribution of the consumer characteristics $\boldsymbol{\nu_2}$. As we see in section 3.6, we solve this computational problem via aggregation by simulation.

Does the random coefficient model addresses the weaknesses in the logit model?

In random coefficient model, the ratio of purchasing product j relative to product l can be expressed from equation (21) as

$$\frac{s_j}{s_l} = \frac{\int_{\nu_{2i}} \frac{\exp(\delta_j + \mu_{ij})}{\sum_{j=0}^{J} \exp(\delta_j + \mu_{ij})} f(\nu_{2i}) d\nu_{2i}}{\int_{\nu_{2i}} \frac{\exp(\delta_l + \mu_{il})}{\sum_{j=0}^{J} \exp(\delta_j + \mu_{ij})} f(\nu_{2i}) d\nu_{2i}},$$

which shows that the denominators of the logit formula are inside the integrals and therefore do not cancel and the ratio depends on all the data, including attributes of alternatives other than product j and l. In other words, it does not exhibit independence of irrelevant alternatives (I.I.A.) property.

The first order derivative of the market shares with respct to the prices under the random coefficient model can be obtained by differentiating (21) with respect to p_l as

$$\frac{\partial s_j}{\partial p_l} = \begin{cases} -\alpha \int_{\nu_{2i}} \frac{s_{ij}(1-s_{ij})}{(y_i-p_j)} \times f(\nu_{2i})d\nu_{2i} & (l=j), \\ \alpha \int_{\nu_{2i}} \frac{s_{ij}s_{il}}{(y_i-p_j)} \times f(\nu_{2i})d\nu_{2i} & (l \neq j). \end{cases} \tag{22}$$

The price elasticities of market shares s_j as defined by equation (21) are thus

$$E_{s_j|p_l} = \frac{\partial s_j p_l}{\partial p_l s_j} = \begin{cases} -\alpha \int_{\nu_{2i}} \frac{s_{ij}(1-s_{ij})}{(y_i-p_j)} f(\nu_{2i})d\nu_{2i} \times \frac{p_j}{s_j} & (l=j), \\ \alpha \int_{\nu_{2i}} \frac{s_{ij}s_{il}}{(y_i-p_j)} f(\nu_{2i})d\nu_{2i} \times \frac{p_l}{s_j} & (l \neq j). \end{cases} \tag{23}$$

where $s_{ij} = \frac{\exp(\delta_j+\mu_{ij})}{\sum_{j=0}^{J}\exp(\delta_j+\mu_{ij})}$ is the probability of individual i purchasing product j in equation (20). This result (23) implies that each individual will have a different price sensitivity, which will be averaged to a mean price sensitivity using the individual specific probabilities of purchase as weights. The price sensitivity will be different for different brands.

What makes the random coefficient respond differently to product characteristics? For example, suppose we observe three products: Nissan Elgrand, Toyota Alphard, and Honda Acty. Elgrand and Alphard are very similar in their characteristics in that they are both minivans, while Alphard and Acty, the latter being a mini vehicle, have the same market shares. Now, suppose the only change is that the price of product Elgrand increases. The logit model predicts that the market shares of both products Alphard and Acty should increase similarly. On the other hand, the random coefficient model allows for the possibility that the market share of product Alphard, the one more similar to product Elgrand, will increase by more. By observing the actual relative change in the market shares of products Alphard and

Acty we can distinguish between the two models. Furthermore, the degree of change will allow us to identify the parameters that govern the distribution of the random coefficients. Thus, the random coefficient model allows for more flexible substitution patterns.

3.5 Cost side specification

Whether one wishes to use logit, nested logit, or random coefficient model of discrete choice of demand, one needs to have cost side specification if one analyzes market equilibrium.

Suppose that there are F firms in the market and each firm produces several products. Let $\mathcal{J}_f$ denote the set of products belonging to firm f. The simple profit function for firm f is given by

$$\pi_f = \sum_{j \in \mathcal{J}_f} (p_j - c_j(q_j, w_j, \omega_j, \gamma)) M s_j(x, \xi, p, \beta, \theta_2). \tag{24}$$

where $c_j(q_j, w_j, \omega_j, \gamma)$ is the marginal cost given as a function of output quantity q_j, observable cost shifter w_j, unobserved cost shifter ω_j of product j, and their associated unknown parameters γ. M represents the potential market size. The quantity M represents the potential market size. From equation (24), the first-order conditions in terms of p_j for the firm f become

$$s_j + \sum_{l \in \mathcal{J}_f} (p_l - c_l) \frac{\partial s_l}{\partial p_j} = 0 \quad \text{for } j \in \mathcal{J}_f. \tag{25}$$

Suppose firm f has $k(f)$ products indexed by $j = J_1^f, \ldots, J_{k(f)}^f$, where $J_1^1 = 1$ and $J_{k(F)}^F = J$. Let us define the matrix Δ^f as

$$\Delta^f = \begin{pmatrix} \frac{\partial s(J_1^f)}{\partial p(J_1^f)} & \cdots & \frac{\partial s(J_{k(f)}^f)}{\partial p(J_1^f)} \\ \vdots & \ddots & \vdots \\ \frac{\partial s(J_1^f)}{\partial p(J_{k(f)}^f)} & \cdots & \frac{\partial s(J_{k(f)}^f)}{p(J_{k(f)}^f)} \end{pmatrix} \quad \text{for } f = 1, \ldots, F, \tag{26}$$

so that the first-order conditions can be expressed in vector form

$$\underbrace{\begin{pmatrix} s_1 \\ \vdots \\ s_J \end{pmatrix}}_{s} + \underbrace{\begin{pmatrix} \Delta^1 & & 0 \\ & \ddots & \\ 0 & & \Delta^F \end{pmatrix}}_{\Delta} \underbrace{\begin{pmatrix} p_1 - c_1 \\ \vdots \\ p_J - c_J \end{pmatrix}}_{p-c} = 0. \tag{27}$$

Assuming $\boldsymbol{\Delta}$ is a nonsingular matrix, solving above condition for $\boldsymbol{c}$ gives the cost side equation

$$\boldsymbol{c} = \boldsymbol{p} + \boldsymbol{\Delta}^{-1}\boldsymbol{s}. \tag{28}$$

Assuming that marginal cost is log-linear in observed cost shifters ($\log(c_j) = \boldsymbol{w}_j\gamma + \omega_j$), we obtain the cost side error term.

$$\tilde{\omega}_j = \log\left(p_j + \{\boldsymbol{\Delta}^{-1}\tilde{s}\}_j\right) - \boldsymbol{w}_j\gamma. \tag{29}$$

We combine the demand equations derived for the random coefficient model (21) and the cost side equations with multi-products firms (29) to describe Japanese automobile market in market equilibrium.

3.6 Estimation algorithm

Estimation proceeds as follows. First we estimate the demand side error term $\tilde{\xi}_j$. Then we derive the cost side error term $\tilde{\omega}_j$. With a set of effective demand and cost side instrumental variables (z_j^d, z_j^c) respectively, we estimate parameters to minimize the inner products $\tilde{\xi}_j(z_j^d)'$ and $\tilde{\omega}_j(z_j^c)'$ between the error terms $(\tilde{\xi}_j, \tilde{\omega}_j)$ and the (z_j^d, z_j^c).

The demand side error term $\tilde{\xi}_j$

We simulate the market share $\tilde{s}_j$ first by drawing n sets of ν_{2i} from $f(\nu_{2i})$, then by calculating the following simulation estimator $\tilde{s}_j$

$$\begin{aligned}\tilde{s}_j &= \frac{1}{n}\sum_{i=1}^{n}\frac{\exp(\delta_j+\mu_{ij})}{1+\sum_{j=1}^{J}\exp(\delta_j+\mu_{ij})} && (30)\\ &\simeq \int_{\nu_{2i}}\frac{\exp(\delta_j+\mu_{ij})}{1+\sum_{j=1}^{J}\exp(\delta_j+\mu_{ij})}f(\nu_{2i})d\nu_{2i}. && (31)\end{aligned}$$

Then we estimate $\boldsymbol{\theta}_2=(\alpha,\sigma_1,\ldots,\sigma_K)'$ so that the observed market share S_j of product j is as close to the simulation estimator $\tilde{s}_j$ obtained above in

$$\delta_j^{h+1}=\delta_j^h+\ln(S_j)-\ln(\tilde{s}_j(p_j,\boldsymbol{x}_j,\delta_j^h,P_n|\boldsymbol{\theta}_2)), \tag{32}$$

where P_n represents the sampled n vectors of $\boldsymbol{\nu}_{2i}$ from the $f(\boldsymbol{\nu}_{2i})$. This method is sometimes referred as contraction mapping. Then we can solve for the demand side error term ξ_j as

$$\tilde{\xi}_j=\tilde{\delta}_j(\boldsymbol{\theta}_2)-\boldsymbol{x}_j\boldsymbol{\beta}. \tag{33}$$

