must all be taken into acount, together with the safety of hostages, the demands of criminals, the repercussions of success or failure in a rescue attempt, and so on. The purpose of this paper is to propose a mathematical model of an optimal hostage rescue problem by using the concept of a sequential stochastic decision processes and examine properties of an optimal rescuing rule. Unfortunately, for our problem we were unable to find any reference material based on a mathematical approach. Accordingly, we can not list any references to be directly cited.

2 Model

Consider the following sequential stochastic decision process with a finite planning horizon. Here, for convenience, let points in time be numbered backward from the final point in time of the planning horizon, time 0, as 0, 1, $\cdots$, and so on. Let the time interval between two successive points, say times t and $t-1$, be called the period t. Here, assume that time 0 is the deadline at which a rescue attempt is considered as the only course of action for some reason, say, the hostage's health condition, the degree of criminal desperation, and so on.

Suppose $i \geq 1$ persons are taken as hostages at any given point in time t, and we have to make a decision on attempting either rescue or no rescue. Let x denote a decision variable of a certain point in time t where $x = 0$ if no rescue attempt and $x = 1$ if rescue attempt, and X_t denote the set of possible decisions of time t, i.e., $X_t = \{0, 1\}$ for $t \geq 1$ and $X_0 = \{1\}$.

Let p $(0 < p < 1)$ be the probability of a hostage being killed if $x = 1$ (Case 3), and let s $(0 \leq s < 1)$ be the probability of criminal(s) surrendering up to the next point in time, i.e., time $t-1$ if $x = 0$ (Case 2), so $1 - s$ is the probability of criminal(s) not surrendering. Further, let q and r $(0 < q < 1,\ 0 \leq r < 1$, and $0 < q + r < 1)$ be the probabilities of a hostage being, respectively, killed (Case 1) or set free (Case 3) up to the next point in time if $x = 0$ and criminal(s) not surrendering; accordingly, $1 - q - r$ is the probability of the hostage being neither killed nor set free. The objective here is to maximize the probability of no hostage being killed. Here, the case of $p = 0$, $p = 1$, $s = 1$, $q = 0$, $q = 1$, $r = 1$, and $q + r = 1$ makes the problem trivial; accordingly, all of which are excluded in the definition of the model.

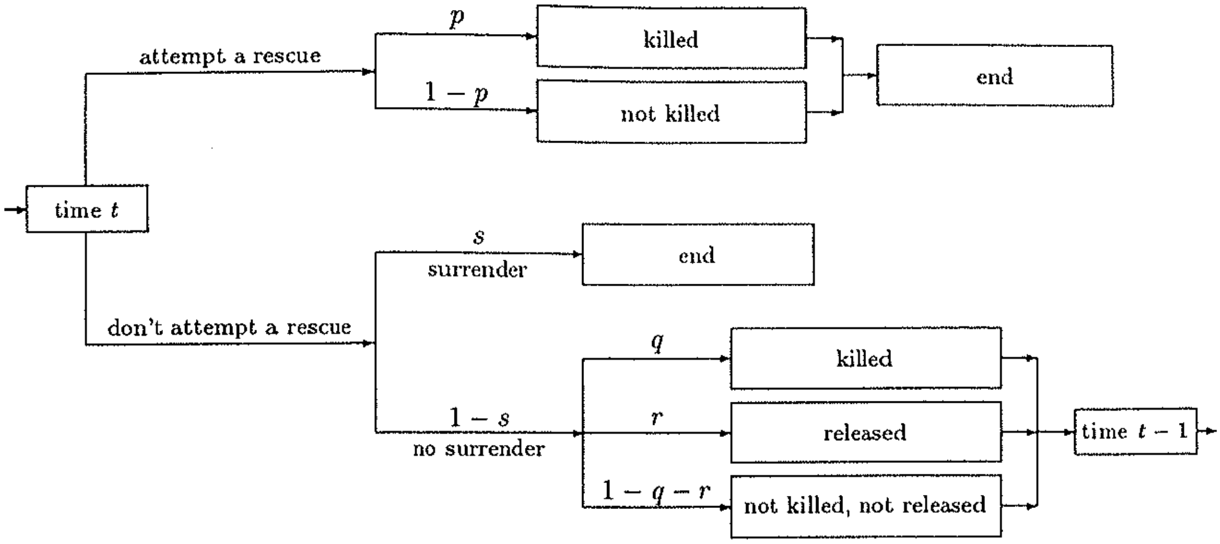

Figure 2.1: Decision Tree

3 Optimal Equation

Let $f_p(m|i)$ be the probability of m hostages being killed among i hostages if rescue attempt is made ($x = 1$), which is given by

$$f_p(m|i) = \binom{i}{m} p^m (1-p)^{i-m}, \qquad i \geq 1, \quad 0 \leq m \leq i. \tag{3.1}$$

Let the probability of no hostage being killed if a rescue attempt is made ($x = 1$) at any time t be denoted by $P(i)$. Then

$$P(i) = f_p(0|i) = (1-p)^i, \qquad i \geq 1. \tag{3.2}$$

Further, let $f_{qr}(k, \ell|i)$ be the probability of k hostages being killed and ℓ hostages being set free among i hostages if no rescue attempt ($x = 0$) and criminal(s) not surrendering up to the next point in time, which is given by

$$f_{qr}(k, \ell|i) = \frac{i!}{k!\ell!(i-k-\ell)!} q^k r^\ell (1-q-r)^{i-k-\ell}, \qquad i \geq 1, \quad 0 \leq k+\ell \leq i. \tag{3.3}$$

Now, let $v_t(i)$ be the maximum probability of no hostage being killed, starting from time $t \geq 0$ with i hostages, expressed as

$$v_0(i) = P(i), \qquad i \geq 1, \tag{3.4}$$

$$v_t(i) = \max\{P(i), W_t(i)\}, \qquad i \geq 1, \quad t \geq 1, \tag{3.5}$$