No. 862

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by

Tsutomu Suzuki

April 2000

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Tsutomu Suzuki

Assistant Professor, Institute of Policy and Planning Sciences
University of Tsukuba
1-1-1 Tennodai, Tsukuba, Ibaraki 305-8573 Japan
E-mail: tsutomu@sk.tsukuba.ac.jp
Phone: +81-298-53-5186 Fax: +81-298-55-3849

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Abstract

In large metropolitan areas in the industrialized world commuting times are extraordinarily lengthy. While the large volume of commuting could partially be an inevitable result of a vast regional extension, it could also be a result of inefficient matching of homes and workplaces.

Many researchers have challenged the measurement of excess commuting which is calculated by the discrepancy between actual and minimum average commuting time. However, we could not reach a discussion on how urban structure (distributions of employees' homes and workplaces) influences excess commuting, because the exact extent of excess was not determined due to imperfect methodology of measurement (basically a zonal approach that yields aggregation biases).

In this paper we use continuous urban structure instead of zonal model, try to calculate unbiased excess commuting with joint distribution of homes and workplaces developed by Vaughan (1974), and describe the relationship between urban structure and commuting distance explicitly and theoretically for generalized home-workplace assignment pattern. First, we simplify the quadrivariate distribution model to a model with three important parameters: spread of homes, spread of workplaces, and spatial correlation of homes and workplaces. Second, we show that average commuting distance and excess commuting can be evaluated explicitly by the above three parameters, and if we assume the actual assignment to be the neighbor assignment, the excess increase as the extent of decentralization of workplace increases. Third, by applying this model, we explain the difference of excess commuting between US cities and Japanese cities.

According to our results, we first conclude that the decentralization of workplaces can increase or decrease average commuting distance depending on spatial correlation. We can decrease commuting distance if neighbor assignment is assumed, to an extent which is not so drastic, however. In this case, an attempt to encourage minisum assignment should be an effective way to reduce commuting time. Second, we found that excess commuting increases as decentralization of workplaces is promoted. This means that the possibility of decreasing distance can be larger if we promote a decentralization policy. Third, we found that the extent of excess is larger for US cities because they generally have more balanced urban structure than Japanese cities.

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1. Introduction

In large metropolitan areas in the industrialized world commuting times are extraordinarily lengthy. While the large volume of commuting could partially be an inevitable result of a vast regional extension, it could also be a result of inefficient matching of homes and workplaces. Long distance commuting causes not only physical exhaustion of workers but also enormous burden to the environment in a sense that workers must rely on much fuel in order to travel to work everyday. Reduction of commuting distance is supposed to be one of serious political goals in some metropolises such as Tokyo.

As a matter of course, commuting starts from homes and terminates at workplaces. Therefore, the distribution of homes and workplaces plays an important role in reducing commuting distance. Giuliano and Small (1993) concluded that urban structure does not affect commuting distance as an empirical result from the analyses of commuting behavior in Los Angeles. Cervero and Wu (1997, 1998) supported Giuliano and Small's (1993) results by showing the evidence that employment decentralization has failed to shorten average commuting distances in San Francisco Bay Area. However, this fact is not obvious before the analysis. On the one hand, Gordon, Kumer and Richardson (1989) found supportive result by analyzing the Standard Metropolitan Statistical Areas (SMSAs) of the United States. They verified that polycentric and dispersed metropolitan areas facilitate shorter commuting times. On the other hand, Næss and Sandberg (1996) studied the journey-to-work in Greater Oslo, Norway, and found that the commuting trips are on average somewhat shorter for workplaces in inner areas than businesses at the urban fringe, contrary to the results for US cities. It is rather natural to consider that there exists some relationship between workers' distribution and commuting characteristics. In this paper we deal with how urban structure influences commuting distance.

Many researchers have challenged the measurement of excess ('wasteful' in the previous studies) commuting, which is calculated by the discrepancy between actual and minimum average commuting time. The latter is obtained by solving the transportation problem using linear programming technique, while the distribution of homes and workplaces are constrained to be unchangeable. Excess commuting may be interpreted as what we can afford to reduce by rearranging the location of homes or workplaces. If we have much excess, we can reduce commuting distance (or time) drastically by controlling matching of homes and workplaces.

However, we could not reach a discussion on how urban structure, that is, distributions of employees' homes and workplaces, influences excess commuting. Merriman *et al.* (1995) tried to simulate that urban structural change towards the decentralization of employment cores results in the increase of excess. This is because the exact extent of excess was not determined due to imperfect methodology of measurement, which is basically a zonal approach that yields aggregation biases. Merriman *et al.* (1995) still suffers from and cannot sweep biases away in their analysis.

In this paper we use continuous urban structure instead of zonal model, try to calculate unbiased excess commuting with joint distribution model of homes and workplaces, and describe the relationship between urban structure and commuting distance explicitly and theoretically for generalized home-workplace assignment pattern.