The cost side error term $\tilde{\omega}_j$

With the random n draws $\boldsymbol{\nu}_{2i}$ from $F(\boldsymbol{\nu}_{2i})$, we also approximate to (22) by

$$\left(\widetilde{\frac{\partial s_j}{\partial p_l}}\right)=\begin{cases}-\alpha\frac{1}{n}\sum_{i=1}^{n}\tilde{s}_{ij}(1-\tilde{s}_{ij})/(y_i-p_j) & (l=j),\\ \alpha\frac{1}{n}\sum_{i=1}^{n}\tilde{s}_{ij}\tilde{s}_{il}/(y_i-p_j) & (l\neq j).\end{cases} \tag{34}$$

Thus $\boldsymbol{\Delta}^f$ in (26) becomes

$$\boldsymbol{\Delta}^f = \frac{\alpha}{n}\sum_i\begin{pmatrix}\frac{-\tilde{s}_i(J_1^f)\{1-\tilde{s}_i(J_1^f)\}}{y_i-p(J_1^f)} & \frac{\tilde{s}_i(J_2^f)\tilde{s}_i(J_1^f)}{y_i-p(J_1^f)} & \cdots & \frac{\tilde{s}_i(J_{k(f)}^f)\tilde{s}_i(J_1^f)}{y_i-p(J_1^f)}\\ \frac{\tilde{s}_i(J_1^f)\tilde{s}_i(J_2^f)}{y_i-p(J_2^f)} & \ddots & \ddots & \vdots\\ \vdots & \ddots & \ddots & \frac{\tilde{s}_i(J_{k(f)}^f)\tilde{s}_i(J_{k(f)-1}^f)}{y_i-p(J_{k(f)-1}^f)}\\ \frac{\tilde{s}_i(J_1^f)\tilde{s}_i(J_{k(f)}^f)}{y_i-p(J_{k(f)}^f)} & \cdots & \frac{\tilde{s}_i(J_{k(f)-1}^f)\tilde{s}_i(J_{k(f)}^f)}{y_i-p(J_{k(f)}^f)} & \frac{-\tilde{s}_i(J_{k(f)}^f)\{1-\tilde{s}_i(J_{k(f)}^f)\}}{y_i-p(J_{k(f)}^f)}\end{pmatrix}$$

(35)

where $\tilde{s}_i(j) = \tilde{s}_{ij}$ and $p(j) = p_j$. Therefore, using (35) we can compute the cost side error term ω_j as

$$\tilde{\omega}_j = \log\left(p_j + \{\tilde{\boldsymbol{\Delta}}^{-1}\tilde{s}\}_j\right) - \boldsymbol{w}_j\gamma. \tag{36}$$

To minimize the inner products $(\tilde{\xi}_j(z_j^d)', \tilde{\omega}_j(z_j^c)')$ or to satisfy the orthogonality conditions as sometimes mentioned, we use the generalized method of moments (GMM) estimation technique to obtain the estimates of parameters $\boldsymbol{\theta}_1 = (\boldsymbol{\beta}', \boldsymbol{\gamma}')'$ and $\boldsymbol{\theta}_2 = (\alpha, \sigma_1, \ldots, \sigma_K)'$. Let $\boldsymbol{z}_j^d = (z_{j1}^d, \ldots, z_{jM_1}^d)$ and $\boldsymbol{z}_j^c = (z_{j1}^c, \ldots, z_{jM_2}^c)$ be the vector of M_1 and M_2 elements of instrumental variables for product j to be used for the demand and the cost side equations respectively. Set

$$\boldsymbol{h}(\boldsymbol{\theta}, \boldsymbol{v}_j) = \begin{pmatrix} \tilde{\xi}_j(\boldsymbol{z}_j^d)' \\ \tilde{\omega}_j(\boldsymbol{z}_j^c)' \end{pmatrix}, \tag{37}$$

where $\boldsymbol{v}_j = (\boldsymbol{x}_j, p_j, \boldsymbol{w}_j, \boldsymbol{z}_j^d, \boldsymbol{z}_j^c)$ and $\boldsymbol{\theta} = (\boldsymbol{\theta}_1', \boldsymbol{\theta}_2')'$. Then the orthogonal conditions can be written as

$$\mathrm{E}[\boldsymbol{h}(\boldsymbol{\theta}, \boldsymbol{v}_j)] = \boldsymbol{0}. \tag{38}$$

Define the sample moments for $\boldsymbol{h}(\boldsymbol{\theta}, \boldsymbol{v}_j)$ as

$$\boldsymbol{H}(\boldsymbol{\theta}, \boldsymbol{v}) = \frac{1}{J}\sum_{j=1}^{J} \boldsymbol{h}(\boldsymbol{\theta}, \boldsymbol{v}_j). \tag{39}$$

The GMM estimator $\hat{\boldsymbol{\theta}}$ is the value of $\boldsymbol{\theta}$ that minimizes the scalar

$$Q(\boldsymbol{\theta}) = \boldsymbol{H}(\boldsymbol{\theta}, \boldsymbol{v})'\Omega^{-1}\boldsymbol{H}(\boldsymbol{\theta}, \boldsymbol{v}), \tag{40}$$

where Ω^{-1} is the weighting matrix whose optimal value is the inverse of the asymptotic covariance of the sample mean $\boldsymbol{H}(\boldsymbol{\theta}_0, \boldsymbol{v})$ with the true parameter $\boldsymbol{\theta}_0$, that is

$$\Omega = \lim_{J\to\infty} J \cdot \mathrm{E}[\boldsymbol{H}(\boldsymbol{\theta}_0, \boldsymbol{v}) \cdot \boldsymbol{H}(\boldsymbol{\theta}_0, \boldsymbol{v})']. \tag{41}$$

4 Data

The data used for the analysis are the sales volume, the vehicle characteristics, and the manufacturer's suggested retail prices for 100 most popular model vehicles—passenger cars, luxury cars, specialty cars, minivans, SUVs, and mini vehicles—in terms of sales in 2002. They covered 92% of the market share of all the vehicle sold in 2002. We did not use the remaining models with very small shares because doing so enabled us to reduce computational time considerably without changing the nature of the results. The data on sales volume are complied by Motor magazine Inc. and listed in their publication "Motor Magazine". Their data came from the Japan Automobile Dealers Association, Japan Automobile Importers Association, and Japan Mini Vehicle Association. The vehicles characterictics and the suggested retail prices came from "Domestic & Import Cars Buying Guide" published by Japan Automobile Federation Publishing Co. and a web-page "Car sensor" (http://www.isize.com/carsensor/cgi-bin/CS/CSTOP.cgi?STID=CS0GNAVI &TRCD=TR003) that Recruit Inc. provides. If a model has multiple trim lines, then we chose the median trim as a representative for the model. If there are even number of trim lines, hoever, we chose the one above the median trim.

We use the following characteristics in the demand equation: Horsepower (HP) gives us a measure of the degree of power and acceleration of a vehicle; Fuel mileage measured in terms of kilometers per liter; Safety dummy variables indicating if air bags on both driver and passenger side as well as ABS are standard; Vehicle segment dummy variables indicating if it is a minivan or not and if it is a mini vehicle or not.

We did not include some variables used in the previous studies to minimize the problems of multicollinearity. For example, the size of vehicles measured in terms of length×width and the mileage are highly correlated. Since mini vehicles are regulated to have maximum length of 3,395mm and width of 1,475mm, all the manufacturers designed their mini vehicles to fully take advantage of the regulation. As a result, they tend to have almost identical length×width. We decided to drop the length×width varaible because there are more variations in the mileage than this variable. Similarly, we did not use displacement for they tend to be concentrated just below the threshold values according to which vehicle taxes are determined.

We use all of the variables above as cost shifters in cost-side equation plus the log of total sales volume to account for the economy of scale.

These explanatory variables or their variations are widely used in previous studies such as BLP (1995), Sudhir (2001), and Petrin (2002). Reliability is missing in our study because we do not have objective reliability statistics available to us. Reflecting the domestic tax advantage, we introduced dummy variable indicating if it is a mini vehicle or not.

As for a potential market size, we follow Sudhir (2001) and calculated it to be approximately 8.23 million vehicles by multiplying the number of vehicles per household (1.094 vehicles) by the number of households in millions (47 million) and by dividing by the mean age of vehicles on road in terms of years (6.23 years).

