INCENTIVES FOR INNOVATION UNDER DIFFERENTIATED GOODS MARKETS

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Abstract: *This paper explores how the speed of innovation adoption may differ between the Cournot (quantity-setting type) and Bertrand (price-setting type) differentiated goods markets. The extent of cost reduction is indexed by a parameter representing the ease of imitation. Innovation adoption occurs earlier (resp. later) in Cournot competition than in Bertrand competition if the degree of production differentiation is sufficiently high (resp. low). Social welfare at the optimum will be higher in Bertrand competition than in Cournot competition if the degree of production differentiation is sufficiently high.* (JEL Classification Codes: D43, L13 O31)

Key words: *Cournot and Bertrand duopoly, differentiated goods markets, innovation adoption*

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1. INTRODUCTION

Most firms in developing countries will often resort to imitation to succeed in business. Mansfield, Schwartz and Wagner [1981] reported that in spite of patent systems, new technologies tend to be imitated within a period as brief as four years. In an empirical study on the speed at which various kinds of information about new products and processes leak out, Mansfield [1985] found that a firm's information about new process or product innovations finds its way into the hands of rival firms within an average of 12 to 18 months.[1] Mansfield also made the interesting observation that differences in the rate of diffusion of technological information do not play an important role in explaining differences in the ease of imitation across industries. Two theoretical questions arise: First, how does the speed of imitation differ across industries or market structures? Second, does a more competitive market structure lead to earlier imitation? If we recognize differences in market structures across industries that Mansfield [1985] did not focus on, then the discrepancies in the speed of imitation (innovation diffusion) can simply be explained by these differences in market structures. Nonetheless, as far as we know, no theoretical approach to these questions has yet been attempted.

We explore how the speed of innovation adoption may differ between two common theoretical market structures, i.e., Cournot (quantity-setting) competition and Bertrand (price-setting) competition, and then compare the results with the social optimum. We first modify a model of Reinganum [1981a, 1981b] so that we can examine the speed of innovation adoption in a model of duopoly producing differentiated goods and then compare the results in both types of market competition. In our model, it is assumed that one firm has invented a new technology for its own use and a second firm can imitate (or adopt) the new technology at a cost which declines over time. The imitator may not achieve the same level of reduction in the unit cost of production as the inventor does. The extent of cost reduction is indexed by a parameter representing the ease of imitation.[2]

We shall first show that innovation adoption occurs earlier (resp. later) in Cournot competition than in

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[1] Mansfield [1981] noted that information about process innovation tends to leak out more slowly than information about product innovation because new processes can be developed with less communication and interaction with other firms than new products (ibid. p. 219).

[2] Mansfield, Schwartz and Wagner [1981] measured the ease of imitation by the ratio of imitation costs to innovation costs. By relating the ease of imitation to the imitator's ability to reduce the cost of production, we are able to analyze how the ease of imitation affects the incentives for imitation and social welfare.

Bertrand competition if the degree of production differentiation is sufficiently high (resp. low). The intuition supporting these results is as follows. The total effect of the innovation may be positive, but the direct and indirect effects are not uniformly positive in either market structure. Specifically, the direct effect is positive and higher in Bertrand competition, while the indirect effect is positive in Cournot competition and negative in Bertrand competition. Second, we shall show that in both competitions, the speed of innovation adoption is too slow for the first adopter relative to the social optimum, irrelevant of the degree of product differentiation. However, it is too slow (resp. too fast) for the second adopter relative to the social optimum if the degree of production differentiation is sufficiently high (resp. low). Furthermore, social welfare at the optimum will be higher in Bertrand competition than in Cournot competition if the degree of production differentiation is sufficiently high.

An adopting firm maximizes its own intertemporal profits, represented by the sum of the discount present value of the (temporal) market equilibrium profits, to choose the optimal adoption time. The timing issue arises because the cost of adoption is assumed to decline over time. The adopter takes into account the benefits and losses of delaying the adoption. The important point is that not the *total* profit but the *marginal* benefit determines the speed of innovation adoption that is different between the two market structures. It was often pointed out that Bertrand competition is inherently more competitive than Cournot competition because the former has a lower price and larger output.[3] This does not necessarily imply that Bertrand competition leads to stronger incentives for innovation, as proposed in the existing literature.

If two firms have symmetric demand functions and initially have equal unit costs of production, then one firm who first introduces a new (superior) technology to lower the cost can enjoy the larger market share than the other who still operates with an old (inferior) technology. When products are almost homogenous, the first firm can monopolize the market in Bertrand competition, while this may not the case in Cournot competition. This is why patterns of the speed of innovation adoption may differ between the two firms as well as across industries, which has not been pointed out in the existing literature. The second adopter can only have the smaller market share than the first adopter due to the inability of perfect imitation.

Most papers considering adoption of cost-reducing innovation have assumed that the inventor is not a member of the adopting industry and also assumed homogenous goods markets. This paper instead assumed that the inventor is a member of the industry and also assumed differentiated goods markets. Quirmbach [1986] compared the noncooperative and welfare-optimal diffusion rates in a model of Cournot oligopoly in a case of

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[3] See Singh and Vives [1984], Vives [1985] and Cheng [1985].

linear demand functions, assuming homogenous goods markets, and explored how the additional number of adopting firms influences the diffusion rate. However, Quirmbach did not take into account how the ease of imitation influences the adoption times and social welfare. Petrakis [1994], assuming differentiated goods markets, compared Cournot and Bertrand equilibrium adoption times with the socially optimal adoption time in a case of linear demand functions. However, Petrakis did not focus on the effect of imitation on the adoption timing as well as on social welfare.

The reminder of this paper is organized as follows. Section 2 constructs a basic model. The Cournot and Bertrand subgames are incorporated into a game of adoption of a new technology. Section 3 compares the optimal adoption times between Cournot and Bertrand market structures. Section 4 provides welfare analysis. Concluding remarks are given in Section 5.

2. A MODEL

We consider an economy with an imperfectly competitive sector, each firm producing a differentiated good, and a competitive numeraire sector. Following Dixit [1979] and Singh and Vives [1984], we define the utility for the representative consumer as $U(q_1, q_2, M) = V(q_1, q_2) + M$, where q_i ($i = 1, 2$) denotes firm i's output (good i) and M denotes a numeraire good. There are no income effects on the duopoly industry. We assume competitive consumers. Maximizing $U - p_1 q_1 - p_2 q_2$ with respect to q_i, where p_i denotes good i's price, we have $\partial U / \partial q_i = p_i$ ($i = 1, 2$), which defines the demand function for good i.

Direct demand systems are given by $q_i = d_i(p_i, p_j)$ ($i, j = 1, 2; i \neq j$). We assume that goods are gross substitutes, i.e., $\partial d_i / \partial p_j > 0$, the direct demand function for good i is downward sloping with respect to good i's price, i.e., $\partial d_i / \partial p_i < 0$, and the own effect is larger than the cross effect, i.e., $| \partial d_i / \partial p_i | > \partial d_i / \partial p_j > 0$. Inverse demand systems are given by $p_i = p_i(q_i, q_j)$. Taking the total differential of the direct demand function $q_i = d_i(p_i, p_j)$ with respect to output q_i and holding the above assumptions, we find that the inverse demand function is downward sloping with respect to each output, i.e., $\partial p_i / \partial q_i < 0$ and $\partial p_j / \partial q_i < 0$, and the own effect is larger than the cross effect, i.e., $| \partial p_i / \partial q_i | > | \partial p_i / \partial q_i |$. A detailed calculation can be found in part (I) of Appendix.