First, we simplify the quadrivariate distribution model to a model with three important parameters: spread of homes, spread of workplaces, and spatial correlation of homes and workplaces. Second, we show that average commuting distance and excess commuting can be evaluated explicitly by the above three parameters, and if we assume the actual assignment to be the neighborhood assignment, the excess increases as the extent of decentralization of workplace increases. Third, by applying this model, we try to explain the difference of excess commuting between US cities and Japanese cities.

2. Brief review of debate on excess commuting

Excess commuting is defined as the difference between the actual and the required average commuting given the distribution of homes and workplaces. By measuring excess commuting, we can determine the extent to which the volume of commuting is an inevitable result of the functioning of a vast interconnected economic system and the extent to which it is the result of inefficient matching of workers' homes and workplaces.

Hamilton (1982) started with the test of excess commuting by using a monocentric model in which the workers were assumed to be distributed as a Clark-type model. He concluded that the amount of excess should be 80 percent of the actual commutings or over for several US metropolises. He focused on the fact that commuters do not necessarily minimize their commuting costs as often as assumed to be realized in the models of urban economics.

White (1988), on the other hand, adopted a zonal approach. She divided the targeted area into several zones, and grasped commuters' flow by an origin-destination matrix. In her approach the minimum average commuting distance is derived by solving a transportation problem which is a kind of linear programming. After testing for some US cities, she evaluated the amount of excess to be about 10 percent; still her method included aggregation bias. Hamilton (1989) and Small and Song (1992) corrected the bias and found the excess to be over 60 percent.

Excess commuting in the Tokyo metropolitan area was studied by Merriman, Ohkawara and Suzuki (1995). The urbanized area within 60 kilometers from the center of Tokyo was divided into 211 jurisdictional zones and the computed minimized average commuting time was 42 minutes using the zonal approach, while the actual average commuting time was 49 minutes. Thus the excess was 15 percent which is significantly smaller than that of US cities. When we use commuting distance to measure separation of homes and workplaces instead of commuting time, the excess increases to 36 percent. In the US cities, however, the excess amounts to over 60 percent also in distance. Most recently, Kim (1995) developed models which predict the commuting distances of two-worker households and estimated excess commuting in Los Angeles, whereas Frost, Linneker and Spence (1998) tested excess commuting in a selection of British cities.

Is the smallness of excess in Japanese cities generally observed? In Table 1, the excess in Tokyo, Osaka, Nagoya, and Chicago metropolitan areas, and San Francisco Bay Area are shown. We can observe a tendency that Japanese cities have longer commuting distance or time and smaller excess commuting than US cities. If this is true, there is no longer much room for reducing commuting distance in Japanese cities.

Polycentrism or dispersion of jobs has been heavily discussed by many researchers. Gordon, Richardson and Wong (1986) examined the change of distribution of homes and workplaces in Los Angeles metropolitan area, and revealed that the pattern of dispersal of employment is not simple dispersion but spatial concentration around a few major employment centers. Giuliano and Small (1991) identified 32 centers within Los Angeles region by using the 1980 Census journey-to-work data. Small and Song (1994) estimated monocentric and polycentric density functions for population and employment, and found that the polycentric models fit statistically better than monocentric models and that the employment shift toward a more polycentric pattern. McDonald and Prather (1994) also treated the journey-to-work data of the 1980 Census for Chicago urbanized area, and found three strong subcenters which attract workers' destinations.

Thus, in most large cities in the world, urban structure tends to become multinuclear employment systems. However, whether or not it brings about the increase or decrease in commuting distance depends on the characteristic of each city. Peng (1997) showed that there is no significant change in vehicle miles traveled (VMT) by changing jobs-housing balance in the Portland, Oregon, metropolitan area.

In this paper, we deductively show that commuting distance can increase or decrease depending on the jobs-housing balance and on the correlation of locations of homes and workplaces with a theoretical framework.

Traditional zonal approach is based on the network representation associated with a discrete set of coordinates such as the location of jurisdictional centers. Every point in geographic space is assigned to a particular zone, and every zone is associated with one or more nodes in the network. This approach often causes an aggregation problem such as biases in measuring excess commuting.

If urban space is represented as a field, we no longer need to consider a zonal system or a transportation network. The distributions of population, employment, and spatial interaction such as commuting trip are given as continuous distributions. This approach will simply and economically estimate the spatial pattern of travel in a city, and it is particularly suited for macroscopic studies. It is rather a complement to traditional transportation planning methods and not an alternative. In this paper we discuss such a continuous field of urban space hereafter.

3. Description of the relation between urban structure and commuting distance

3.1 Joint distribution of homes and workplaces

In this chapter we define the word 'home' as the home end of a work trip, that is, 'employed resident.' The word 'origin' can be used for the beginning point of work trip instead of 'home.' Similarly, we define the word 'workplace' as the destination end of a work trip, that is, 'employed establishment.' The total number of homes and the total number of workplaces are assumed to be equal to P.

Vaughan (1987) summarized the notion of the joint distribution of homes and workplaces which he formulated in 1974. Usually, as it is often treated by transportation planners, administrative zones are used as a unit to describe the movement of travelers from some part of a city to another. However, the continuous approach adopted here uses mathematical expressions, which gives the number of trips between any two arbitrarily selected unit zones.

