SOCIAL WELFARE FUNCTION FOR RESTRICTED PREFERENCE DOMAIN

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Abstract. We consider the social preference ordering in a society where each individual's preference domain is restricted to a subset of the whole set of alternatives. We show that the social welfare function satisfying unrestricted domain property, independence of irrelevant alternatives and weak Pareto optimality is always dictatorial when at least one individual is entitled to express his/her preference on the whole set of alternatives.

1. Introduction

We often encounter the problem of aggregating opinions of individuals in a society. Arrow [2] introduced the social choice theory for this problem, and gave the monumental impossibility theorem: a social welfare function which satisfies unrestricted domain property, independence of irrelevant alternatives and Pareto optimality is dictatorial. From then onward, the difficulty of the problem has been well recognized, and a variety of impossibility theorems in the Arrow's framework have been developed. The reader is recommended referring to Sen [14].

This paper studies the existence and properties of a social preference ordering when individual's preference domain is restricted: one expresses one's preference on one's alternative set that is a subset of the whole set of alternatives. This modification can be viewed as a relaxation of the unrestricted domain property in the Arrow's framework. For relaxation of the unrestricted domain property, there are many researches such as Blair and Muller [3], Bordes and Le Breton [4], Fishburn and Kelly [5], Kalai, Muller and Satterthwaite [8] and Redekop [10]. In particular, Kalai and Muller [7], Ritz [11] and Ritz [12] deal with a restriction on permissible preferences for individuals instead of profile restrictions.

Ando, Ohara, and Yamamoto [1] deals with the problem in a society where individuals evaluate mutually. The set of alternatives coincides with the set of individuals, and each individual expresses one's preference ordering on the whole set of individuals except oneself. They studied properties of the social welfare function, and proved that an outcome of social welfare function satisfying unrestricted domain property, independence of irrelevant alternatives and Pareto optimality can be cyclic, hence cannot be a weak order, meaning the nonexistence of social preference ordering.

In order to avoid the paradoxical outcome of Ando, Ohara, and Yamamoto [1], we add several individuals who express their preference on the whole set of alternatives.

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Unlike their model, the set of alternatives can be any finite set instead of the set of individuals. Accordingly, unrestricted domain property, independence of irrelevant alternatives and weak Pareto optimality are defined. We will show in Theorem 3.1 that the social welfare function satisfying these three axioms is dictatorial. More precisely, someone who expresses his/her preference on the whole set of alternatives is a dictator.

This model is, for example, applied to a vote in a class. Take a vote for a relay team (or a leader or a sub leader). Each pupil submits his/her preference on all of his/her classmates except him/herself. The teacher, however, is entitled to expresses his/her preference on all pupils in the class. The main theorem implies that the teacher is a dictator under three axioms.

In Section 2, the framework of the model and notations are introduced. In Section 3, we give the main theorem. In Section 4, a special case will be discussed. Section 5 summarizes the results.

2. Notations and Framework

Let us denote the set of alternatives by X and assume that there are at least three alternatives, i.e., $|X| \geq 3$. A binary relation $\succsim$ on X is called a weak ordering if it satisfies the following conditions:

(i) reflexivity : for any $x \in X$, $x \succsim x$,
(ii) completeness : for any pair of alternatives x and y, either $x \succsim y$ or $y \succsim x$ holds,
(iii) transitivity : if $x \succsim y$ and $y \succsim z$, then $x \succsim z$.

We write $x \sim y$ when both $x \succsim y$ and $y \succsim x$ hold while we write $x \succ y$ when $x \succsim y$ and $y \not\succsim x$.

Let $N = \{1, ..., n\}$ be the set of all individuals and assume that $n \geq 2$. In Arrow's framework, each individual is interested in all of the alternatives, and hence his/her preference is defined as a weak ordering on the whole set X. However, there might be some individuals who are not interested in all the alternatives. To express such a situation we consider the set of alternatives that are of interest to individual i, and denote it by X_i. Then individual i has his/her preference being a weak ordering on X_i. We denote by W_i the set of all weak orderings defined on X_i. Let P be a subset of $W_1 \times \cdots \times W_n$. We call an element $p \in P$ a profile, and denote by $\succsim_i^p$ the preference of individual i at profile p. A social welfare function, which will be denoted by f hereafter, is a mapping that assigns a weak ordering on X to a profile $p \in P$, i.e., $f : P \to W$, where W is the set of all weak orderings on X, and we denote by $\succsim^p$ the weak ordering determined by f at profile p.