We denote the unit cost of production for firm i by c_i. We shall formulate momentary price (Bertrand) and quantity (Cournot) competitions, as follows:

(i) Bertrand competition: Firm i maximizes the profit $g_i(p_i, p_j, c_i) = (p_i - c_i)d_i(p_i, p_j)$ with respect to the price p_i, taking p_j as given. The first-order condition, given by $\partial g_i(p_i(p_j), p_j, c_i) / \partial p_i = 0$, defines the best-response

function $R_i^B(p_j) = p_i(p_j)$. We assume that the second-order condition is concave in own prices, i.e., $\partial^2 g_i/\partial p_i^2 < 0$. Then, the second-order condition, given by $\partial^2 g_i(p_i(p_j), p_j, c_i)/\partial p_i^2 < 0$, is satisfied. We also assume that prices are strategic complements, i.e., $\partial^2 g_i(p_i(p_j), p_j, c_i)/\partial p_i \partial p_j > 0$. The Bertrand-Nash market equilibrium is defined at a point where the two best-response curves intersect. We assume that the Bertrand-Nash equilibrium uniquely exists. A sufficient condition for uniqueness is $|dR_i^B(p_j)/dp_j| < 1$, or equivalently, $|\partial^2 g_i/\partial p_i^2| > \partial^2 g_i/\partial p_i \partial p_j > 0$.[4] The reaction functions are upward sloping since $dR_i^B(p_j)/dp_j = (-1)\{(\partial^2 g_i/\partial p_i^2)/(\partial^2 g_i/\partial p_i \partial p_j)\} > 0$ holds. The equilibrium is locally stable since $\Delta^B = (\partial^2 g_i/\partial p_i^2)(\partial^2 g_j/\partial p_j^2) - (\partial^2 g_i/\partial p_i \partial p_j)(\partial^2 g_j/\partial p_i \partial p_j) > 0$ holds. Henceforth we denote the market equilibrium prices by $(p_i^B, p_j^B) = (p_i(c_i, c_j), p_j(c_i, c_j))$ and the market equilibrium profits by $(\pi_i^B, \pi_j^B) = (g_i(p_i(c_i, c_j), p_j(c_i, c_j), c_i), g_j(p_i(c_i, c_j), p_j(c_i, c_j), c_j))$. It holds that $\partial^2 \pi_i^B/\partial p_i^2 = 2(\partial d_i/\partial p_i) + (p_i - c_i)(\partial^2 d_i/\partial p_i^2)$ (< 0) and $\partial^2 \pi_i^B/\partial p_i \partial p_j = \partial d_i/\partial p_j + (p_i - c_i)(\partial^2 d_i/\partial p_i \partial p_j)$ (> 0).

(ii) Cournot competition: Firm i maximizes the profit $f_i(q_i, q_j, c_i) = \{p_i(q_i, q_j) - c_i\}q_i$ with respect to the output q_i, taking q_j as given. The first-order condition, given by $\partial f_i(q_i(q_j), q_j, c_i)/\partial q_i = 0$, defines the best-response function $R_i^C(q_j) = q_i(q_j)$. We assume that the second-order condition is concave in own outputs, i.e., $\partial^2 f_i/\partial q_i^2 < 0$. Then, the second-order condition, given by $\partial^2 f_i(q_i(q_j), q_j, c_i)/\partial q_i^2 < 0$, is satisfied. We also assume that quantities are strategic substitutes, i.e., $\partial^2 f_i(q_i(q_j), q_j, c_i)/\partial q_i \partial q_j < 0$. The Cournot-Nash market equilibrium is defined at a point where the two best-response curves intersect. We assume that the Cournot-Nash equilibrium uniquely exists. A sufficient condition for uniqueness is $|dR_i^C(q_j)/dq_j| < 1$, or equivalently, $|\partial^2 f_i/\partial q_i^2| > |\partial^2 f_i/\partial q_i \partial q_j|$.[5] The reaction functions are downward sloping since $dR_i^C(q_j)/dq_j = (-1)\{(\partial^2 f_i/\partial q_i^2)/(\partial^2 f_i/\partial q_i \partial q_j)\} < 0$ holds. The equilibrium is locally stable since $\Delta^C = (\partial^2 f_i/\partial q_i^2)(\partial^2 f_j/\partial q_j^2) - (\partial^2 f_i/\partial q_i \partial q_j)(\partial^2 f_j/\partial q_i \partial q_j) > 0$ holds. Henceforth we denote the market equilibrium outputs by $(q_i^C, q_j^C) = (q_i(c_i, c_j), q_j(c_i, c_j))$ and the market equilibrium profits by $(\pi_i^C, \pi_j^C) = (f_i(q_i(c_i, c_j), q_j(c_i, c_j), c_i), f_j(q_i(c_i, c_j), q_j(c_i, c_j), c_j))$. It holds that $\partial^2 \pi_i^C/\partial q_i^2 = 2(\partial p_i/\partial q_i) + q_i(\partial^2 p_i/\partial q_i^2)$ (< 0) and $\partial^2 \pi_i^C/\partial q_i \partial q_j = \partial p_i/\partial q_j + q_i(\partial^2 p_i/\partial q_i \partial q_j)$ (< 0).

Let and assume that $\partial^2 d_i/\partial p_i^2 = x(p_1^B, p_2^B) \leq 0$, $\partial^2 d_i/\partial p_i \partial p_j = y(p_1^B, p_2^B) \geq 0$, $\partial^2 p_i/\partial q_i^2 = X(q_1^C, q_2^C) \leq 0$ and $\partial^2 p_i/\partial q_i \partial q_j = Y(q_1^C, q_2^C) \leq 0$. We also assume that $|x| \geq |y|$ and $|X| \geq |Y|$ so that the sufficient condition for uniqueness of the Bertrand-Nash equilibrium and that of the Cournot-Nash equilibrium should be satisfied, i.e., $|\partial^2 \pi_i^B/\partial p_i^2| > |\partial^2 \pi_i^B/\partial p_i \partial p_j|$ and $|\partial^2 \pi_i^C/\partial q_i^2| > |\partial^2 \pi_i^C/\partial q_i \partial q_j|$, respectively.

Henceforth we focus on the case where the demand functions are symmetric. Let $\partial d_1/\partial p_1 = \partial d_2/\partial p_2 =$

$a(p_1^B, p_2^B)$, $\partial d_1/\partial p_2 = \partial d_2/\partial p_1 = b(p_1^B, p_2^B)$, $\partial p_1/\partial q_1 = \partial p_2/\partial q_2 = A(q_1^C, q_2^C)$ and $\partial p_1/\partial q_2 = \partial p_2/\partial q_1 = B(q_1^C, q_2^C)$ (<0), where $|a| > |b|$ and $|A| > |B|$. We now introduce *the degree of product differentiation* by $\delta = (\partial d_1/\partial p_2)(\partial d_2/\partial p_1)/(\partial d_1/\partial p_1)(\partial d_2/\partial p_2) = B^2/A^2$, where $0 < \delta < 1$.[6] It follows that $b = (\sqrt{\delta})A\sqrt{\delta}$ since $a = A$ and $b = (\sqrt{\delta})B$. In Bertrand competition, we have $\partial^2\pi_i^B/\partial p_i^2 = 2a$ $(x/a)d_i$ (<0) and $\partial^2\pi_i^B/\partial p_i\partial p_j = a\sqrt{\delta}$ $(y/a)d_i$ (>0). In Cournot competition, we have $\partial^2\pi_i^C/\partial q_i^2 = 2A + Xq_i$ (<0) and $\partial^2\pi_i^C/\partial q_i\partial q_j = A\sqrt{\delta} + Yq_i$ (<0). Henceforth we assume that the unit costs of production are initially identical. Then, it holds at $(c_1, c_2) = (c, c)$ that $d_1 = d_2 > 0$ and $q_1 = q_2 > 0$.

Singh and Vives [1984] and Vives [1984] showed the results, summarized in the following theorem:[7]

Theorem 1. *The outputs are larger more in Bertrand competition than in Cournot competition.*

Theorem 1 means in our context that $d_i > q_i$.