Individual tastes v_{ik} for the k-th product characterisics were drawn from a standard normal distribution. The individual income in the demand equation is drawn from a variable-cell width histogram of 2002 income titled Family Income and Expenditure Survey published by Stastistics Bureau Within Ministry of Public Management, Home Affairs, Posts and Telecommunications. In both individual tastes and income, the sample size is one hundred.

Because vehicle prices and market shares are endogeneous and are correlated with the error terms ξ_j and ω_j, we need instrumental variables. Following BLP (1995) and Sudhir (2001), we use the exogenous product characteristics in the demand and pricing equation; the average of the exogenous vehicle characteristics over vehicles produced by the same firm for the market segment (passenger cars, specialty cars, luxury cars, minivans, SUVs, mini vehicles) to which the vehicle belongs; the average of the exogenous vehicle characteristics offered by other firms for the market segment to which the vehicle belongs. Instruments for the two vehicle segment—minivans and mini vehicles—dummy are not constructed for obvious reason.

Consequently, we estimate thirteen coefficients on the demand side. They are the mean as well as the random utility coefficients of intercept, HP, mileage, safety dummy, minivan dummy, mini vehicle dummy, plus that of log(disposable income). On the cost side, we estimate seven coefficients of intercept, HP, mileage, safety dummy, minivan dummy, mini vehicle dummy, and log(total sales).

5 Results and Discussions

The results of the estimation in both logit and the random coefficient models of discrete choice of demand are in Tables 1 and 2 for the 50 and 100 best-selling model vehicles. Generally speaking, as the number of analyzed models increases, the fit of both—random coefficient as well as logit—models deteriorates as evidenced by its more than proportional increases in chi-square statistics. Notice that the 50 best-selling models already covers about 75% of the market share, while the 100 best-selling models' coverage reaches close to 92%. This disproportionate increase in chi-square statistics is probably because the model with approximately 14 logit or 20 random coefficient parameters is too simplistic to capture the outlying market condition that exists for the 51 to 100 best-selling vehicles. We focus on the estimation result for the 100 best-selling model vehicles below.

Demand side

The β coefficients measure the average preference, while the σ coefficients measure the heterogeneity in preferences. As expected log(Income − Price) has positive coefficient, indicating the price sensitivity of Japanese consumers. Notice that the log specification ensures that higer-income consumers are less sensitive to price than lower-income counterparts.

As expected, consumers on average prefer higher horsepower and more mini vehicle, however in the 100 best-selling models, they do not necessarily prefer fuel efficiency or safety. They on average do not prefer minivans, however, we observed significant heterogeneity here in that some consumers strongly prefer minivans.

We found that demand for mini vehicles may not be ignorable in Japan. Mini vehicles provide a convenient, economical mode of transportation for commuting, shopping, and running errands. These combined features coupled with their favorable tax status have been behind their immense popularity especially as a second car in recent years.

It is surprising that the Japanese consumers on average do not prefer minivan, but these results are consistent with Petrin (2002) who studied the U.S. automobile market. Since consumers are quite heterogeneous in their valuation of minivan, this implies that the potential market size of minivans may not be as big as some of the automobile manufacturers are counting on.

Cost

It costs more to produce high horsepower vehicles, and fuel-efficient vehicles, and the economy of scale is very important in automobile manufacturing. Contrary to our expectation, the safety dummy is insignificant probably because airbags on both driver and passenger sides and ABS are such prevalent features of automobiles that their installation costs add very little.

Issues

In this study, we are able to handle 100 vehicle models and 100 random drawings of heteroscedastic consumers. It is important to note that asymptotic results such as the asymptotic t-values and chi-square statistics obtained by the generalized method of moments are appropriate if and only if the number of models increases. We know that the results could be improved if we use more models, increase the number of random drawings, but they may not be feasible unless we improve the simulation methods, or minimization algorithm.

	Random Coefficient Model				Logit Model	
	β		σ		β	
Demand Side	**Estimate**	**t-value**	**Estimate**	**t-value**	**Estimate**	**t-value**
Intercept	-6.499**	-2.428	0.000	0.000	-6.643**	-2.594
Horse Power	1.987*	1.898	0.000	0.000	2.077**	2.250
Mileage(km/l)	1.444*	1.228	0.432	0.307	1.596**	2.992
Safety	0.493*	1.344	0.000	0.000	0.488	0.226
Minivan	1.019*	1.842	0.001	0.000	1.053*	2.036
Mini-vehicle	1.230**	2.234	0.000	0.000	0.842**	2.961
	α				α	
$\ln(y_i - \text{price}_j)$	32.940*	1.186			34.799*	1.544
Cost Side	γ				γ	
Intercept	7.657**	3.077			7.443**	2.705
Horse Power	1.554**	3.076			1.475**	3.413
Mileage(km/l)	2.138*	1.392			2.007*	1.994
Safety	0.544	0.789			0.540	0.206
Minivan	0.926*	1.165			0.906	0.906
Mini-vehicle	0.755**	2.228			0.709**	2.308
ln(Sales Volume)	-2.877**	-3.529			-2.762**	-2.927
Objective Function	11.877				11.849	
Degree of Freedom	8				14	

$*$: $|\text{t-value}| > 1.0$

$**$: $|\text{t-value}| > 2.0$

Table 1: The estimation result for the 50 best-selling models

	Random Coefficient Model				Logit Model	
	β		σ		β	
Demand Side	**Estimate**	**t-value**	**Estimate**	**t-value**	**Estimate**	**t-value**
Intercept	-4.536**	-2.121	0.000	0.000	-5.266**	-2.366
Horse Power	1.058**	3.185	0.038	0.037	1.226**	2.776
Mileage(km/l)	0.664	0.793	0.927	0.770	1.186**	2.882
Safety	0.363	0.931	0.203	0.153	0.398	0.483
Minivan	-5.679*	-1.913	6.515**	2.864	1.175*	1.918
Mini-vehicle	0.321*	1.112	0.000	0.000	0.368*	1.169
	α				α	
$\ln(y_i - \text{price}_j)$	35.093**	3.391			38.410**	3.757
Cost Side	γ				γ	
Intercept	3.990**	2.370			3.976**	3.670
Horse Power	0.734**	4.108			0.729**	2.457
Mileage(km/l)	0.562*	1.434			0.491	0.713
Safety	0.385	0.860			0.393	0.970
Minivan	0.547	0.585			0.608*	1.163
Mini-vehicle	0.213	0.563			0.232	0.437
ln(Sales Volume)	-1.405**	-4.304			-1.375**	-5.383
Objective Function	28.400				28.123	
Degree of Freedom	8				14	

$*$: $|\text{t-value}| > 1.0$

$**$: $|\text{t-value}| > 2.0$

Table 2: The estimation result for the 100 best-selling models

6 Conclusions and future work

In this paper we analyzed the recent new vehicle sales in Japan using the publicly available model-by-model sales data. We follow in a tradition of applied economic and industrial organization literature that seeks to uncover basic parameters of demand and supply. We employ as the demand framework the so-called random coefficient model of discrete choice of demand. The model can account for heterogeneous preferences of the utility maximizing consumers and is able to realize more reasonable substitution patterns between similar products.

We do find the not so surprizing result that the Japanese consumers on the average prefer higher horsepower and are price-sensitive. Contrary to our expectation, neither mileage nor safety does not matter very much when selecting vehicles. Our analysis also uncovers minivans' conflicting utility in that they are likely to decrease the utility of average consumers, but for some consumers having the third-row seat, however small, is a desirable vehicle characteristic. This phenomena cannot be captured at all if we restrict ourselves to the widely-used logit model of discrete choice of demand.

An interesting question still unresolved is at what level of income, the utility of mini vehicles and compact vehicles such as Toyota Vitz, Nissan March, and Honda Fit can be reversed. Obviously these vehicles with their larger displacement and more sophisticated safety features should appeal to some Japanese consumers.

Also adding consumer-level micro data on the consumer profiles and purchasing patterns such as Petrin (2002) and Berry, Levinsohn, and Pakes (2003) did will evaluate the Japanese automobile market equilibrium more accurately.