Some axioms on the social welfare function are introduced. The first axiom means that each individual is allowed to have any preference one wishes.

Axiom 2.1 (Unrestricted Domain Property (UDP))**.** The domain P of the social welfare function f is $P = W_1 \times \cdots \times W_n$.

Given a set $A \subseteq X$ of alternatives, let $N(A)$ be the set of all individuals whose preference domain contains A, i.e.,

$$N(A) = \{\, i \in N \mid A \subseteq X_i \,\}.$$

Axiom 2.2 (Weak Pareto Optimality (WPO))**.** If

$$x \succ_i^p y \text{ for all } i \in N(\{x,y\}) \text{ implies } x \succ^p y,$$

holds for any alternatives $x, y \in X$ and for any profile $p \in P$, then the social welfare function f is said to have weak Pareto optimality.

Axiom 2.3 (Independence of Irrelevant Alternatives (IIA))**.** If

$$\succsim_i^p |\{x,y\} = \succsim_i^q |\{x,y\} \text{ for all } i \in N(\{x,y\}) \text{ implies } \succsim^p |\{x,y\} = \succsim^q |\{x,y\}$$

holds for any alternatives $x, y \in X$ and for any profiles $p, q \in P$, then the social welfare function f is said to satisfy independence of irrelevant alternatives.

Definition 2.4 (Dictator)**.** An individual $i \in N$ is called a dictator if $x \succ_i^p y$ implies $x \succ^p y$ for any alternatives $x, y \in X_i$ and for any profile $p \in P$. If there exists a dictator, then the social welfare function f is said to be dictatorial.

3. Impossibility Theorem

We will show in this section that the social welfare function satisfying Axioms (UDP), (WPO) and (IIA) is dictatorial in the presence of an individual of $N(X)$. The main theorem is as follows.

Theorem 3.1. *Suppose that $N(X) \neq \phi$. If a social welfare function f satisfies Axioms (UDP), (WPO) and (IIA), then an individual in $N(X)$ is a dictator.*

We first define $(x \succ y)$-decisive coalition and decisive coalition for the proof of Theorem 3.1.

Definition 3.2 ($(x \succ y)$-decisive coalition)**.** Let $x, y \in X$ be a pair of distinct alternatives. A nonempty subset of individuals $M \subseteq N(\{x,y\})$ is said to be an $(x \succ y)$-decisive coalition if for any profile $p \in P$

$$x \succ_i^p y \text{ for all } i \in M \text{ and } y \succ_j^p x \text{ for all } j \in N(\{x,y\}) \backslash M \text{ imply } x \succ^p y.$$

Definition 3.3 (decisive coalition)**.** A nonempty subset of individuals $M \subseteq N$ is said to be a decisive coalition if M is an $(x \succ y)$-decisive coalition for some pair of distinctive alternatives $x, y \in X$.

To prove Theorem 3.1 we utilize these definitions and the following lemmas.

Lemma 3.4. *Assume Axiom (IIA) and let $M \subseteq N$ be a nonempty subset of $N(\{x,y\})$ for some distinct pair of alternatives x and y. If there is a profile $p \in P$ such that*

$$x \succ_i^p y \text{ for all } i \in M,\ y \succ_j^p x \text{ for all } j \in N(\{x,y\}) \backslash M \text{ and } x \succ^p y.$$

then M is an $(x \succ y)$-decisive coalition.

Proof. Let q be an arbitrary profile such that $x \succ_i^q y$ for $i \in M$ and $y \succ_j^q x$ for $j \in N(\{x,y\}) \backslash M$. Then $\succsim_i^p |\{x,y\} = \succsim_i^q |\{x,y\}$ for all $i \in N(\{x,y\})$. Applying Axiom (IIA), we have $x \succ^q y$, meaning that M is an $(x \succ y)$-decisive coalition. □

Lemma 3.5. *Suppose that $N(X) \neq \phi$ and Axioms (UDP), (WPO) and (IIA). Then any $(x \succ y)$-decisive coalition contains an individual i with $X_i \backslash \{x,y\} \neq \phi$.*