We suppose that firm 1 is an inventor who developed a new technology and firm 2 an imitator. We also suppose that each firm can reduce the current unit cost of production c_i to $c_i - z_i$ by adopting a new technology at time $t = \tau_i \in [0, \infty)$. We assume that firm 2 can imperfectly imitate the technology so that its cost-reduction size is smaller than firm 1's. We denote a parameter that represents the ease of imitation by θ, so that $z_1 = z$ and $z_2 = \theta z$, where $\theta \in (0, 1]$. We denote the cost of adoption by $N(t)$. We assume that the cost declines over time but at a decreasing rate:

Assumption 1. $dN(t)/dt < 0$, $d^2N(t)/dt^2 > 0$

Justification of Assumption 1 is that the technology, although too expensive at the initial stage, will be improved due to ongoing basic research, which reduces these costs.[8]

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[6] Singh and Vives [1984] and Shy [1995] defined it in the case of linear demand functions. The degree of product differentiation can be rewritten, by incorporating own-price elasticity of demand $\varepsilon_{ii} = (p_i/d_i)(\partial d_i/\partial p_i) < 0$ and cross-price elasticity of demand $\varepsilon_{ij} = (p_j/d_i)(\partial d_i/\partial p_j) > 0$ $(i, j = 1, 2; i \neq j)$, as $\delta = \varepsilon_{12}\varepsilon_{21}/\varepsilon_{11}\varepsilon_{22}$. A higher (resp. lower) δ means a lower (resp. higher) $\varepsilon_{12}\varepsilon_{21}$ or a higher (resp. lower) $\varepsilon_{11}\varepsilon_{22}$.

[7] See Singh and Vives [1984, Proposition 3] and Vives [1985, Proposition 1].

[8] See Katz and Shapiro [1987].

We suppose that firm 1, anticipating the imitation subgame, develops a new technology at τ_1, and firm 2, taking the development time as given, adopts the technology at τ_2. Both firms maximize the intertemporal profits, defined by the present discounted value of the temporal profits, with respect to time. The intertemporal profit for firm 1 is

$$\Pi_1 = \int_0^{\tau_1} e^{-\rho t}\, \pi_1(c_1, c_2)\, dt \; + \int_{\tau_1}^{\tau_2} e^{-\rho t}\, \pi_1(c_1 - \delta_1, c_2)\, dt \; + \int_{\tau_2}^{\infty} e^{-\rho t}\, \pi_1(c_1 - \delta_1, c_2 - \delta_2)\, dt$$

$$- e^{-\rho \tau_1} N(\delta_1)$$

where ρ is a market interest rate. The intertemporal profit for firm 2 is

$$\Pi_2 = \int_0^{\tau_1} e^{-\rho t}\, \pi_2(c_1, c_2)\, dt \; + \int_{\tau_1}^{\tau_2} e^{-\rho t}\, \pi_2(c_1 - \delta_1, c_2)\, dt \; + \int_{\tau_2}^{\infty} e^{-\rho t}\, \pi_2(c_1 - \delta_1, c_2 - \delta_2)\, dt$$

$$- e^{-\rho \tau_2} N(\delta_2)$$

The first-order conditions for maximizing the intertemporal profits, which define the privately optimal adoption times, are given by

(1) $\rho N(\delta_1^*) - dN(\delta_1^*)/d\delta_1 = \pi_1(c_1 - \delta_1, c_2) - \pi_1(c_1, c_2)$

(2) $\rho N(\delta_2^*) - dN(\delta_2^*)/d\delta_2 = \pi_2(c_1 - \delta_1, c_2 - \delta_2) - \pi_2(c_1 - \delta_1, c_2)$

The second-order conditions are satisfied under Assumption 1, which implies that the objective functions for both firms are concave with respect to time.

In regard to (1) and (2), both firms equalize the marginal cost of innovation, represented by the left-hand sides, with the (private) marginal benefit from innovation, represented by the right-hand sides. Note that the marginal cost of innovation is decreasing, while the marginal benefit from innovation is constant, over time. The privately optimal adoption time for each firm is given at a point where the two curves meet. We assume that

Assumption 2. $\rho N(0) - dN(0)/dt = +\infty$, $\rho N(\infty) - dN(\infty)/dt = 0$

Under Assumption 2, we can eliminate two corner-solution cases:[9] (a) The marginal benefit from innovation is higher than the marginal cost of innovation at $t = 0$. (b) The marginal cost of innovation is higher than the marginal benefit from innovation for all t. In case (a), firms may adopt immediately, although they could gain more by waiting a little moment. In case (b), on the contrary, they may not adopt. Thus, we have a unique interior solution since the marginal benefit from innovation should be positive.[10]

3. THE INCENTIVES FOR INNOVATION

Clearly, the higher is the marginal benefit from innovation, the earlier is the privately optimal adoption time for the imitator. In this sense, we measure the incentive for innovation by the increment of the adopter's equilibrium profit, as Bester and Petrakis [1993] did. In this section we compare the incentives for innovation and show how the *privately* optimal adoption times differ between Cournot and Bertrand market structures.

We have $\{ \pi_i^B(c_i, c_{-i}, .) - \pi_i^B(c, .)\} - \{ \pi_i^C(c_i, c_{-i}, .) - \pi_i^C(c, .)\} = \{ \pi_i^B(c_i, c_{-i}, .) - \pi_i^C(c_i, c_{-i}, .)\} - \{ \pi_i^B(c, .) - \pi_i^C(c, .)\}$ for a given c_j. Then, by The Mean Value Theorem there exists some $m \in (c_i, c_{-i}, c_i)$ that satisfies $[\{ \pi_i^B(c_i, c_{-i}, .) - \pi_i^C(c_i, c_{-i}, .)\} - \{ \pi_i^B(c, .) - \pi_i^C(c, .)\}]/(c_{-i}) = d\{ \pi_i^B(m, .) - \pi_i^C(m, .)\}/dc_i$. Thus, we have only to explore the sign of $d\{ \pi_i^B(m, .) - \pi_i^C(m, .)\}/dc_i$.

We have the following proposition:

Proposition 1. *Innovation adoption occurs for both firms earlier (resp. later) in Cournot competition than in Bertrand competition if products are highly differentiated (resp. almost homogenous), that is, the degree of product differentiation is close to zero (resp. unity).[11]*

Proof. It holds that $d\pi_i^B/dc_i = \partial\pi_i^B/\partial c_i + (\partial\pi_i^B/\partial p_j)(\partial p_j^B/\partial c_i) = d_i[\ a\sqrt{\delta} - \{ a\sqrt{\delta} - (y/a)d_i\}/\beta^B - 1]$, where $\partial\pi_i^B/\partial c_i = (-d_i)$ by the Hotelling's lemma and $\partial p_j^B/\partial c_i = (-1/\beta^B)(\partial d_i/\partial p_i)(\partial^2\pi_j^B/\partial p_i\partial p_j) > 0$,

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[9] Reinganum [1989] employed the same assumption (ibid. Assumption 11).

[10] In view of (1) and (2), both τ_1^* and τ_2^* are determined as an intersection point of the two reaction functions in $\tau_1 - \tau_2$ plane, satisfying $\tau_1^* < \tau_2^*$. Since τ_1^* and τ_2^* are independent of each other, a pair of the privately optimal adoption times (τ_1^*, τ_2^*) constitutes the Nash equilibrium adoption times.

[11] The definition of 'highly differentiated or almost homogenous products' is owed by Shy [1995, pp. 136-137].

and d $\pi_i^C/dc_i = \partial\pi_i^C/\partial c_i + (\partial\pi_i^C/\partial q_j)(\partial q_j^C/\partial c_i) = q_i\{A\sqrt{\delta}(A\sqrt{\delta} + Yq_i)/\gamma^C - 1\}$, where $\partial\pi_i^C/\partial c_i = (-q_i)$ by the Hotelling's lemma and $\partial q_j^C/\partial c_i = (-1/\gamma^C)(\partial^2\pi_j^C/\partial q_i\partial q_j) > 0$. With δ close to 0, $|d\pi_i^B/dc_i| = d_i > |d\pi_i^C/dc_i| = q_i$ by Theorem 1. Thus, in view of (1) and (2), the optimal adoption time is solved as an interior solution in both competitions: $(0 <) \tau_i^{*B} < \tau_i^{*C} < +\infty$. With δ close to 1, $|d\pi_i^B/dc_i| = 0 < |d\pi_i^C/dc_i| = q_i$. Thus, in view of (1) and (2), the optimal adoption time is solved as an interior solution in Cournot competition but is solved as a corner solution in Bertrand competition: $(0 <) \tau_i^{*C} < \tau_i^{*B} = +\infty$. *Q.E.D.*

Bester and Petrakis [1993] and Petrakis [1994] showed in a model of linear demand functions where products are sufficiently substitutable that the incentives for innovation are stronger in Cournot competition than in Bertrand competition. This result implies that innovation adoption occurs earlier in Cournot competition than in Bertrand competition. Petrakis [1994] had the same results, except that only the linear demand case was explored and the ease of imitation was not taken into account. Note that we have shown the result in Proposition 1 beyond the case of linear demand functions.