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A Proof of the equation(4): Logit model of demand

Under the utility maximization principle, the probability that consumer i chooses a product j, s_{ij} is

$$
\begin{aligned}
& \Pr(\delta_j + \epsilon_{ij} > \delta_k + \epsilon_{ik}, j \neq k) \\
= & \Pr(\epsilon_{ik} < \epsilon_{ij} + \delta_j - \delta_k, j \neq k) \\
= & \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\epsilon_{ij}+\delta_j-\delta_0} \cdots \int_{-\infty}^{\epsilon_{ij}+\delta_j-\delta_{j-1}} \int_{-\infty}^{\epsilon_{ij}+\delta_j-\delta_{j+1}} \cdots \int_{-\infty}^{\epsilon_{ij}+\delta_j-\delta_J} f(\epsilon_{i0} \ldots \epsilon_{ij} \ldots \epsilon_{iJ}) \right. \\
& \left. \times d\epsilon_{i0} \ldots d\epsilon_{ij-1} d\epsilon_{ij+1} \ldots d\epsilon_{iJ} \right] d\epsilon_{ij} \\
= & \int_{-\infty}^{\infty} \left[\frac{\partial}{\partial \epsilon_{ij}} \left\{ \int_{-\infty}^{\epsilon_{i0}} \cdots \int_{-\infty}^{\epsilon_{ij}} \cdots \int_{-\infty}^{\epsilon_{iJ}} f(\epsilon_{i0} \cdots \epsilon_{ij} \cdots \epsilon_{iJ}) \right. \right. \\
& \left. \left. \times d\epsilon_{i0} \cdots d\epsilon_{ij} \cdots d\epsilon_{iJ} \right\} \right] \Bigg|_{\epsilon_{i0}=\epsilon_{ij}+\delta_j-\delta_0,\ldots,\epsilon_{ij}=\epsilon_{ij}+\delta_j-\delta_j,\ldots,\epsilon_{iJ}=\epsilon_{iJ}+\delta_j-\delta_J} d\epsilon_{ij} \\
= & \int_{-\infty}^{\infty} \left[\frac{\partial}{\partial \epsilon_{ij}} F\left(\epsilon_{i0} \ldots \epsilon_{ij} \ldots \epsilon_{iJ}\right) \right] \Bigg|_{\epsilon_{i0}=\epsilon_{ij}+\delta_j-\delta_0,\ldots,\epsilon_{ij}=\epsilon_{ij}+\delta_j-\delta_j,\ldots,\epsilon_{iJ}=\epsilon_{iJ}+\delta_j-\delta_J} d\epsilon_{ij} \\
= & \int_{-\infty}^{\infty} F_j(\epsilon_{ij} + \delta_j - \delta_0, \ldots, \epsilon_{ij}, \ldots, \epsilon_{ij} + \delta_j - \delta_J) d\epsilon_{ij}. \qquad (42)
\end{aligned}
$$

where F_j denotes the partial derivative of F with respect to its jth argument. That is, this equation (42) means a joint density except ϵ_{ij} is integrated from each $-\infty$ to $\epsilon_{ik} + \delta_j - \delta_k \quad (k = 0, \ldots, J, k \neq j)$.

$\epsilon_{ik} \quad (i = 1, \ldots, n, \text{and } k = 0, \ldots, J)$ are assumed to be independently identically distributed with an extreme value distribution of type 1 whose cumulative joint distribution function is

$$F(\epsilon_{i0}, \ldots, \epsilon_{ij}, \ldots, \epsilon_{iJ}) = \Pi_{k=0}^{J} \exp[-\exp(-\epsilon_{ik})]. \qquad (43)$$

and F_j is

$$
\begin{aligned}
\frac{\partial}{\partial \epsilon_{ij}} F(\epsilon_{i0}, \ldots, \epsilon_{ij}, \ldots, \epsilon_{iJ}) = & \exp[-\exp(-\epsilon_{ij})]\{-\exp(-\epsilon_{ij}) \times (-1)\} \\
& \times \Pi_{k=0,k\neq j}^{J} \exp[-\exp(-\epsilon_{ik})]. \qquad (44)
\end{aligned}
$$

Substitute, $\epsilon_{i0} = \epsilon_{ij} + \delta_j - \delta_0, \ldots, \epsilon_{ij} = \epsilon_{ij} + \delta_j - \delta_j, \ldots, \epsilon_{iJ} = \epsilon_{ij} + \delta_j - \delta_J$, then F_j is

$$\begin{aligned}
& F_j(\epsilon_{ij} + \delta_j - \delta_0, \ldots, \epsilon_{ij} \ldots \epsilon_{ij} + \delta_j - \delta_J) \\
&= \exp(-\epsilon_{ij}) \exp[-\exp(-\epsilon_{ij})] \times \Pi_{k=0,k\neq j}^{J} \exp[-\exp(-(\epsilon_{ij} + \delta_j - \delta_k))] \\
&= \exp(-\epsilon_{ij}) \times \Pi_{k=0}^{J} \exp[-\exp(-\epsilon_{ij} + \delta_j - \delta_k)]. \qquad (45)
\end{aligned}$$

Using them, therefore

$$\begin{aligned}
s_{ij} &= \int_{-\infty}^{\infty} F_j(\epsilon_{ij} + \delta_j - \delta_0, \ldots, \epsilon_{ij}, \ldots, \epsilon_{ij} + \delta_j - \delta_J) d\epsilon_{ij} \\
&= \int_{-\infty}^{\infty} \exp(-\epsilon_{ij}) \times \Pi_{k=0}^{J} \exp[-\exp(-\epsilon_{ij} + \delta_j - \delta_k)] d\epsilon_{ij}
\end{aligned}$$

Suppose $\exp(-\epsilon_{ij}) = t$, then $-\exp(-\epsilon_{ij}) d\epsilon_{ij} = dt$, and $-t d\epsilon_{ij} = dt$, so

$$\begin{aligned}
s_{ij} &= \int_{\infty}^{0} t \times \exp\left[-t \times \sum_{k=0}^{J} \exp(\delta_k - \delta_j)\right] (-1/t) dt \\
&= \int_{0}^{\infty} \exp\left[-t \times \sum_{k=0}^{J} \exp(\delta_k - \delta_j)\right] dt \\
&= \left[\frac{\exp[-t \times \sum_{k=0}^{J} \exp(\delta_k - \delta_j)]}{-\sum_{k=0}^{J} \exp(\delta_k - \delta_j)}\right]_0^{\infty} \\
&= \frac{1}{\sum_{k=0}^{J} \exp(\delta_k - \delta_j)} \\
&= \frac{\exp(\delta_j)}{\sum_{k=0}^{J} \exp(\delta_k)}. \qquad (46)
\end{aligned}$$

δ_j do not vary with the consumers, due to this, the probability that consumer i chooses product j is equal to

$$s_j = \frac{\exp(\delta_j)}{\sum_{k=0}^{J} \exp(\delta_k)}. \qquad (47)$$

for all consumers. Hence this is the probability that product j is purchased in the market, in othor words, the market share of product j.

B Nested logit model of demand

In contrast to the logit model, the nested logit model or "tree extreme value" model (McFadden, 1978; et al.) preserves the assumption that consumer tastes have an extreme value distribution but allows consumer tastes to be correlated in a restricted fashion across products j. This allows for more reasonable consumer tastes substitution patterns as compared to the logit model.

Let the number of alternative $J+1$ be partitioned into $T+1$ subset denorted $B^0, \ldots, B^T$. The utility that consumer i obtains from alternative j in subset $B^t \quad (t = 0, \ldots, T)$ is denoted by

$$u_{ij} \;=\; \delta_j + \epsilon_{ij}, \qquad i = 1, \ldots, n, \quad j = 0, \ldots, J, \quad j \in B^t \tag{48}$$

as specified in logit model. In the nested logit model, assuming that ϵ_{ij} for products j are distributed in accordance with a Gumbel's type 2 bivariate distribution (sometimes refered as a generalized extreme value distribution). That is, the joint cumulative distribution of stochastic term ϵ_{ij} for products j is assumed to be

$$F(\epsilon_{i0}, \epsilon_{i1}, \ldots, \epsilon_{iJ}) = \exp\left\{ -\sum_{t=0}^{T} \left(\sum_{j \in B^t} \exp^{-\epsilon_{ij}/\lambda_t} \right)^{\lambda_t} \right\}, \tag{49}$$

where λ_t is the correlation indicators within subset (group) r. In this specification, all ϵ_{ij} within each subset are correlated with each other, but between products j in different subsets B^t, there is no correlation between ϵ_{ij}.

Then, the market share of product j nested subset $B^r \quad (0 \leq r \leq T)$ is given by

$$s_j = \frac{\exp(\delta_j/\lambda_r) \left(\sum_{k \in B^r} \exp(\delta_k/\lambda_r)\right)^{\lambda_r - 1}}{\sum_{t=0}^{T} \left(\sum_{k \in B^t} \exp(\delta_k/\lambda_t)\right)^{\lambda_t}}. \quad (0 \leq k \leq J) \tag{50}$$

This proof is in Appendix C.