Proof. Let M be an $(x \succ y)$-decisive coalition and assume that $X_i = \{x, y\}$ for all $i \in M$. Choose an arbitrary alternative, say z, of $X \backslash \{x, y\}$. Then M intersects none of $N(\{x, y, z\})$, $N(\{y, z\})$ and $N(\{x, z\})$, each of which contains $N(X)$ and hence is nonempty. Let $p \in P$ be a profile such that

$$\begin{cases} x \succ_i^p y & \text{for } i \in M \\ y \succ_i^p z \succ_i^p x & \text{for } i \in N(\{x, y, z\}) \\ y \succ_i^p z & \text{for } i \in N(\{y, z\}) \backslash N(\{x, y, z\}) \\ z \succ_i^p x & \text{for } i \in N(\{x, z\}) \backslash N(\{x, y, z\}) \\ y \succ_i^p x & \text{for } i \in N(\{x, y\}) \backslash (M \cup N(\{x, y, z\})). \end{cases}$$

Since M is an $(x \succ y)$-decisive coalition, we have

$$x \succ^p y. \tag{3.1}$$

Concerning the pair of y and z, $y \succ_i^p z$ holds for all $i \in N(\{y, z\})$, implying

$$y \succ^p z \tag{3.2}$$

by Axiom (WPO). In the same way we see

$$z \succ^p x. \tag{3.3}$$

Clearly (3.2) and (3.3) together contradict (3.1). □

Lemma 3.6. *Suppose that $N(X) \neq \phi$ and Axioms (UDP), (WPO) and (IIA). Then there is a decisive coalition consisting of a single individual.*

Proof. For a pair of distinct alternatives x and y, $N(\{x, y\})$ is clearly an $(x \succ y)$-decisive coalition from Axiom (WPO). Therefore there is at least one decisive coalition.

Let M be a decisive coalition that is minimal with respect to set inclusion partial order, and suppose it is an $(x \succ y)$-decisive coalition. We will show that the assumption $|M| \geq 2$ leads to a contradiction. We have seen in Lemma 3.5 that $X_i \backslash \{x, y\} \neq \phi$ for some individual $i \in M$. Let z be an arbitrary alternatives of $X_i \backslash \{x, y\}$. For $i \in M$, $z \in X_i \backslash \{x, y\}$ and $M \setminus \{i\} \neq \emptyset$ thus constructed, we consider a profile $p \in P$ such that

$$\begin{cases} z \succ_i^p x \succ_i^p y & \\ x \succ_j^p y \succ_j^p z & \text{for } j \in (M \setminus \{i\}) \cap N(\{x, y, z\}) \\ x \succ_j^p y & \text{for } j \in (M \setminus \{i\}) \backslash N(\{x, y, z\}) \\ y \succ_j^p z \succ_j^p x & \text{for } j \in (N \backslash M) \cap N(\{x, y, z\}) \\ y \succ_j^p x & \text{for } j \in (N \backslash M) \cap (N(\{x, y\}) \backslash N(\{x, y, z\})) \\ y \succ_j^p z & \text{for } j \in (N \backslash M) \cap (N(\{y, z\}) \backslash N(\{x, y, z\})) \\ z \succ_j^p x & \text{for } j \in (N \backslash M) \cap (N(\{x, z\}) \backslash N(\{x, y, z\})). \end{cases}$$

Since M is an $(x \succ y)$-decisive coalition, we have

$$x \succ^p y. \tag{3.4}$$

The following two cases are possible.
Case A: $z \succ^p y$.

Since $z \succ_i^p y$ and $y \succ_j^p z$ for all $j \in N(\{y,z\})\backslash\{i\}$, we conclude that $\{i\}$ alone is a $(z \succ y)$-decisive coalition from Lemma 3.4. This contradicts the minimality assumption of M.

Case B: $y \succsim^p z$.

First note that

$$x \succ^p z \tag{3.5}$$

by (3.4) and the transitivity. We will show that $(M \setminus \{i\}) \cap N(\{x,y,z\})$ is an $(x \succ z)$-decisive coalition. Suppose $(M \setminus \{i\}) \cap N(\{x,y,z\}) = \phi$. Then $z \succ_i^p x$ for all $i \in N(\{x,z\})$. This implies $z \succ^p x$ by Axiom (WPO), which contradicts (3.5). Therefore $(M \setminus \{i\}) \cap N(\{x,y,z\}) \neq \phi$. By the construction of p and (3.5) we see that $(M \setminus \{i\}) \cap N(\{x,y,z\})$ is an $(x \succ z)$-decisive coalition and this fact again contradicts the minimality of M. □

Lemma 3.7. *Suppose that $N(X) \neq \phi$ and Axioms (UDP), (WPO) and (IIA). Then there is an individual of N(X) who alone forms a decisive coalition.*

