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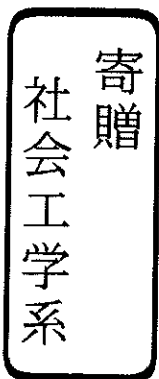
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AN OPTIMAL HOSTAGE RESCUE PROBLEM

by

FENGBO SHI

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AN OPTIMAL HOSTAGE RESCUE PROBLEM

Fengbo Shi
University of Tsukuba
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Abstract

We propose the following mathematical model of optimal rescue problems concerning hostages. Suppose i persons are taken as hostages at any given point in time t , and we have to make a decision on attempting either rescue or no rescue. Let p be the probability of a hostage being killed in a rescue attempt, and let s be the probability of criminal(s) surrendering up to the following point in time if no rescue attempt is made. Further, let q and r be the probabilities of a hostage being, respectively, killed or set free if no rescue attempt and criminal(s) not surrendering up to the following point in time. The objective here is to maximize the probability of no hostage being killed, starting from time t . Several properties of an optimal rescuing rule in the model are clarified and prescribed.

Keywords: Dynamic programming; Stochastic processes; Hostage rescue

1 Introduction

Throughout history acts involving, hostage taking have occurred for different reasons, e.g., social inequality, poverty, religious problems, racial problems, and so on. The problem has become an urgent issue to be tackled worldwide. Typical examples in recent years include:

- Case 1. A 17-year-old youth wielding a knife hijacked a bus on the Sanyo Expressway and killed a 68-year-old hostage. After 15 hours, the police stormed the bus, the other hostages were rescued, and the hijacker was arrested (May 4, 2000).
- Case 2. An armed man took a Finance Ministry official hostage in the Tokyo Stock Exchange building and demanded a meeting with the Finance Minister. He surrendered to the police after a tense, five and half hour standoff (January 12, 1998).
- Case 3. Fourteen guerrillas stormed the home of the Japanese ambassador to Peru and took about three hundred people hostage, including diplomats and government officials attending the emperor's birthday party. All but one of the hostages were rescued while all the rebels were killed when special forces stormed the building (December 17, 1996).
- Case 4. A man with a knife broke into a house and took a 2-year-old boy hostage. The police finally rushed into the house, set the uninjured boy free, and arrested the criminal (December 1, 1995).

Although not being available for accurate statistics, it could be said that different scenarios of similar plots are always occurring all over the world. The most important decision for the person in charge of crisis settlement is the timing to enact rescue of the hostages, especially after all possible negotiations have broken down. Wrestling with the problem, needless to say, involves many factors, political, economical, sociological, psychological, and so on, and they

must all be taken into account, together with the safety of hostages, the demands of criminals, the repercussions of success or failure in a rescue attempt, and so on. The purpose of this paper is to propose a mathematical model of an optimal hostage rescue problem by using the concept of a sequential stochastic decision processes and examine properties of an optimal rescuing rule. Unfortunately, for our problem we were unable to find any reference material based on a mathematical approach. Accordingly, we can not list any references to be directly cited.

2 Model

Consider the following sequential stochastic decision process with a finite planning horizon. Here, for convenience, let points in time be numbered backward from the final point in time of the planning horizon, time 0, as 0, 1, \dots , and so on. Let the time interval between two successive points, say times t and $t - 1$, be called the period t . Here, assume that time 0 is the deadline at which a rescue attempt is considered as the only course of action for some reason, say, the hostage's health condition, the degree of criminal desperation, and so on.

Suppose $i \geq 1$ persons are taken as hostages at any given point in time t , and we have to make a decision on attempting either rescue or no rescue. Let x denote a decision variable of a certain point in time t where $x = 0$ if no rescue attempt and $x = 1$ if rescue attempt, and X_t denote the set of possible decisions of time t , i.e., $X_t = \{0, 1\}$ for $t \geq 1$ and $X_0 = \{1\}$.

Let p ($0 < p < 1$) be the probability of a hostage being killed if $x = 1$ (Case 3), and let s ($0 \leq s < 1$) be the probability of criminal(s) surrendering up to the next point in time, i.e., time $t - 1$ if $x = 0$ (Case 2), so $1 - s$ is the probability of criminal(s) not surrendering. Further, let q and r ($0 < q < 1$, $0 \leq r < 1$, and $0 < q + r < 1$) be the probabilities of a hostage being, respectively, killed (Case 1) or set free (Case 3) up to the next point in time if $x = 0$ and criminal(s) not surrendering; accordingly, $1 - q - r$ is the probability of the hostage being neither killed nor set free. The objective here is to maximize the probability of no hostage being killed. Here, the case of $p = 0$, $p = 1$, $s = 1$, $q = 0$, $q = 1$, $r = 1$, and $q + r = 1$ makes the problem trivial; accordingly, all of which are excluded in the definition of the model.