Decomposition in the nested logit model

To interpret equation (50), we can decompose it,

$$
\begin{aligned}
&\frac{\exp(\delta_j/\lambda_r)\left(\sum_{k\in B^r}\exp(\delta_k/\lambda_r)\right)^{\lambda_r-1}}{\sum_{t=0}^{T}\left(\sum_{k\in B^t}\exp(\delta_k/\lambda_t)\right)^{\lambda_t}} \\
&= \frac{\exp(\delta_j/\lambda_r)}{\left(\sum_{k\in B^r}\exp(\delta_k/\lambda_r)\right)^{1-\lambda_r}\left(\sum_{t=0}^{T}(\sum_{k\in B^t}\exp(\delta_k/\lambda_t))\right)^{\lambda_t}} \\
&= \frac{\exp(\delta_j/\lambda_r)}{\sum_{k\in B^r}\exp(\delta_k/\lambda_r)} \times \frac{\left(\sum_{k\in B^r}\exp(\delta_k/\lambda_r)\right)^{\lambda_r}}{\sum_{t=0}^{T}\left(\sum_{k\in B^t}\exp(\delta_k/\lambda_t)\right)^{\lambda_t}}.
\end{aligned}
\tag{51}
$$

Assuming that δ_j, the part of the utility, is given by

$$
\delta_j = \delta_r + \lambda_r\delta_{j|r} \quad (j \in B^r).
\tag{52}
$$

the δ_r indicates mean avarage utility all over subset B_r and $\lambda_r\delta_{j|r}$ that indicate product j's deviation from δ_r is the indirect utility of product j conditioned on a group r.

We substitute (52) for the right hand side in (51),

$$
\begin{aligned}
\frac{\exp(\delta_j/\lambda_r)}{\sum_{k\in B^r}\exp(\delta_k/\lambda_r)} &= \frac{\exp(\delta_r/\lambda_r+\delta_{j|r})}{\sum_{k\in B^r}\exp(\delta_r/\lambda_r+\delta_{k|r})} \\
&= \frac{\exp(\delta_{j|r})}{\sum_{k\in B^r}\exp(\delta_{k|r})}.
\end{aligned}
\tag{53}
$$

and

$$
\begin{aligned}
\frac{\left(\sum_{k\in B^r}\exp(\delta_k/\lambda_r)\right)^{\lambda_r}}{\sum_{t=0}^{T}(\sum_{k\in B^t}\exp(\delta_k/\lambda_t))^{\lambda_t}} &= \frac{\left(\sum_{k\in B^r}\exp(\delta_r/\lambda_r+\delta_{k|r})\right)^{\lambda_r}}{\sum_{t=0}^{T}(\sum_{k\in B^t}\exp(\delta_t/\lambda_t+\delta_{k|t}))^{\lambda_t}} \\
&= \frac{\left(\exp(\delta_r/\lambda_r)\sum_{k\in B^r}\exp(\delta_{k|r})\right)^{\lambda_r}}{\sum_{t=0}^{T}\left(\exp(\delta_t/\lambda_t)\sum_{k\in B^t}\exp(\delta_{k|t})\right)^{\lambda_t}} \\
&= \frac{\exp(\delta_r)\left(\sum_{k\in B^r}\exp(\delta_{k|r})\right)^{\lambda_r}}{\sum_{t=0}^{T}\exp(\delta_t)\left(\sum_{k\in B^t}\exp(\delta_{k|t})\right)^{\lambda_t}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\exp\left(\delta_r + \lambda_r \log \sum_{k\in B^r} \exp(\delta_{k|r})\right)}{\sum_{t=0}^{T} \exp\left(\delta_t + \lambda_t \log \sum_{k\in B^t} \exp(\delta_{k|t})\right)} \\
&= \frac{\exp(\delta_r + \lambda_r L_r)}{\sum_{t=0}^{T} \exp(\delta_t + \lambda_t L_t)}. \qquad (54)
\end{aligned}
$$

where $L_r = \log\left(\sum_{k\in B^r} \exp(\delta_{k|r})\right)$ is called the inclusive value and interpreted as the expected value of the maximum utility obtained from the choice over all products conditioned on a group r.

Therefore, equation (51) is rewritten by

$$
\frac{\exp(\delta_j/\lambda_r)\left(\sum_{k\in B^r} \exp(\delta_k/\lambda_r)\right)^{\lambda_r - 1}}{\sum_{t=0}^{T}\left(\sum_{k\in B^t} \exp(\delta_k/\lambda_t)\right)^{\lambda_t}} = \bar{s}_{j|r} \times \bar{s}_r. \qquad (55)
$$

where

$$
\bar{s}_{j|r} = \frac{\exp(\delta_{j|r})}{\sum_{k\in B^r} \exp(\delta_{k|r})}, \quad \bar{s}_r = \frac{\exp(\delta_r + \lambda_r L_r)}{\sum_{t=0}^{T} \exp(\delta_t + \lambda_t L_t)}.
$$

$\bar{s}_{j|r}$ denotes the market share of product j in the subset r and $\bar{s}_r$ denotes the market share of the subset r.

Linear equation in the nested logit model

Next, we use equation (51), and suppose denominators following

$$
D_r = \sum_{k\in B^r} \exp(\delta_k/\lambda_r), \quad D_t = \sum_{k\in B^t} \exp(\delta_k/\lambda_t).
$$

so, we can rewrite as

$$
\bar{s}_{j|r} = \frac{\exp(\delta_j/\lambda_r)}{D_r}, \quad \bar{s}_r = \frac{D_r^{\lambda_r}}{\sum_{t=0}^{T} D_t^{\lambda_t}}.
$$

With $\delta_0 \equiv 0$, $D_0 = 1$ and so

$$
s_0 = \frac{1}{\sum_{t=0}^{T} D_t^{\lambda_t}} \qquad (56)
$$

Taking logs of s_j and s_0,

$$
\ln(s_j) - \ln(s_0) = \ln \frac{\exp(\delta_j/\lambda_r)}{D_r^{1-\lambda_r}\left[\sum_{t=0}^{T} D_t^{\lambda_t}\right]} - \ln \frac{1}{\sum_t D_t^{\lambda_t}}
$$

$$
\begin{aligned}
&= \ln\left(\exp(\delta_j/\lambda_r)\right) - \left(\ln D_r^{1-\lambda_r} + \ln\left(\sum_t D_t^{\lambda_t}\right)\right) + \ln\left(\sum_t D_t^{\lambda_t}\right) \\
&= \frac{\delta_j}{\lambda_r} - \ln D_r^{1-\lambda_r} \\
&= \frac{\delta_j}{\lambda_r} - (1-\lambda_r)\ln D_r. \qquad (57)
\end{aligned}
$$

And, we rewrite $\bar{s}_r = D_r^{\lambda_r} \cdot s_0$, taking log of this,

$$
\ln D_r = \frac{\ln(\bar{s}_r) - \ln(s_0)}{\lambda_r}. \qquad (58)
$$

Substituting this into equation (57),

$$
\ln(s_j) - \ln(s_0) = \frac{\delta_j}{\lambda_r} - \frac{(1-\lambda_r)}{\lambda_r}\left(\ln(\bar{s}_r) - \ln(s_0)\right). \qquad (59)
$$

Solving equation (59) for δ_j gives

$$
\begin{aligned}
\delta_j &= \lambda_r\left(\ln(s_j) - \ln(s_0)\right) + (1-\lambda_r)(\ln(\bar{s}_r) - \ln(s_0)) \\
&= \lambda_r\left(\ln(s_j) - \ln(\bar{s}_r)\right) + \ln(\bar{s}_r) - \ln(s_0). \qquad (60)
\end{aligned}
$$

Suppose, λ_r=$1-\sigma_r$, the right hand side of (60)

$$
\begin{aligned}
\lambda_r\left(\ln(s_j) - \ln(\bar{s}_r)\right) + \ln(\bar{s}_r) - \ln(s_0) &= (1-\sigma_r)\left(\ln(s_j) - \ln(\bar{s}_r)\right) + \ln(\bar{s}_r) - \ln(s_0) \\
&= \ln(s_j) - \ln(s_0) - \sigma_r \ln\left(\frac{s_j}{\bar{s}_r}\right). \qquad (61)
\end{aligned}
$$

Substituting $\delta_j = \boldsymbol{x}_j\boldsymbol{\beta} - \alpha p_j + \xi_j$ into equation (61),

$$
\ln(s_j) - \ln(s_0) = \boldsymbol{x}_j\boldsymbol{\beta} - \alpha p_j + \sigma_r \ln\left(\frac{s_j}{\bar{s}_r}\right) + \xi_j. \qquad (62)
$$

So that estimates of α, β and σ_r can be obtained from a linear instrumental variables regression of differences in log market shares on product characteristics, prices, and log of within group share.

How the nested logit model of demand addresses the problems in the logit model?

The nested logit model allows for somewhat more flexible substitutionpatterns. However, in some cases, problems the logit model has still remain.