Proof. Let $\{i\}$ be a decisive coalition demonstrated in Lemma 3.6, and assume that it is an $(x \succ y)$-decisive coalition. Note that $x, y \in X_i$. Suppose $i \notin N(X)$, i.e., $X\backslash X_i \neq \phi$, and let z be an arbitrary alternative of $X\backslash X_i$. Note also that z is distinct from x and y. Now consider a profile $p \in P$ such that

$$\begin{cases} x \succ_i^p y & \\ y \succ_j^p z \succ_j^p x & \text{for } j \in N(\{x,y,z\}) \\ y \succ_j^p x & \text{for } j \in N(\{x,y\})\backslash(\{i\} \cup N(\{x,y,z\})) \\ y \succ_j^p z & \text{for } j \in N(\{y,z\})\backslash N(\{x,y,z\}) \\ z \succ_j^p x & \text{for } j \in N(\{x,z\})\backslash N(\{x,y,z\}). \end{cases}$$

Since $\{i\}$ is an $(x \succ y)$-decisive coalition, we have

$$x \succ^p y. \tag{3.6}$$

We also have $y \succ^p z$ and $z \succ^p x$ by Axiom (WPO). This is contrary to (3.6). Therefore we conclude that $i \in N(X)$. □

Lemma 3.8. *Suppose that $N(X) \neq \phi$ and Axioms (UDP), (WPO) and (IIA). If an individual of N(X) is a decisive coalition, it is an $(x \succ y)$-decisive coalition for any pair of distinct alternatives x and $y \in X$.*

Proof. Suppose that $i \in N(X)$ forms an $(x \succ y)$-decisive coalition, and let z be an arbitrary alternative different from x and y. We first show that $\{i\}$ is an $(x \succ z)$-decisive coalition and then show that it is a $(z \succ y)$-decisive coalition.

(I) $\{i\}$ is an $(x \succ z)$-decisive coalition:

First consider a profile $p \in P$ such that

$$\begin{cases} x \succ_i^p y \succ_i^p z & \\ y \succ_j^p z \succ_j^p x & \text{for } j \in N(\{x,y,z\})\backslash\{i\} \\ y \succ_j^p x & \text{for } j \in N(\{x,y\})\backslash N(\{x,y,z\}) \\ y \succ_j^p z & \text{for } j \in N(\{y,z\})\backslash N(\{x,y,z\}) \\ z \succ_j^p x & \text{for } j \in N(\{x,z\})\backslash N(\{x,y,z\}). \end{cases}$$

Since $\{i\}$ is an $(x \succ y)$-decisive coalition, we see

$$x \succ^p y. \tag{3.7}$$

Note that $y \succ^p z$ since $y \succ^p_j z$ for all $j \in N(\{y,z\})$. This together with (3.7) implies $x \succ^p z$. Since $x \succ^p_i z$ and $z \succ^p_j x$ for all $j \in N(\{x,z\})\backslash\{i\}$, we conclude that $\{i\}$ is an $(x \succ z)$-decisive coalition.

(II) $\{i\}$ is a $(z \succ y)$-decisive coalition:

Next, consider a profile $q \in P$ such that

$$\begin{cases} z \succ^q_i x \succ^q_i y & \\ y \succ^q_j z \succ^q_j x & \text{for } j \in N(\{x,y,z\})\backslash\{i\} \\ y \succ^q_j x & \text{for } j \in N(\{x,y\})\backslash N(\{x,y,z\}) \\ y \succ^q_j z & \text{for } j \in N(\{y,z\})\backslash N(\{x,y,z\}) \\ z \succ^q_j x & \text{for } j \in N(\{x,z\})\backslash N(\{x,y,z\}). \end{cases}$$

Since $\{i\}$ is an $(x \succ y)$-decisive coalition, we have $x \succ^q y$. Furthermore, from Axiom (WPO), we also have $z \succ^q x$, and hence by the transitivity, we have $z \succ^p y$. Observe that $z \succ^q_i y$ and $y \succ^p_j z$ for all $j \in N(\{y,z\})\backslash\{i\}$. This means that $\{i\}$ is $(z \succ y)$-decisive coalition.