Let us now provide the intuition of Proposition 1. By the Envelope Theorem, we have d $\pi_i/dc_i = \partial\pi_i/\partial c_i + (\partial\pi_i/\partial x_j)(\partial x_j/\partial c_i)$, so that d $\pi_i/d(\tau c_i) = \partial\pi_i/\partial(\tau c_i) + (\partial\pi_i/\partial x_j)\{\partial x_j/\partial(\tau c_i)\}$. The total effect of a cost-reducing innovation, d $\pi_i/d(\tau c_i)$, is decomposed into two effects: the direct effect or *the cost-minimizing effect*, $\partial\pi_i/\partial(\tau c_i)$, and the indirect effect or *the strategic effect*, $(\partial\pi_i/\partial x_j)\{\partial x_j/\partial(\tau c_i)\}$, where the valuable x represents the market equilibrium output q or price p.[12] First, by the Hotelling's lemma and Theorem 1, we have $\partial\pi_i^B/\partial(\tau c_i) = d_i > \partial\pi_i^C/\partial(\tau c_i) = q_i$. Hence, the sign of the cost-minimizing effect is positive in both competitions and is higher in Bertrand competition than in Cournot competition. Second, it holds that $\partial p_j^B/\partial c_i = (-1/\gamma^B)(\partial d_i/\partial p_i)(\partial^2\pi_j^B/\partial p_i\partial p_j) = (\partial p_j^B/\partial p_i^B)(\partial p_i^B/\partial c_i) = \{dR_j^B(p_i)/dp_i\}(\partial p_i^B/\partial c_i) > 0$ since $\partial p_i^B/\partial c_i = (1/\gamma^B)(\partial d_i/\partial p_i)(\partial^2\pi_j^B/\partial p_j^2) > 0$ and $\partial q_j^C/\partial c_i = (-1/\gamma^C)(\partial^2\pi_j^C/\partial q_i\partial q_j) = (\partial q_j^C/\partial q_i^C)(\partial q_i^C/\partial c_i) = \{dR_j^C(q_i)/dq_i\}(\partial q_i^C/\partial c_i) > 0$ since $\partial q_i^C/\partial c_i = (1/\gamma^C)(\partial^2\pi_j^C/\partial q_j^2) < 0$. Since (by symmetry) sign$(\partial\pi_i^B/\partial p_j)$ = sign$(\partial\pi_j^B/\partial p_i) > 0$ and sign$(\partial\pi_i^C/\partial q_j)$ = sign$(\partial\pi_j^C/\partial q_i) < 0$, we have sign$[(\partial\pi_i^B/\partial p_j)\{\partial p_j^B/\partial(\tau c_i)\}]$ = (-1)sign$[(\partial\pi_j^B/\partial p_i)(\partial p_i^B/\partial c_i)\{dR_j^B(p_i)/dp_i\}] < 0$. On the other hand, sign$[(\partial\pi_i^C/\partial q_j)\{\partial q_j^C/\partial(\tau c_i)\}]$ = (-1)sign$[(\partial\pi_j^C/\partial q_i)(\partial q_i^C/\partial c_i)\{dR_j^C(q_i)/dq_i\}] > 0$. Hence, the sign of the strategic effect is positive in the case of strategic substitutes and negative in the case of strategic complements. In the case of strategic complements, the direct effect should be larger than the indirect effect in total. As Proof of Proposition 1 suggests, if the degree of

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[12] See Brander and Spencer [1983] and Tirole [1988].

product differentiation   is close to zero (resp. unity), then the strategic effect will be smaller (resp. larger) than the cost-minimizing effect.

Bester and Petrakis [1993] referred to a 'market share effect' reflecting how differences in market structure can influence the incentives for innovation through determination of the equilibrium output. They showed that this effect is stronger with closely substitutable products—hence the incentives for innovation are stronger—in Bertarnd competition than in Cournot competition. Qiu [1997] pointed out that the market share effect is less important because the identicalness of all players makes both the pre-innovation and post-innovation equilibria symmetric. Qiu also showed that the incentives for innovation are stronger in Cournot competition than in Bertarnd competition, regardless of substitutability of the products. However, these papers were modeled in timeless settings.

4. WELFARE ANALYSIS

How does an increase in the ease of imitation affect social welfare? How is the speed of imitation evaluated from the social optimality viewpoint? We shall analyze these interesting problems and make the relevant discussions.[13] We denote consumer's surplus by   and producer's surplus by   . Total surplus is denoted by   =   +   . Note that in regard to consumer's surplus, $^B = U(d_1(p_1{}^B, p_2{}^B), d_2(p_1{}^B, p_2{}^B), M)$ $p_1{}^B d_1(p_1{}^B, p_2{}^B)$ $p_2{}^B d_2(p_1{}^B, p_2{}^B)$ and $^C = U(q_1{}^C, q_2{}^C, M)$ $p_1(q_1{}^C, q_2{}^C)q_1{}^C$ $p_2(q_1{}^C, q_2{}^C)q_2{}^C$. We define social welfare by the (net) present discounted value of total surplus, as follows:[14]

$$
= \int_0^{\tau_1} e^{-\rho t} \quad (c_1, c_2)\,\mathrm{d}t + \int_{\tau_1}^{\tau_2} e^{-\rho t} \quad (c_1 \quad {}_1, c_2)\,\mathrm{d}t + \int_{\tau_2} e^{-\rho t} \quad (c_1 \quad {}_1, c_2 \quad {}_2)\,\mathrm{d}t
$$

$$
e^{-\rho \tau_1} N(\quad {}_1) \quad e^{-\rho \tau_2} N(\quad {}_2)
$$

A social planner maximizes   with respect to $_i$ on behalf of firm i and compares the privately optimal adoption time(s) $_i{}^*$ with socially optimal adoption time(s) $_i{}^{**}$.

---

[13] Barzel [1968] compared a monopoly regime with a competition regime in regard to the speed of innovation adoption and evaluated it from the viewpoint of social optimality.

[14] Quirmbach [1986] defined social welfare as post-innovation total surplus.

The first-order conditions for maximizing social welfare, which define the socially optimal adoption times, τ_i^{**}, are given by

(3) $N(\tau_1^{**})\ dN(\tau_1^{**})/d\tau_1 =\ (c_1 - \gamma_1, c_2)\ (c_1, c_2)$

(4) $N(\tau_2^{**})\ dN(\tau_2^{**})/d\tau_2 =\ (c_1 - \gamma_1, c_2 - \gamma_2)\ (c_1 - \gamma_1, c_2)$

In regard to (3) and (4), the right-hand sides show the *social* marginal benefits from innovation and the left-hand sides show the *social* marginal costs of innovation. Henceforth we focus on the case where the right-hand sides are positive.

We explore how an increase in the ease of imitation affects on social welfare. Taking the total differential of with respect to , we have $d\ /d\ = (\ /\ _2)(\ _2^{**}/\ ) +\ /\ $, where the second term in the right-hand side vanishes since by (4) it holds that $/\ = e^{-\rho\tau_2}\,[\{\ N(\ _2)\ dN(\ _2)/d\ _2\}\ \{\ (c_1 - \gamma_1,$ $c_2 - \gamma_2)\ (c_1 - \gamma_1, c_2)\}] = 0$ at $_2 = \ _2^{**}$. Thus, the total effect an increase in the ease of imitation, $d\ /d\ $, consists of the indirect effect alone, $(\ /\ _2)(\ _2^{**}/\ )$.

