$i \le i'$. Accordingly, from Eq. (3.13) we immediately get $V_t(i) = V_1(i) < 0$. This completes the induction.

(e) From (c) and Lemma 5.5 we immediately get $i(t)$ exists, and from (b) we have a unique $i(1)$, hence $V_1(i) < 0$ for $i \le i(1) - 1$, $V_1(i(1)) \ge 0$, and $V_1(i) > 0$ for $i > i(1)$. Now, letting $i' = i(1) - 1$, from (d) we have $V_t(i) = V_1(i) < 0$ for $i \le i'$, and from Lemma 5.4 we have $V_t(i(1)) \ge V_1(i(1)) \ge 0$ and $V_t(i) \ge V_1(i) > 0$ for $i > i(1)$. Accordingly, by the definition of $i(t)$ it follows that $i(t) = i(1)$. This implies that $i(t)$ is unique and independent of t for all $t \ge 1$. ∎

Now, noting Eq. (5.2), we immediately have

$$p > (= (<))\ (1-s)z \iff V_1(1) > (= (<))\ 0. \tag{5.18}$$

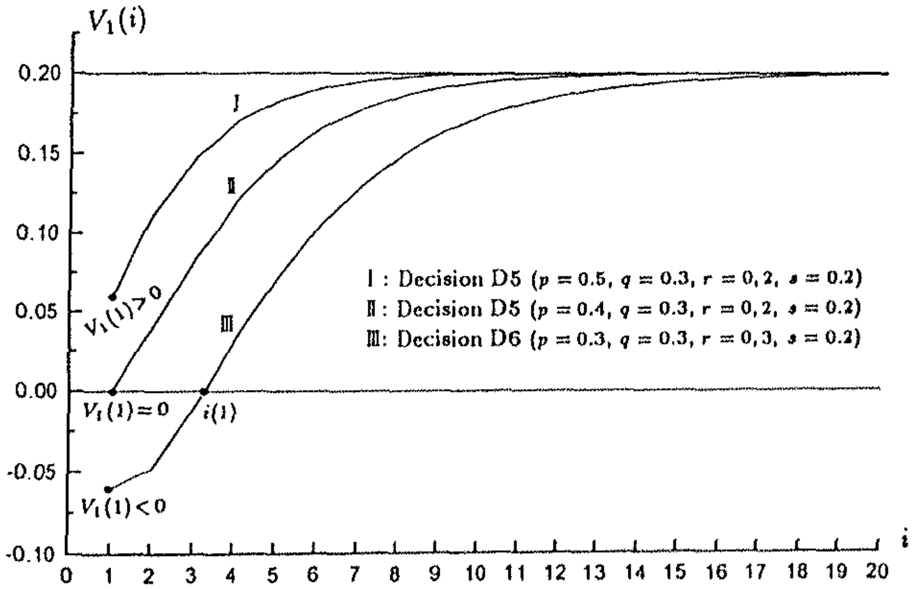

Figure 5.2: The graph of $V_1(i)$ with i $(s > 0)$

6 Concluding Remarks

6.1 Case of $s = 0$

In hostage plots perpetrated by a person who is determined to go through with it no matter what, and not surrender on any terms, he knows that, if arrested, he will be condemned to death or life imprisonment. This can be regarded as the case of $s = 0$. From Lemma 5.6 the optimal decision rule for any given i and t is independent of i and can be described as follows.

D1: *If $p \geq q/(q+r)$, wait up to time $t = 0$, deadline.*

D2: *If $p < q/(q+r)$, attempt a rescue immediatly when a hostage plot occurs.*

6.2 Case of $s > 0$ and $i = 1$

The most typical hostage plots are the case of $i = 1$. In this case, if $p \geq (1-s)z$, then $V_1(1) \geq 0$ from Eq. (5.2), hence it follows from Lemma 5.4 that $V_t(1) \geq 0$ for all $t \geq 1$, and if $p < (1-s)z$, then $V_1(1) < 0$ from Eq. (5.2), hence $V_t(1) < 0$ for all $t \geq 1$ from Lemma 5.3. Accordingly, if $i = 1$, the optimal decision rule for any given t can be stated as follows.

D3: *If $p \geq (1-s)z$, wait up to time 0, deadline.*

D4: *If $p < (1-s)z$, attempt a rescue immediately a hostage plot occurs.*

6.3 Case of $s > 0$

This is the case where a criminal might surrender, which in reality is the most possible case. In this case, if $p \geq (1-s)z$, then $V_t(i) > 0$ for all i and t from Lemma 5.7, and if $p < (1-s)z$, from (e) of Lemma 5.8 there exists a unique i^* such that $V_t(i) < (\geq) 0$ for $i < (\geq) i^*$, where $i^* = i(t) = i(1)$. Then, the optimal decision rule can be stated as follows.

D5: *If $p \geq (1-s)z$, wait up to time 0, deadline, irrespective of i, the number of remaining hostages.*

D6: *If $p < (1-s)z$, wait if $i < i^*$ and attempt a rescue if $i \geq i^*$.*

7 Future Studies

In our paper we propose a basic mathematical model for an optimal rescuing problem involving hostages. Taking different real hostage situations into account, we feel a need to modify the model from the following viewpoints:

1. In many real cases, criminals operate with confused motives. This causes the probabilities p, q, r and s to change randomly from one minute to the next. This consideration leads us to the model in which p, q, r and s are random variables with a known or unknown distribution function $F(p,q,r,s)$. When it is unknown, we can and must update its unknown parameters by using Bays' theorem.
2. In real hostage plots, some courses of action can be considered: whether or not to submit to the demands to be airlifted to another country, to provide a means of escape, to pay the ransom, to release comrades in prison, and so on. Taking such courses of action will influence the probabilities p, q, r and s to a greater or lesser degree. A problem arises as to when to act and what course of action to propose in order to maximize the probability of no hostage being killed.