Firstly, nested logit model allows partial relaxzation of I.I.A. property. I.I.A. holds within nests but not across nests. If product j and l is in a same nest B^r, the odds of the market share is

$$\begin{aligned} \frac{s_j}{s_l} &= \frac{\exp(\delta_j/\lambda_r)\left(\sum_{k\in B^r}\exp(\delta_k/\lambda_r)\right)^{\lambda_r-1}}{\sum_{t=0}^{T}\left(\sum_{k\in B^t}\exp(\delta_k/\lambda_t)\right)^{\lambda_t}} \Big/ \frac{\exp(\delta_l/\lambda_r)\left(\sum_{k\in B^r}\exp(\delta_k/\lambda_r)\right)^{\lambda_r-1}}{\sum_{t=0}^{T}\left(\sum_{k\in B^t}\exp(\delta_k/\lambda_t)\right)^{\lambda_t}} \\ &= \frac{\exp(\delta_j/\lambda_r)}{\exp(\delta_l/\lambda_r)} \qquad (63) \end{aligned}$$

depend on δ_j and δ_l. Next, if product j and m is in a differnt nest, B^r and B^q, the odds of the market share is

$$\begin{aligned} \frac{s_j}{s_m} &= \frac{\exp(\delta_j/\lambda_r)\left(\sum_{k\in B^r}\exp(\delta_k/\lambda_r)\right)^{\lambda_r-1}}{\sum_{t=0}^{T}\left(\sum_{k\in B^t}\exp(\delta_k/\lambda_t)\right)^{\lambda_t}} \Big/ \frac{\exp(\delta_m/\lambda_q)\left(\sum_{k\in B^q}\exp(\delta_k/\lambda_q)\right)^{\lambda_q-1}}{\sum_{t=0}^{T}\left(\sum_{k\in B^t}\exp(\delta_k/\lambda_t)\right)^{\lambda_t}} \\ &= \frac{\exp(\delta_j/\lambda_r)\left(\sum_{k\in B^r}\exp(\delta_k/\lambda_r)\right)^{\lambda_r-1}}{\exp(\delta_m/\lambda_q)\left(\sum_{k\in B^q}\exp(\delta_k/\lambda_q)\right)^{\lambda_q-1}} \qquad (64) \end{aligned}$$

depend on all $\delta_0, \ldots, \delta_J$.

Secondly, the elasticities for the nested logit derived from (15) are

$$E_{s_j|p_l} = \frac{\partial s_j p_l}{\partial p_l s_j} = \begin{cases} -\alpha\frac{1}{\lambda_r}s_j\left[1-(1-\lambda_r)\bar{s}_{j|r} - \lambda_r s_j\right] \times \frac{p_j}{s_j} & (l=j), \\ \alpha s_j\left[\frac{1-\lambda_r}{\lambda_r}\bar{s}_{l|t} + s_j\right] & (l \neq j, l \in B^r), \\ 0 & (l \notin B^r). \end{cases} \qquad (65)$$

for $j \in B^r$. This means that the market share of product j in group r is not affect by the price change of products outside of r even if they are produced by the same firm. And, nested logit helps with the problem of cross elasticity across nests, but does not help with within nests and own elasticity. And in many cases the priori division of products into group, and the assumption of i.i.d shocks within a group, will not be reasonable, either because of segments is not clear or because the segmentation does not fully account for the substitution patterns.

C Proof of the equation (15): Nested logit model

We use the equation (42) used in logit model, s_{ij} is

$$s_{ij} = \int_{-\infty}^{\infty} F_j(\epsilon_{ij} + \delta_j - \delta_0, \ldots, \epsilon_{ij}, \ldots, \epsilon_{ij} + \delta_j - \delta_J) d\epsilon_{ij}. \tag{66}$$

In the case of nested logit, we assume that ϵ_{ik} $(i = 1, \ldots, n, k = 0, \ldots, J)$ is Gumbel's type 2 bivariate distribution. Using (49), the joint distribution of ϵ_{ij} is

$$F(\epsilon_{i0}, \epsilon_{i1}, \ldots, \epsilon_{iJ}) = \exp\left\{ -\sum_{t=0}^{T} \left(\sum_{j \in B^t} \exp^{-\epsilon_{ij}/\lambda_t} \right)^{\lambda_t} \right\} \tag{67}$$

Suppose, $\sum_{t=0}^{T} \left(\sum_{j \in B^t} \exp^{-\epsilon_{ij}/\lambda_t} \right)^{\lambda_t} = G[\exp(-\epsilon_{i0}), \ldots, \exp(-\epsilon_{iJ})]$, then the right hand side of (67) is

$$\begin{aligned} &\exp\left\{ -\sum_{t=0}^{T} \left(\sum_{j \in B^t} \exp^{-\epsilon_{ij}/\lambda_t} \right)^{\lambda_t} \right\} \\ &= \exp\left\{ -G[\exp(-\epsilon_{i0}), \ldots, \exp(-\epsilon_{iJ})] \right\}. \end{aligned} \tag{68}$$

We mention the function G as follows. Consider G satisfying

1. $G[\exp(-\epsilon_{i0}), \exp(-\epsilon_{i1}), \ldots, \exp(-\epsilon_{iJ})] \geq 0$.

2. $\lim_{\epsilon_{ij}\to\infty} G[\exp(-\epsilon_{i0}), \exp(-\epsilon_{i1}), \ldots, \exp(-\epsilon_{iJ})] = \infty \ (j = 0, \ldots, J)$

3. The mixed partial derivatives of G exist and if we differntiatic G with respect to distinct $\exp(-\epsilon_{ij})$ odd times, then it is non negative and even times, it is non positive, or

$$G_{j,k,\cdots,n} = \frac{\partial^{\text{length of } (j,k,\cdots,n)} G}{\partial \exp(-\epsilon_{ij}) \partial \exp(-\epsilon_{ik}) \cdots \partial \exp(-\epsilon_{in})} = \begin{cases} \geq 0 & (\text{if } n \text{ is odd}) \\ \leq 0 & (\text{if } n \text{ is even}) \end{cases}$$

4. The function G is a homogeneous function of degree equal to 1, or $G[a \cdot \exp(-\epsilon_{i0}), \cdots, a \cdot \exp(-\epsilon_{iJ})] = a \cdot G[\exp(-\epsilon_{i0}), \cdots, \exp(-\epsilon_{iJ})]$.

   So, partial derivartive ϵ_{ij}, suppose G_j, is a homogeneous function of degree equal to 0, or

   $G[a \cdot \exp(-\epsilon_{i0}), \cdots, a \cdot \exp(-\epsilon_{iJ})] = G[\exp(-\epsilon_{i0}), \cdots, \exp(-\epsilon_{iJ})]$.

And substitute, $\epsilon_{i0} = \epsilon_{ij} + \delta_j - \delta_0, \ldots, \epsilon_{ij} = \epsilon_{ij} + \delta_j - \delta_j, \ldots, \epsilon_{iJ} = \epsilon_{ij} + \delta_j - \delta_J$

$$\begin{aligned}
&F_j(\epsilon_{ij} + \delta_j - \delta_0, \ldots, \epsilon_{ij}, \ldots, \epsilon_{ij} + \delta_j - \delta_J) \\
&= \frac{\partial F}{\partial G} \cdot \frac{\partial G}{\partial \exp(-\epsilon_{ij})} \cdot \frac{\partial \exp(-\epsilon_{ij})}{\partial \epsilon_{ij}} \\
&= \exp\{-G[\exp(-(\epsilon_{ij} + \delta_j - \delta_0)), \ldots, \exp(-(\epsilon_{ij})), \ldots, \exp(-(\epsilon_{ij} + \delta_j - \delta_J))]\} \times (-1) \\
&\quad \times G_j[\exp(-(\epsilon_{ij} + \delta_j - \delta_0)), \ldots, \exp(-\epsilon_{ij}), \ldots, \exp(-(\epsilon_{ij} + \delta_j - \delta_J))] \\
&\quad \times \exp(-\epsilon_{ij}) \times (-1). \qquad (69)
\end{aligned}$$

where G_j denotes the partial derivative of G with respect to its jth argument $(\exp(-\epsilon_{ij}))$. Since G is a homogeneous function of degree equal to 1,

$$\begin{aligned}
&G[\exp(-(\epsilon_{ij} + \delta_j - \delta_0)), \exp(-(\epsilon_{ij} + \delta_j - \delta_1)), \ldots, \exp(-\epsilon_{ij}), \ldots, \exp(-(\epsilon_{ij} + \delta_j - \delta_J))] \\
&= \exp(-\epsilon_{ij}) G[\exp(-(\delta_j - \delta_0)), \exp(-(\delta_j - \delta_1)), \ldots, 1, \ldots, \exp(-(\delta_j - \delta_J))] \\
&= \exp(-\epsilon_{ij}) G^*. \qquad (70)
\end{aligned}$$

where we denote

$$G[\exp(-(\delta_j-\delta_0)), \exp(-(\delta_j-\delta_1)), \ldots, 1, \ldots, \exp(-(\delta_j-\delta_J))] = G^*.$$

and since G_j is a homogeneous function of degree equal to 0,

$$\begin{aligned}
&G_j[\exp(-(\epsilon_{ij}+\delta_j-\delta_0)), \ldots, \exp(-\epsilon_{ij}), \ldots, \exp(-(\epsilon_{ij}+\delta_j-\delta_J))] \\
&= G_j[\exp(-(\delta_j-\delta_0)), \ldots, 1, \ldots, \exp(-(\delta_j-\delta_J))] \\
&= G_j^*. \qquad (71)
\end{aligned}$$

where we denote

$$G_j[\exp(-(\delta_j-\delta_0)), \ldots, 1, \ldots, \exp(-(\delta_j-\delta_J))] = G_j^*.$$