We proceed to evaluate the privately optimal adoption times in both competitions from the social optimality viewpoint. We have the following theorem:

Theorem 2. $/\ _2 < $ *(resp. >) 0 if and only if* $d\{\ (c_1 - \gamma_1, k) +\ _1(c_1 - \gamma_1, k)\}/dc_2 < $ *(resp. >) 0 at* $(c_1, c_2) = (c\ , k^B)$ *where k is such a value in* $(c_2 - \gamma_2, c_2)$ *that satisfies* $\{\ (c_1 - \gamma_1, c_2 - \gamma_2) +\ _1(c_1 - \gamma_1, c_2 - \gamma_2)\}\ \{\ (c_1 - \gamma_1, c_2) +\ _1(c_1 - \gamma_1, c_2)\} = (\ _2)d\{\ (c_1 - \gamma_1, k) +\ _1(c_1 - \gamma_1, k)\}/dc_2.$

A proof can be found in part (II) of Appendix. Theorem 2 implies that at the privately optimal adoption time $_2 = \ _2^{*}$, social welfare increases (resp. decreases) if an only if consumer will benefit more (resp. less) than the inventor will lose. In like manner, we can show that at $(c_1, c_2) = (c\ , s)$, where s is such a value in $(c_2 - \gamma_2, c_2)$ that satisfies $\{\ (c_1 - \gamma_1, c_2) +\ _2((c_1 - \gamma_1, c_2)\}\ \{\ (c_1, c_2) +\ _2(c_1, c_2)\} = (\ _1)[d\{\ (s, c_2) +\ _2(s, c_2)\}/dc_1]$, it holds that $/\ _1 < $ (resp. >) 0 if and only if $d\{\ (s, c_2) +\ _2(s, c_2)\}/dc_1 < $ (resp. >) 0.

By Theorem 2, we can show whether the privately optimal adoption time for firm 2 will be earlier or later than the socially optimal adoption time in both competitions. (i) First, $d(\ ^B +\ _1^B)/dc_2 = (1/\ ^B)\{a^2\sqrt{\delta}\ d_1$ $2a^2 d_2 + x d_1 d_2\ (x\sqrt{\delta} + y)(d_1)^2\}$. It holds that $d_1 > d_2 = 0$ at $(c_1, c_2) = (c\ , k^B)$, where $c\ < k^B < c$. That is, firm 1 can capture all the market since it can produce with the superior (new) technology while firm 2 still does

with the inferior (current) technology. With close to 0, we have $d(\pi^B + \pi_1^B)/dc_2 < 0$. Hence, the privately optimal adoption time is *later* than the socially optimal adoption time: $\tau_2^{*B} > \tau_2^{**B}$. With close to 1, on the other hand, we have $d(\pi^B + \pi_1^B)/dc_2 > 0$. Hence, the privately optimal adoption time is *earlier* than the socially optimal adoption time: $\tau_2^{*B} < \tau_2^{**B}$. (ii) Second, $d(\pi^C + \pi_1^C)/dc_2 = (1/\Delta^C)\{A^2\sqrt{\delta}\{(2 - \sqrt{\delta})q_2 + q_1\} - B(X - Y)q_1 q_2\}$. It holds that $q_1 > q_2 > 0$ at $(c_1, c_2) = (c\ , k^C)$, where $c\ < k^C < c$. With close to 0, we have $d(\pi^C + \pi_1^C)/dc_2 < 0$. Hence, the privately optimal adoption time is *later* than the socially optimal adoption time: $\tau_2^{*C} > \tau_2^{**C}$. With highly substitutable products, it holds that $d(\pi^C + \pi_1^C)/dc_2 > 0$. Hence, the privately optimal adoption time is *earlier* than the socially optimal adoption time: $\tau_2^{*C} < \tau_2^{**C}$.

In sum up, we have the following proposition:

Proposition 2. *The speed of innovation for the second adopter (i.e., imitator) is too slow (resp. too fast) relative to the social optimum in both Bertrand and Cournot duopoly markets if the degree of product differentiation is close to zero (resp. unity).*

How are the privately optimal adoption times for the first adopter in both competitions evaluated from the social optimality viewpoint? It holds that $d_1 > d_2 = 0$ at $(c_1, c_2) = (c\ , s^B)$, where $c\ < s^B < c$, and $q_1 > q_2 > 0$ at $(c_1, c_2) = (c\ , s^C)$, where $c\ < k^C < c$. (iii) First, $d(\pi^B + \pi_2^B)/dc_1 = (1/\Delta^B)\{a^2\sqrt{\delta}\,d_2 - 2a^2 d_1 + x d_1 d_2 - (x\sqrt{\delta} + y)(d_2)^2\} < 0$, which implies that $\partial\pi^B/\partial\tau_1 < 0$. Hence, the privately optimal adoption time is *later* than the socially optimal adoption time: $\tau_1^{*B} > \tau_1^{**B}$. (iv) Second, $d(\pi^C + \pi_2^C)/dc_1 = (1/\Delta^C)\{A^2\sqrt{\delta}\{(2 - \sqrt{\delta})q_1 + q_2\} - B(X - Y)q_1 q_2\} < 0$, which implies that $\partial\pi^C/\partial\tau_1 < 0$. Hence, the privately optimal adoption time is *later* than the socially optimal adoption time: $\tau_1^{*C} > \tau_1^{**C}$. These results hold irrelevant of the degree of product differentiation.

In sum up, we have the following proposition:

Proposition 3. *The speed of innovation for the first adopter is too slow relative to the social optimum, irrelevant of the degree of product differentiation, in both Bertrand and Cournot duopoly markets.*

Figure 1 illustrates the results in Proposition 2 and Proposition 3.

Quirmbach [1986] had the same results in the case of Cournot competition with homogeneous goods where the number of adopting firms is two and the demand functions are linear (ibid. Proposition 8). Petrakis [1994] also

had the same results in the case of Bertrand and Cournot duopoly with differentiated goods where the demand functions are linear.

It is of quite interest to consider why the privately optimal adoption time can be *earlier* than the socially optimal adoption time in spite of the existence of market power and the resulting distortion. In such a case, the marginal benefit will be more for the imitator than for the consumer, and the marginal benefit for the consumer will be less than marginal loss for the inventor. Moreover, it is not the consumer but the imitator who will appropriate the increment of the social benefit. As a result, the private marginal benefit from innovation may be larger than the social marginal benefit from innovation.

Taking the total differential of (4) and rearranging terms, we have $\partial z_2^{**}/\partial\theta < 0$ under Assumption 1. That is, an increase in the ease of imitation accelerates innovation for the second adopter. With δ close to 0, we have $\partial\pi^B/\partial z_2 < 0$ and $\partial\pi^C/\partial z_2 < 0$. Since $d\pi^B/d\theta = (\partial\pi^B/\partial z_2)(\partial z_2^{**B}/\partial\theta) > 0$ and $d\pi^C/d\theta = (\partial\pi^C/\partial z_2)(\partial z_2^{**C}/\partial\theta) > 0$, an increase in the ease of imitation *improves* social welfare in both competitions. With δ close to 1, we have in this case $\partial\pi^B/\partial z_2 > 0$ and $\partial\pi^C/\partial z_2 > 0$. Since $d\pi^B/d\theta = (\partial\pi^B/\partial z_2)(\partial z_2^{**B}/\partial\theta) < 0$ and $d\pi^C/d\theta = (\partial\pi^C/\partial z_2)(\partial z_2^{**C}/\partial\theta) < 0$, an increase in the ease of imitation *worsens* social welfare in both competitions. We have the following proposition:

Proposition 4. *An increase in the ease of imitation improves (resp. worsens) social welfare in both Bertrand and Cournot duopoly markets if products are highly differentiated (resp. substitutable).*

Let us now compare social welfare at the optimum in both competitions. We first explore the effect on producer's surplus and then the effect on consumer's surplus.