Now let v and w be two distinct alternatives. When $w \neq x$, $\{i\}$ is also an $(x \succ w)$-decisive coalition by the argument (I). Applying argument (II) we have that $\{i\}$ is a $(v \succ w)$-decisive coalition. When $w = x$, choose an arbitrary $z \in X\backslash\{x,y\}$. Then by argument (II), $\{i\}$ is a $(z \succ y)$-decisive coalition. Applying argument (II) and (I) repeatedly, we see that $\{i\}$ is a $(z \succ w)$-decisive coalition and then finally it is a $(v \succ w)$-decisive coalition. □

Proof of Theorem 3.1

Proof. We showed that there is an individual of $N(X)$ who alone forms a decisive coalition in Lemma 3.7. Let $\{i\}$ be a singleton decisive coalition for $i \in N(X)$. Take an arbitrary pair of distinct alternatives x and y, and a profile $p \in P$ such that $x \succ^p_i y$. We will show that $x \succ^p y$. Let

$$\begin{aligned} N_1 &= \{\, j \in N(\{x,y\})\backslash\{i\} \mid x \succ^p_j y \,\}, \\ N_2 &= \{\, j \in N(\{x,y\})\backslash\{i\} \mid y \succ^p_j x \,\}, \\ N_3 &= \{\, j \in N(\{x,y\})\backslash\{i\} \mid x \sim^p_j y \,\}, \text{ and} \\ N_4 &= N\backslash N(\{x,y\}). \end{aligned}$$

Choose an alternative $z \in X\backslash\{x,y\}$ arbitrarily, and consider the following profile

$q \in P$ such that

$$\begin{cases} x \succ_i^q z \succ_i^q y & \\ z \succ_j^q x \succ_j^q y & \text{for } j \in N_1 \cap (N(\{x,y,z\}) \backslash \{i\}) \\ x \succ_j^q y & \text{for } j \in N_1 \backslash N(\{x,y,z\}) \\ z \succ_j^q y \succ_j^q x & \text{for } j \in N_2 \cap N(\{x,y,z\}) \\ y \succ_j^q x & \text{for } j \in N_2 \backslash N(\{x,y,z\}) \\ z \succ_j^q x \sim_j^q y & \text{for } j \in N_3 \cap N(\{x,y,z\}) \\ x \succ_j^q y & \text{for } j \in N_3 \backslash N(\{x,y,z\}) \\ z \succ_j^q y & \text{for } j \in N_4 \cap N(\{y,z\}) \\ z \succ_j^q x & \text{for } l \in N_4 \cap N(\{x,z\}). \end{cases}$$

Note that $\succsim_j^p |\{x,y\} = \succsim_j^q |\{x,y\}$ for all $j \in N(\{x,y\})$. Since $\{i\}$ is an $(x \succ z)$-decisive coalition from Lemma 3.8, we see that $x \succ^q z$. By Axiom (WPO), we also have $z \succ^q y$. Then $x \succ^q y$ by the transitivity. Applying Axiom (IIA) we conclude that $x \succ^p y$, meaning that i is a dictator. □

4. Special Case

Ando, Ohara and Yamamoto [1] consider a social preference ordering in a situation of mutual evaluation. Each individual evaluates all individuals in the society but oneself. Namely, the set of alternatives coincides with the set of individuals in the society, $X = N$, and individual i's preference domain X_i is given by $X_i = N \setminus \{i\}$. They show an impossibility theorem in this situation. One of the crucial roles in their argument is played by the "cyclic profile" c which is the profile defined by

$$\begin{array}{ll} 2 \prec_1^c 3 \prec_1^c \cdots \prec_1^c n & \\ i+1 \prec_i^c i+2 \prec_i^c \cdots \prec_i^c n-1 \prec_i^c n \prec_i^c 1 \prec_i^c \cdots \prec_i^c i-1 & \text{for } i = 2, \cdots, n-1 \\ 1 \prec_n^c 2 \prec_n^c \cdots \prec_n^c n-1. & \end{array}$$

It is readily seen that assuming Axiom (WPO) would lead to a social preference $\succsim^c$ such that

$$1 \prec^c 2 \prec^c \cdots \prec^c n-1 \prec^c n \prec^c 1$$

which is not a weak ordering. Hence the social welfare function is impossible. They show that relaxing Axiom (WPO) in several ways would not lead to a positive result under Axioms (UDP) and (IIA). To exclude the controversial cyclic profile add an individual who is entitled to evaluate all the individuals in the society. Then from Theorem 3.1 we see that the social welfare function is dictatorial and the additional individual is a dictator.

5. Concluding Remarks

We consider a society where each individual's preference domain is restricted to a subset of the whole set of alternatives. We have shown the impossibility theorem that a social welfare function is always dictatorial whenever at least one individual has an unrestricted preference domain.

One of possible future research themes would be strategy-proofness (see [6, 13]). A natural question to answer would be "if a nonmanipulable voting is always dictatorial when individuals have restricted preference domain?" such as mutual evaluation situation.

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