Substituting equation (70) for the first term on the equation (69), and (71) for the second term on the equation (69),

$$\begin{aligned}
&F_j(\epsilon_{ij}+\delta_j-\delta_0, \ldots, \epsilon_{ij}, \ldots, \epsilon_{ij}+\delta_j-\delta_J) \\
&= \exp[-\exp(-\epsilon_{ij})G^*] \cdot G_j^* \cdot \exp(-\epsilon_{ij}) \\
&= \exp[-\exp(-(\epsilon_{ij}-\ln G^*))] \exp(-(\epsilon_{ij}-\ln G^*))G_j^*/G^* \\
&= \exp[-\epsilon_{ij}^* - \exp(-\epsilon_{ij}^*)]G_j^*/G^*. \qquad (72)
\end{aligned}$$

above $\epsilon_{ij}^*{=}\epsilon_{ij} - \ln G^*$, therefore s_j is

$$\begin{aligned}
s_j &= \int_{-\infty}^{\infty} \exp[-\epsilon_{ij}^* - \exp(-\epsilon_{ij}^*)]G_j^*/G^* d\epsilon_{ij}^* \\
&= G_j^*/G^* \int_{-\infty}^{\infty} \exp[-\epsilon_{ij}^* - \exp(-\epsilon_{ij}^*)]d\epsilon_{ij}^* \\
&= G_j^*/G^*. \qquad (73)
\end{aligned}$$

where $\int_{-\infty}^{\infty} \exp[-\epsilon_{ij}^* - \exp(-\epsilon_{ij}^*)]d\epsilon_{ij}^*$ is equal to 1, because

$$\begin{aligned}
&\int_{-\infty}^{\infty} \exp[-\epsilon_{ij}^* - \exp(-\epsilon_{ij}^*)]d\epsilon_{ij}^* \\
&= \int_{-\infty}^{\infty} \exp(-\epsilon_{ij}^*) \exp(-\exp(-\epsilon_{ij}^*))d\epsilon_{ij}^*. \qquad (74)
\end{aligned}$$

Suppose, $\exp(-\epsilon^*_{ij}) = t$, then $d\epsilon^*_{ij} = -\frac{1}{\exp(-\epsilon^*_{ij})}dt$, and $d\epsilon^*_{ij} = -\frac{1}{t}dt$, the right hand side of (74) is

$$\begin{aligned} \int_{-\infty}^{\infty} \exp(-\epsilon^*_{ij}) \exp(-\exp(-\epsilon^*_{ij}))d\epsilon^*_{ij} &= \int_{\infty}^{0} t \times \exp(-t) \times (-\frac{1}{t})dt \\ &= [-\exp(-t)]_0^{\infty} \\ &= = 1. \end{aligned} \quad (75)$$

Furthermore, using property that G is a homogeneous function degree equal to 1, G_j is degree equal to 0,

$$\begin{aligned} G^* &= G[\exp(-(\delta_j - \delta_0)), \exp(-(\delta_j - \delta_1)), \ldots, \exp(-(\delta_j - \delta_j)), \ldots, \exp(-(\delta_j - \delta_J))] \\ &= \exp(-\delta_j)G[\exp(\delta_0), \exp(\delta_1), \ldots, \exp(\delta_j), \ldots, \exp(\delta_J)]. \end{aligned} \quad (76)$$

$$\begin{aligned} G^*_j &= G_j[\exp(-(\delta_j - \delta_0)), \exp(-(\delta_j - \delta_1)), \ldots, \exp(-(\delta_j - \delta_j)), \ldots, \exp(-(\delta_j - \delta_J))] \\ &= G_j[\exp(\delta_0), \exp(\delta_1), \ldots, \exp(\delta_j), \ldots, \exp(\delta_J)]. \end{aligned} \quad (77)$$

Therefore, substituting (76) and (77) for (73) obtains

$$s_j = \frac{G_j[\exp(\delta_0), \exp(\delta_1), \ldots, \exp(\delta_j), \ldots, \exp(\delta_J)]}{\exp(-\delta_j)G[\exp(\delta_0), \exp(\delta_1), \ldots, \exp(\delta_j), \ldots, \exp(\delta_J)]}. \quad (78)$$

Replace the function G with its original notation,

$$G[\exp(\delta_0), \ldots, \exp(\delta_j), \ldots, \exp(\delta_J)] = \sum_{t=0}^{T} \left(\sum_{k \in B^t} \exp(\delta_k/\lambda_t) \right)^{\lambda_t}. \quad (79)$$

Since product j is nested within r-th subject B^r, differetiating (79) with respect $\exp(-\epsilon_{ij})$,

$$
\begin{aligned}
& G_j[\exp(\delta_0), \ldots, \exp(\delta_j), \ldots, \exp(\delta_J)] \\
&= \left. \frac{\partial G}{\partial \exp(-\epsilon_{ij})} \right|_{\exp(-\epsilon_{i0})=\exp(\delta_0),\cdots,\exp(-\epsilon_{iJ})=\exp(\delta_J)} \\
&= \left. \frac{\partial \sum_{t=0}^{T} \left(\sum_{k\in B^t} \exp(-\epsilon_{ik}/\lambda_t)\right)^{\lambda_t}}{\partial \exp(-\epsilon_{ij})} \right|_{\exp(-\epsilon_{i0})=\exp(\delta_0),\cdots,\exp(-\epsilon_{iJ})=\exp(\delta_J)} \\
&= \left. \frac{\partial \sum_{k\in B^r} \exp(-\epsilon_{ik}/\lambda_r)^{\lambda_r}}{\partial \exp(-\epsilon_{ij})} \right|_{\exp(-\epsilon_{i0})=\exp(\delta_0),\cdots,\exp(-\epsilon_{iJ})=\exp(\delta_J)} \\
&= \left. \frac{\partial \sum_{k\in B^r} \exp(-\epsilon_{ik}/\lambda_r)^{\lambda_r}}{\partial \sum_{k\in B^r} \exp(-\epsilon_{ik}/\lambda_r)} \cdot \frac{\partial \sum_{k\in B^r} \exp(-\epsilon_{ik}/\lambda_r)}{\partial \exp(-\epsilon_{ij})} \right|_{\exp(-\epsilon_{i0})=\exp(\delta_0),\cdots,\exp(-\epsilon_{iJ})=\exp(\delta_J)} \\
&= \lambda_r \left(\sum_{k\in B^r} \exp(-\epsilon_{ik}/\lambda_r) \right)^{\lambda_r - 1} \cdot \left. \frac{\partial \sum_{k\in B^r} \exp(-\epsilon_{ik})^{1/\lambda_r}}{\partial \exp(-\epsilon_{ij})} \right|_{\exp(-\epsilon_{i0})=\exp(\delta_0),\cdots,\exp(-\epsilon_{iJ})=\exp(\delta_J)} \\
&= \lambda_r \left(\sum_{k\in B^r} \exp(-\epsilon_{ik}/\lambda_r) \right)^{\lambda_r - 1} \cdot \left. \frac{1}{\lambda_r} \exp(-\epsilon_{ij})^{\frac{1}{\lambda_r}-1} \right|_{\exp(-\epsilon_{i0})=\exp(\delta_0),\cdots,\exp(-\epsilon_{iJ})=\exp(\delta_J)} \\
&= \sum_{k\in B^r} \exp(\delta_k/\lambda_r)^{\lambda_r - 1} \times \exp(\delta_j)^{\frac{1}{\lambda_r}-1} \\
&= \sum_{k\in B^r} \exp(\delta_k/\lambda_r)^{\lambda_r - 1} \cdot \exp(\delta_j)^{\frac{1}{\lambda_r}} \cdot \exp(-\delta_j) \qquad (80)
\end{aligned}
$$