First, in which competition does producer's surplus increase more due to the unit cost reduction by one firm? That is, whether is larger, $|d\pi^B/dc_i|$ or $|d\pi^C/dc_i|$? We have the following theorem:

Theorem 3. $|d\pi^B/dc_i| > |d\pi^C/dc_i|$ *with δ close to zero.*

Proof. We obtain $d\pi^B/dc_i = d(\pi_i^B + \pi_j^B)/dc_i = d\pi_i^B/dc_i + d\pi_j^B/dc_i = (1/\pi^B)\{a^2\ d_i + 2a^2\sqrt{\delta}\ d_j\ x\sqrt{\delta}\ (d_j)^2 + y\sqrt{\delta}\ d_id_j\ \pi^Bd_i\}$, where $d\pi_i^B/dc_i = \partial\pi_i^B/\partial c_i + (\partial\pi_i^B/\partial p_j)(\partial p_j^B/\partial c_i) = d_i[\ a\sqrt{\delta}\ \{\ a\sqrt{\delta}\ (y/a)d_i\}/\pi^B\ 1]$ and $d\pi_j^B/dc_i = (\partial\pi_j^B/\partial p_i)(\partial p_i^B/\partial c_i) = (1/\pi^B)[a\sqrt{\delta}\ \{2a\ (x/a)d_j\}d_j] > 0$, and $d\pi^C/dc_i = d(\pi_i^C + \pi_j^C)/dc_i = d\pi_i^C/dc_i + d\pi_j^C/dc_i = (1/\pi^C)\{\ A^2\ q_i + 2A^2\sqrt{\delta}\ q_j + AX\sqrt{\delta}\ (q_j)^2\ AY\sqrt{\delta}\ q_iq_j\ \pi^Cq_i\}$,

where $d\pi_i^C/dc_i = (\partial\pi_i^C/\partial c_i) + (\partial\pi_i^C/\partial q_j)(\partial q_j^C/\partial c_i) = q_i\{ A\sqrt{\delta}\,(A\sqrt{\delta} + Yq_i)/\Omega^C - 1\}$ and $d\pi_j^C/dc_i = (\partial\pi_j^C/\partial q_i)(\partial q_i^C/\partial c_i) = (1/\Omega^C)[A\sqrt{\delta}\,(2A + Xq_j)q_j] > 0$. With θ close to 0, therefore, we have $|d\pi^B/dc_i| = d_i > |d\pi^C/dc_i| = q_i$ by Theorem 1. *Q.E.D.*

According to Theorem 3, if the degree of product differentiation is sufficiently high, then producer's surplus increases more in Bertrand competition than in Cournot competition.

Second, in which competition does consumer's surplus increase more due to the unit cost reduction by one firm? That is, which is larger, $|d\pi^B/dc_i|$ or $|d\pi^C/dc_i|$? We have the following theorem:

Theorem 4. *Assume that $|x|$ and $|X|$ are sufficiently small. Then, $|d\pi^B/dc_i| > |d\pi^C/dc_i|$ for all $0 \le \theta < 1$.*

Proof. We obtain $d\pi^B/dc_i = (\partial\pi/\partial d_i)(dp_i^B/dc_i) + (\partial\pi/\partial d_j)(dp_j^B/dc_i) = (1/\Omega^B)\{ -2a^2 d_i - a^2\sqrt{\delta}\,d_j + xd_id_j - y\sqrt{\delta}\,(d_j)^2\} < 0$, where $dp_i^B/dc_i = (1/\Omega^B)(2a^2 - xd_j) > 0$ and $dp_j^B/dc_i = (1/\Omega^B)(a^2\sqrt{\delta} + yd_j) > 0$. We also obtain $d\pi^C/dc_i = (\partial\pi/\partial q_i)(dp_i^C/dc_i) + (\partial\pi/\partial q_j)(dp_j^C/dc_i) = (1/\Omega^C)\{ -A^2(2-\theta)q_i - A^2\sqrt{\delta}\,q_j - (AX - AY\sqrt{\delta}\,)q_iq_j - (AX\sqrt{\delta} - AY)(q_j)^2\} < \text{or} > 0$, where $dp_i^C/dc_i = (\partial p_i/\partial q_j)(dq_j^C/dc_i) + (\partial p_i/\partial q_i)(dq_i^C/dc_i) = (1/\Omega^C)\{ A\sqrt{\delta}\,(A\sqrt{\delta} + Yq_j) + A(2A + Xq_j)\} > 0$ and $dp_j^C/dc_i = (\partial p_j/\partial q_i)(dq_i^C/dc_i) + (\partial p_j/\partial q_j)(dq_j^C/dc_i) = (1/\Omega^C)\{A\sqrt{\delta}\,(2A + Xq_j) - A(A\sqrt{\delta} + Yq_j)\} > \text{or} < 0$. However, with θ close to 0 (resp. < 1), it holds that $dp_j^C/dc_i < (\text{resp.} >)\ 0$. If the demand functions are less convex, that is, $|x|$ and $|X|$ are sufficiently small, then, substituting from $a = \theta A$, we have $d\pi^B/dc_i \approx (-1/\Omega^C)(2A^2 d_i + A^2\sqrt{\delta}\,d_j) < 0$ and $d\pi^C/dc_i \approx (-1/\Omega^C)\{(2-\theta)A^2 q_i + A^2\sqrt{\delta}\,q_j\} < 0$, hence $|d\pi^B/dc_i| > |d\pi^C/dc_i|$ for all $0 \le \theta < 1$ by Theorem 1. *Q.E.D.*

According to Theorem 4, consumer's surplus increases more in Bertrand competition than in Cournot competition, no matter how the degree of product differentiation may be.

In sum up, $|d\Omega^B/dc_i| > |d\Omega^C/dc_i|$ since $d\Omega^B/dc_i = d\pi^B/dc_i + d\pi^B/dc_i$ and $d\Omega^C/dc_i = d\pi^C/dc_i + d\pi^C/dc_i$. Formally, we have the following theorem:

Theorem 5. *Assume that $|x|$ and $|X|$ are sufficiently small. Then, $|d\Omega^B/dc_i| > |d\Omega^C/dc_i|$ with θ close to zero.*

A formal proof can be found in part (III) of Appendix.

According to Theorem 5, total surplus increases more in Bertrand competition than in Cournot competition if

the demand functions are less convex and the degree of product differentiation is sufficiently high. The socially optimal adoption times are then solved as interior solutions and they are earlier in Bertrand competition than in Cournot competition, i.e., $(0 <)$ $\tau_i^{**B} <$ $\tau_i^{**C} (< + \infty)$.

Hence, we have the following proposition:

Proposition 5. *Social welfare at the optimum will be higher in Bertrand competition than in Cournot competition if the degree of production differentiation is sufficiently high.*

A proof can be found in part (IV) of Appendix.

Singh and Vives [1984], Vives [1985], Cheng [1985] and Qiu [1997] showed that Bertrand competition is more efficient than Cournot competition in the sense that the price is lower and the output is larger, which implies that total surplus is higher in Bertrand competition than in Cournot competition. Proposition 5 is an extension of their analyses to the framework in which the timing issue arises.

5. CONCLUDING REMARKS

We have explored the differences between the Cournot and Bertrand market structures in the speed of innovation adoption and then evaluated the results from the social optimality viewpoint. We have pointed out that market structure (or industry) patterns may generate the differences. This point sharply contrasts with Mansfield [1985] that found no significant differences across industries.

Imitation increases consumer's surplus as well as producer's surplus. We have shown that in highly differentiated industries, the consumer will benefit more than the innovator (leader) will lose. As a result, the private marginal benefits from innovation for the imitator (follower) will be larger than the social marginal benefits from innovation. The privately optimal adoption times for the follower will be earlier than the socially optimal adoption times and vice versa. We have also shown that innovation adoption will be worth more socially in Bertarnd competition than in Cournot competition.

In both market structures (industries), if products are highly differentiated, then the private marginal benefit from innovations for both firms are larger than the social marginal benefits from innovation. If products are almost homogenous, then the private marginal benefit from innovation is smaller for the innovator (leader) and larger for the imitator (follower) than the social marginal benefit from innovation. A social planner should force

the follower to *pay a tax* in the case of homogenous goods industries, as Petrakis [1994] suggested, or alternatively, to *pay a fee* to the leader, at which level the private and social marginal benefits from innovation are equalized. With payment, the privately optimal adoption time would be earlier for the leader but later for the follower than that without payment. The socially optimal adoption time, however, would not be affected.

APPENDIX

(I) *Properties of the inverse demand functions* Taking the total differential of the direct demand systems $q_i = d_i(p_i, p_j)$ $(i, j = 1, 2; i \neq j)$, we have $\partial p_1/\partial q_1 = (1/\Delta)(\partial d_2/\partial p_2) < 0$, $\partial p_1/\partial q_2 = (-1/\Delta)(\partial d_1/\partial p_2) < 0$, $\partial p_2/\partial q_1 = (-1/\Delta)(\partial d_2/\partial p_1) < 0$ and $\partial p_2/\partial q_2 = (1/\Delta)(\partial d_1/\partial p_1) < 0$, where $\Delta = (\partial d_1/\partial p_1)(\partial d_2/\partial p_2) - (\partial d_1/\partial p_2)(\partial d_2/\partial p_1) > 0$.