Substitute equation (79) and (80) to equation (78),

$$
\begin{aligned}
s_{ij} &= \frac{\sum_{k\in B^r} \exp(\delta_k/\lambda_r)^{\lambda_r-1} \cdot \exp(\delta_j)^{\frac{1}{\lambda_r}} \cdot \exp(-\delta_j)}{\exp(-\delta_j) \sum_{t=0}^{T} \left(\sum_{k\in B^t} \exp(\delta_k/\lambda_t)\right)^{\lambda_t}} \\
&= \frac{\exp(\delta_j/\lambda_r) \left(\sum_{k\in B^r} \exp(\delta_k/\lambda_r)\right)^{\lambda_r-1}}{\sum_{t=0}^{T} \left(\sum_{k\in B^t} \exp(\delta_k/\lambda_t)\right)^{\lambda_t}}. \qquad (81)
\end{aligned}
$$

δ_j and λ_t do not vary with the consumers, due to this, the probability that consumer i chooses product j is equal to

$$
s_j = \frac{\exp(\delta_j/\lambda_r) \left(\sum_{k\in B^r} \exp(\delta_k/\lambda_r)\right)^{\lambda_r-1}}{\sum_{t=0}^{T} \left(\sum_{k\in B^t} \exp(\delta_k/\lambda_t)\right)^{\lambda_t}}. \qquad (82)
$$

for all consumers, and where the parameter λ_t $(0 \leq \lambda_t \leq 1)$ is a measure of the correlation of unobserved utility within subset B^t. More precisely,

$(1 - \lambda_t)$ is a measure of correlation since λ_t itself drops as the correlation rises.

D Other tables

	Toyota	Honda	Nissan	Suzuki	Daihatsu	Mitsubishi	Mazda	Subaru
1994	33.7	8.5	18.0	7.4	5.1	9.2	6.3	5.1
1995	30.9	9.1	18.6	8.0	5.3	10.2	4.8	4.8
1996	30.1	12.0	17.4	8.5	6.4	7.7	4.2	5.2
1997	29.5	14.4	16.6	8.5	6.1	7.7	5.0	4.6
1998	27.8	14.4	16.8	8.8	7.1	7.9	5.9	4.7
1999	27.8	14.7	13.7	9.9	8.2	7.8	6.1	5.3
2000	28.5	16.2	11.8	10.0	9.1	7.0	6.0	5.2
2001	28.1	18.4	11.5	10.1	8.9	6.7	5.3	4.7
2002	29.8	18.8	12.3	10.3	8.1	5.8	4.9	3.8

Table 3: Market share in Japan (1994-2002)(%)

No	Vehicle name	Trim	Maker	Sales(units)
1	Fit	1.3A	Honda	250,790
2	Corolla	1.5X	Toyota	226,222
3	Wagon R	FM aero	Suzuki	159,691
4	Move	CL	Daihatsu	149,192
5	March	5door 12c	Nissan	139,332
6	Life	DunkTS	Honda	137,299
7	ek-Wagon	MX package	Mitsubishi	131,456
8	Hijet	Special	Daihatsu	110,721
9	Ist	1.5S	Toyota	103,579
10	Vitz	1300 clavier	Toyota	100,801
11	Noah	L	Toyota	97,080
12	Estima	3.0 Aeras	Toyota	95,765
13	Mira	Pico	Daihatsu	95,744
14	Pleo	LS	Subaru	80,853
15	MR Wagon	N-1	Suzuki	78,295
16	Voxy	Z	Toyota	77,958
17	Carry	KC	Suzuki	77,657
18	Cube	SX	Nissan	75,215
19	Mobilio	1.5A	Honda	72,242
20	Alto Lapin	G	Suzuki	72,057
21	Stepwgn	2.0K	Honda	71,128
22	Alto	5door N-1	Suzuki	70,165
23	Sambar	Dias Wagon	Subaru	69,847
24	Max	RS	Daihatsu	69,661
25	Every Wagon	Joypop turbo	Suzuki	69,366

Table 4: Japan' top 100 sellers in 2002 (No.1 - No.25)

No	Vehicle name	Trim	Maker	Sales(units)
26	Stream	iL	Honda	64,289
27	Demio	1500 casual	Mazda	63,050
28	Serena	25X	Nissan	60,492
29	Premio	1.8X	Toyota	58,800
30	Funcargo	1.5G	Toyota	57,525
31	Mark II	Grande	Toyota	57,447
32	Acty	660SDX	Honda	54,112
33	Alphard	2.4AS 4WD	Toyota	53,428
34	Wingroad	1.8S	Nissan	53,407
35	Legacy	2.0 touringwagon GT	Subaru	52,608
36	Odyssey	2.3 absolute	Honda	52,366
37	Ipsum	240u G selection	Toyota	51,939
38	Crown	Majesta400	Toyota	51,615
39	Allion	A18	Toyota	49,975
40	Sunny	1500 Super saloon	Nissan	49,121
41	That's	3AT	Honda	45,443
42	bB	1.5Z	Toyota	43,820
43	MPV	2300 Sport 4WD	Mazda	43,436
44	Elgrand	X 4WD	Nissan	40,439
45	Vamos	Turbo	Honda	39,379
46	Moco	T	Nissan	36,970
47	Lancer	1500MX touring	Mitsubishi	34,075
48	Bluebird Sylphy	18Vi	Nissan	32,690
49	X-Trail	2.0X	Nissan	31,199
50	Minica	5door voice	Mitsubishi	30,850

Table 5: Japan' top 100 sellers in 2002 (No.26 - No.50)

No	Vehicle name	Trim	Maker	Sales(units)
51	Kei	G-type 4WD	Suzuki	29,865
52	Liberty	G Navipackage	Nissan	28,406
53	Forester	X20	Subaru	26,921
54	Impreza	WRX	Subaru	25,059
55	Platz	1.5x	Toyota	24,893
56	Caldina	2000ZT	Toyota	24,885
57	Civic	1.5G	Honda	24,341
58	Duet	1.3S	Toyota	22,600
59	Allex	XS 150G	Toyota	21,424
60	Terios Kid	Custom S edition	Daihatsu	21,140
61	Mark II	Grande25	Toyota	21,062
62	Landcruiser	100VX	Toyota	20,982
63	Atenza	5door 23S	Mazda	20,795
64	Atrai Wagon	Touring-turbo	Daihatsu	20,597
65	Primera	18C	Nissan	18,796
66	Stagea	250RX	Nissan	18,376
67	Premacy	1800 G 4WD	Mazda	18,301
68	Swift	1300 SG-X 4WD	Suzuki	18,163
69	AZ Wagon	FZ-T	Mazda	17,521
70	CR-V	2.0fullmark iL	Honda	17,289
71	Crown	2000 loyal extra	Toyota	17,177
72	Gaia	2000 aero	Toyota	16,739
73	Familia	Swagon RS	Mazda	15,975
74	Naked	G	Daihatsu	15,778
75	Jimny	XG	Suzuki	14,885

Table 6: Japan' top 100 sellers in 2002 (No.51 - No.75)

No	Vehicle name	Trim	Maker	Sales(units)
76	Celsior	4300B eR version	Toyota	14,602
77	Rav4	5door aero	Toyota	13,711
78	KlugerV	2.4FOUR	Toyota	13,641
79	Opa	1.8i	Toyota	13,513
80	Altezza	RS200 Z edition	Toyota	13,498
81	Airtrek	20V	Mitsubishi	13,435
82	Accord Wagon	Wagon 24T	Honda	13,294
83	Vista	N200	Toyota	13,016
84	Pajero Mini	R	Mitsubishi	12,886
85	Dunk	TR	Honda	12,408
86	Wagon R	Solio1.3 WELL S	Suzuki	12,257
87	Colt	1500standard	Mitsubishi	11,759
88	MPV	2.0jive	Mazda	11,414
89	Skyline	250GT	Nissan	11,033
90	Accord	20EL	Honda	10,828
91	Legacy	GT30	Subaru	10,565
92	Cedric	300LV	Nissan	10,262
93	Scrum	Wagon standard	Mazda	9,809
94	Harrier	300G	Toyota	9,520
95	Toppo BJ	M-T	Mitsubishi	9,273
96	Hilux Surf	2700 SSR-G	Toyota	9,138
97	Succeed	TX4AT	Toyota	8,706
98	Brevis	Ai250	Toyota	8,634
99	Cruze1.3	1.3X	Suzuki	8,338
100	Hiace	2.4 super custom G	Toyota	8,275

Table 7: Japan' top 100 sellers in 2002 (No.76 - No.100)