(II) *Proof of Theorem 2* It follows that $\partial \theta/\partial \tau_2 = e^{-\rho\tau_2^{**}}[\{\pi(c_1 - \delta_1, c_2 - \delta_2) + \pi_1(c_1 - \delta_1, c_2 - \delta_2)\} - \{\pi(c_1 - \delta_1, c_2) + \pi_1(c_1 - \delta_1, c_2)\}]$. There exists some $k \in (c_2 - \delta_2, c_2)$ that satisfies $\{\pi(c_1 - \delta_1, c_2 - \delta_2) + \pi_1(c_1 - \delta_1, c_2 - \delta_2)\} - \{\pi(c_1 - \delta_1, c_2) + \pi_1(c_1 - \delta_1, c_2)\} = (-\delta_2)d\{\pi(c_1 - \delta_1, k) + \pi_1(c_1 - \delta_1, k)\}/dc_2$ by the Mean Value Theorem. Hence, $\partial \theta/\partial \tau_2 < (\text{resp.} >) 0$ holds as $d\{\pi(c_1 - \delta_1, k) + \pi_1(c_1 - \delta_1, k)\}/dc_2 < (\text{resp.} >) 0$. Q.E.D.

(III) *Proof of Theorem 5* We have $d\pi^B/dc_i = (1/\Gamma^B)[\{-(2 - \theta)a^2 d_i + a^2\sqrt{\delta} d_j + (x - y\sqrt{\delta})d_i d_j - (x\sqrt{\delta} - y)(d_j)^2\} - \Gamma^B d_i]$ and $d\pi^C/dc_i = (1/\Gamma^C)[\{-2A^2 q_i + A^2\sqrt{\delta} q_j - AXq_i q_j + AY(q_j)^2\} - \Gamma^C q_i]$. Hence, $|d\pi^B/dc_i| = d_i + (1/\Gamma^B)\{(2 - \theta)a^2 d_i - a^2\sqrt{\delta} d_j - (x - y\sqrt{\delta})d_i d_j + (x\sqrt{\delta} - y)(d_j)^2\}$ and $|d\pi^C/dc_i| = q_i + (1/\Gamma^C)\{2A^2 q_i - A^2\sqrt{\delta} q_j + AXq_i q_j - AY(q_j)^2\}]$. With $\theta = 0$, since $|d\pi^B/dc_i| = d_i + (1/\Gamma^B)\{(2a^2 d_i - xd_i d_j - y)(d_j)^2\}$ and $|d\pi^C/dc_i| = q_i + (1/\Gamma^C)\{2A^2 q_i + AXq_i q_j - AY(q_j)^2\}$, we have only to investigate the sign of $|d\pi^B/dc_i| - |d\pi^C/dc_i| = (d_i - q_i) + [\{2A^2/(\Gamma^B/\Gamma^2)\}d_i - (2A^2/\Gamma^C)q_i] + \{(-x/\Gamma^B)d_i d_j - (AX/\Gamma^C)q_i q_j\} + \{(y/\Gamma^B)(d_j)^2 + (AY/\Gamma^C)(q_j)^2\}$, where $d_i - q_i > 0$ by Theorem 1 and $(y/\Gamma^B)(d_j)^2 + (AY/\Gamma^C)(q_j)^2 \geq 0$. First, in order to investigate the sign of $\{2A^2/(\Gamma^B/\Gamma^2)\}d_i - (2A^2/\Gamma^C)q_i$, we should show the following lemma:

Lemma A1. *If $|x|$ is sufficiently small, then $\Gamma^B/\Gamma^2 \geq \Gamma^C$ for all $0 \leq \theta < 1$.*

Proof. Substituting from $a = \Gamma A$ into $\Gamma^B = \{2a - (x/a)d_i\}\{2a - (x/a)d_j\} - \{-a\sqrt{\delta} - (y/a)d_i\}\{-a\sqrt{\delta} - (y/a)d_j\}$ (> 0), we have: $\Gamma^B/\Gamma^2 = (4 - \theta)A^2 - (1/\Gamma^2)(2x + y\sqrt{\delta})(d_i + d_j) + (1/\Gamma^4)\{(x/A)^2 - (y/A)^2\}d_i d_j$. Comparing it with $\Gamma^C = (4 - \theta)A^2 + (2AX - AY\sqrt{\delta})(q_i + q_j) + (X^2 - Y)^2 q_i q_j$ (> 0), we have $\Gamma^B/\Gamma^2 \geq \Gamma^C$ for

all $0 < 1$ if $|x|$ is sufficiently small. *Q.E.D.*

In view of Lemma A1, we have $\{2A^2/(^B/^2)\}d_i (2A^2/^C)q_i > 0$ since $d_i > q_i$. Second, in order to investigate the sign of $\{(x/^B)d_id_j (AX/^C)q_iq_j\}$, we should show the following lemma, which proof is straightforward:

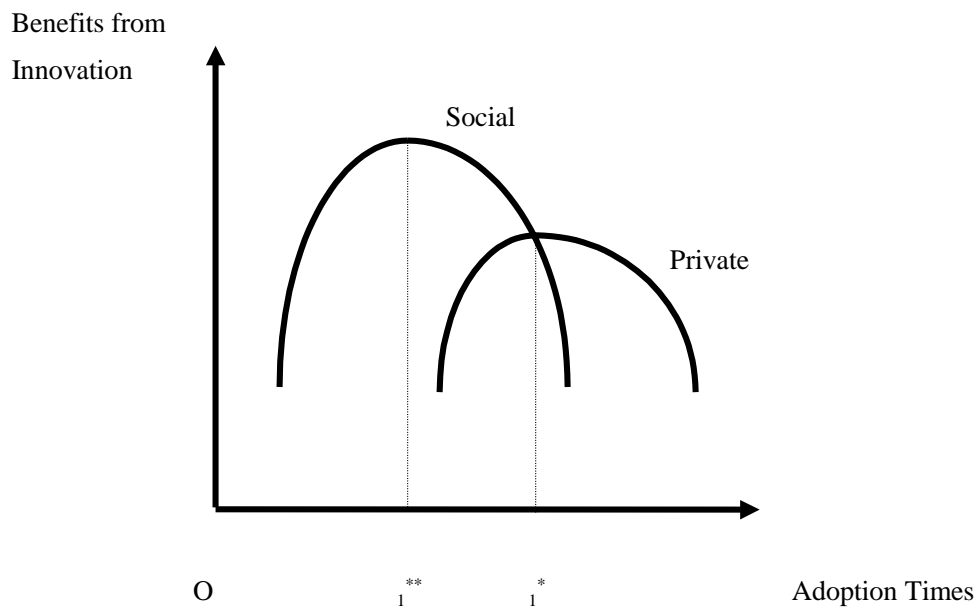**Lemma A2.** *If $|X|$ is sufficiently small, then $|x|/^B AX/^C$ for all $0 < 1$.*

In view of Lemma A1, we have $(x/^B)d_id_j (AX/^C)q_iq_j > 0$ since $d_i > q_i$. By Lemma A1 and Lemma A2, therefore, we have $|d^B/dc_i| > |d^C/dc_i|$. *Q.E.D.*

(IV) *Proof of Proposition 5* First, $^B ^C = \{U(d_1(p_1^B, p_2^B), d_2(p_1^B, p_2^B), M) c_1d_1(p_1^B, p_2^B) c_2d_2(p_1^B, p_2^B)\}$ $\{U(q_1^C, q_2^C, M) c_1q_1^C c_2q_2^C\}$. Let total surplus as $S(z_1, z_2) = U(z_1, z_2, M) c_1z_1 c_2z_2$ and take the total differential of $S(z_1, z_2)$. Then, $dS(z_1, z_2) > $ (resp. $<$) 0 if and only if $(p_1 c_1)/(p_2 c_2) > $ (resp. $<$) dz_2/dz_1, where $p_i = U/z_i > c_i$. We have $dS(z_1, z_2) > 0$ since $d_i > q_i$ implies that $dz_i > 0$, hence $S(d_1, d_2) > S(q_1, q_2)$. That is, ex ante total surplus is larger in Bertrand competition than in Cournot competition: $^B(c_1, c_2) > ^C(c_1, c_2)$. Second, social welfare at the social optimum ($_1^{**}$, $_2^{**}$) is written as: $= (1/)[(c_1, c_2) + e^{-\rho\tau_1^{**}}\{ dN(_1^{**})/dt\} + e^{-\rho\tau_2^{**}}\{ dN(_2^{**})/dt\}]$. We have already shown in the main text that $_i^{**B} < _i^{**C}$ if the demand functions are less convex and the degree of production differentiation is sufficiently high. Thus, under Assumption 1, we have $^B > ^C$. *Q.E.D.*
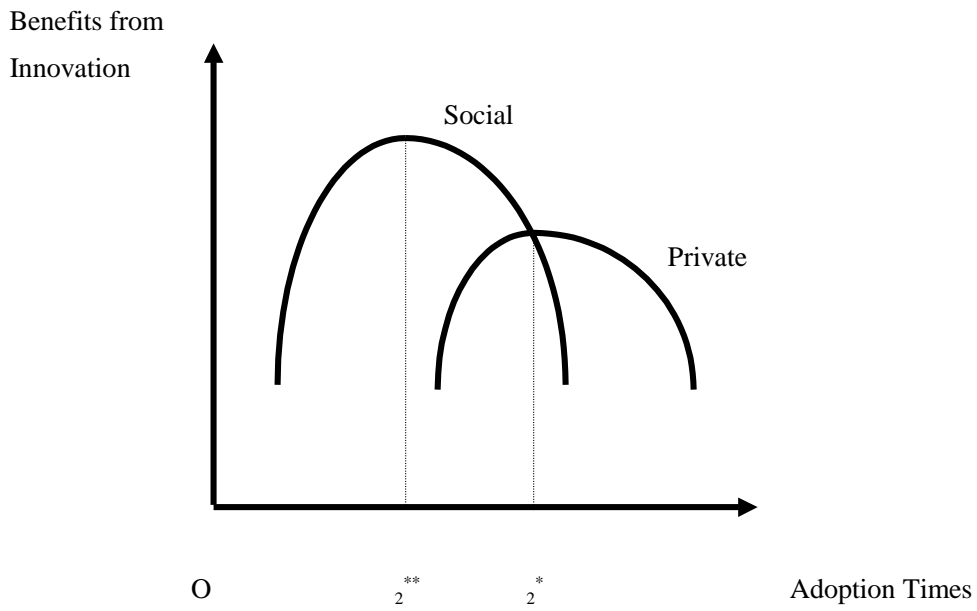
REFERENCES

Barzel, Y., 1968, "Optimal Timing of Innovations," *Review of Economics and Statistics*, vol. 50, pp. 348-355.

Bester, H. and E. Petrakis, 1993, "The Incentives for Cost Reduction in a Differentiated Industry," *International Journal of Industrial Organization*, vol. 11, pp. 519-534.

Brander, J. A. and B. J. Spencer, 1983, "Strategic Commitment with R&D: The Symmetric Case," *Bell Journal of Economics*, vol. 14, pp. 225-235.

Cheng, L., 1985, "Comparing Bertrand and Cournot Equilibria: A Geometric Approach," *Rand Journal of Economics*, vol. 16, pp. 146-152.

Dixit, A. K., 1979, "A Model of Duopoly Suggesting a Theory of Entry Barriers," *Bell Journal of Economics*, vol. 10, pp. 20-32.

Friedman, J. W., 1983, *Oligopoly Theory*, Cambridge University Press

Katz, M. L. and C. Shapiro, 1987, "R&D Rivalry with Licensing or Imitation," *American Economic Review*, vol. 77, pp. 402-420.

Mansfield, E., M. Schwartz and S. Wagner, 1981, "Imitation Costs and Patents: An Empirical Study," *Economic Journal*, vol. 91, pp. 907-918.

Mansfield, E., 1985, "How Rapidly Does New Industrial Technology Leak Out?", *Journal of Industrial Economics*, vol. 34, pp. 217-223.

Petrakis, E., 1994, "Technology Diffusion in a Differentiated Duopoly," Department of Economics Working Paper No. 94-33, Universidad Carlos III de Madrid.

Qiu, L. D., 1997, "On the Dynamic Efficiency of Bertrand and Cournot Equilibria," *Journal of Economic Theory*, vol. 75, pp. 213-229.

Quirmbach, H. C., 1986, "The Diffusion of New Technology and the Market for an Innovation," *Rand Journal of Economics*, vol. 17, pp. 33-47.

Reinganum, J. F., 1981a, "On the Diffusion of New Technology: A Game Theoretic Approach," *Review of Economic Studies*, vol. 48, pp. 395-405.

Reinganum, J. F., 1981b, "Market Structure and the Diffusion of New Technology," *Bell Journal of Economics*, vol. 12, pp. 618-624.

Reinganum, J. F., 1989, "The Timing of Innovation: Research, Development, and Diffusion," in *Handbook of Industrial Organization*, vol. 1, Chapter 14, edited by R. Schmalensee and R. D. Willig, North-Holland.

Singh, N. and X. Vives, 1984, "Price and Quantity Competition in a Differentiated Duopoly," *Rand Journal of Economics*, vol. 15, pp. 546-554.

Shy, O., 1996, *Industrial Organization: Theory and Applications*, M.I.T. Press.

Tirole, J., 1988, *The Theory of Industrial Organization*, M.I.T. Press.

Vives, X., 1985, "On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation," *Journal of Economic Theory*, vol. 36, pp. 166-175.
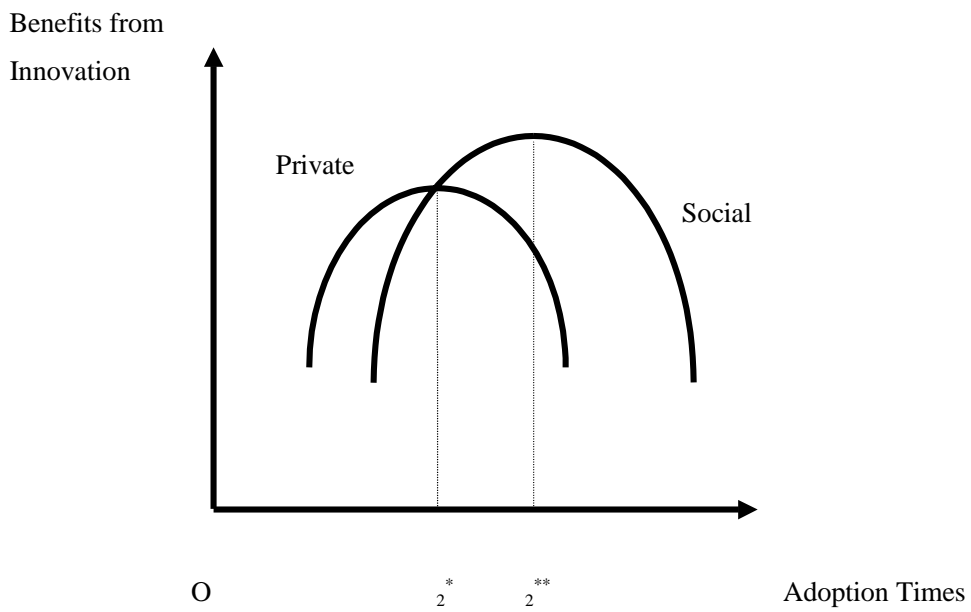
Benefits from
Innovation

Social

Private

O $\qquad$ $_1^{**}$ $\qquad$ $_1^{*}$ Adoption Times

(I) Adoption times for firm 1

Figure 1

Social

Private

O $\quad\quad\quad 2^{**}\quad\quad 2^{*}$ $\quad\quad$ Adoption Times

(II) Adoption times for firm 2: with highly differentiated products

Benefits from
Innovation

Private

Social

O $\quad\quad\quad 2^{*}\quad\quad 2^{**}$ $\quad\quad$ Adoption Times

(III) Adoption times for firm 2: with highly substitutable products

Figure 1