Plenty of continuous urban models have been heavily researched. Clark (1951) approximates residential densities by a negative exponential function, and Alonso (1964) suggested a structure of rents declining with distance from the city center. Not only for the distribution of population or rents but also for spatial interaction the geometrical representation was associated with a continuous coordinate system. Angel and Hyman (1972) established a radially symmetric continuous model for representing spatial distributions of accessibility to jobs and residences, and of traffic flow. They introduced the notion of trip density function, the number of trips from a unit area at an origin to a unit area at a destination, which is close to the notion of joint distribution of homes and workplaces in the paper.

The earlier work on the trip length between two points in a continuous context was established by Haight (1964). Fairthorne (1965) treated the average distance between pairs of points and applied it to measure the accessibility between homes and workplaces with various routing systems including direct routing.

The analysis in this paper is based on Vaughan (1974) joint distribution of homes and workplaces. Wilkins (1969) made the first step to a realistic model by assuming that homes and workplaces were distributed according to Sherratt's law, but still assuming that homes and workplaces were uncorrelated. Vaughan (1974) realized that a spatial correlation effect could be introduced by using the quadrivariate normal distribution that allows homes and workplaces to remain individually distributed in accordance with Sherratt's law. This continuous model of work trips can be utilized as components of urban commute travel models.

Vaughan's (1974) joint distribution of homes and workplaces can be written in a general quadrivariate normal distribution as

$$f_{hw}(x_h, x_w, y_h, y_w) = \frac{P}{(2\pi)^2\sqrt{|\mathbf{V}|}} \exp\left[-\frac{1}{2}(z - \boldsymbol{\mu})^{\mathrm{T}} \mathbf{V}^{-1} (z - \boldsymbol{\mu})) \right] \tag{1}$$

where (x_h, y_h) and (x_w, y_w) represent the location of homes and workplaces by Cartesian coordinates, P denotes the total number of employees in the city, $z^{\mathrm{T}} = (x_h, x_w, y_h, y_w)$ is the location vector, $\boldsymbol{\mu}^{\mathrm{T}} = (\mu_{x_h}, \mu_{x_w}, \mu_{y_h}, \mu_{y_w})$ denotes the location vector of centroids, and

$$\mathbf{V} = \begin{pmatrix} \sigma_{x_h}^2 & Cov(x_h, x_w) & Cov(x_h, y_h) & Cov(x_h, y_w) \\ Cov(x_h, x_w) & \sigma_{x_w}^2 & Cov(x_w, y_h) & Cov(x_w, y_w) \\ Cov(x_h, y_h) & Cov(x_w, y_h) & \sigma_{y_h}^2 & Cov(y_h, y_w) \\ Cov(x_h, y_w) & Cov(x_w, y_w) & Cov(y_h, y_w) & \sigma_{y_w}^2 \end{pmatrix} \tag{2}$$

is the variance-covariance matrix of the coordinates, in which, for example, the standard deviation of x_h is denoted by $\sigma_{x_h}^2$ and the covariance of two coordinates x_h and y_w is denoted by $Cov(x_h, y_w)$. $\sigma_{x_h}^2$ indicates the spread of homes in the x direction and can be derived by the following equation.

$$\begin{aligned} \sigma_{x_h}^2 &= E[(x_h - \mu_{x_h})^2] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x_h - \mu_{x_h})^2 f_{hw}(x_h, x_w, y_h, y_w) \mathrm{d}x_h \mathrm{d}x_w \mathrm{d}y_h \mathrm{d}y_w \end{aligned} \tag{3}$$

Similar definitions are given for $\sigma^2_{x_w}$, $\sigma^2_{y_h}$, and $\sigma^2_{y_w}$. In the same way, the covariance of two variables, for instance, x_h, and y_w, is given by

$$\begin{aligned} Cov(x_h, y_w) &= E[(x_h - \mu_{x_h})(y_w - \mu_{y_w})] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x_h - \mu_{x_h})(y_w - \mu_{y_w})f_{hw}(x_h, x_w, y_h, y_w)\mathrm{d}x_h\mathrm{d}x_w\mathrm{d}y_h\mathrm{d}y_w \end{aligned} \quad (4)$$

with similar definitions for other two different pairs of coordinates. The marginal densities of homes and of workplaces derived from (1) become Sherratt's normal distributions (nonuniform, infinite boundary, radially symmetric, exponentially decreasing functions), which are convenient to handle mathematically. Blumenfeld (1972), and Blumenfeld and Weiss (1974) showed that Sherratt's model was superior for UK cities for the distribution of homes. They also found a good fit for the distribution of workplaces.

3.2 Simplification

The above quadrivariate normal joint distribution model is simplified by Vaughan(1974). The model contains 14 parameters, but under reasonable assumptions the number of parameters is reduced to three. First simplification is centralization. The centroid of homes and the centroid of workplaces can be coincided by putting

$$\boldsymbol{\mu} = \mathbf{0} \quad (5)$$

without losing generality.

Second, we can simplify the joint distribution model by assuming identical direction of city growth. If the directions of growth of homes and workplaces are the same, the covariance of the x and y coordinates of a home and a workplace can be set to be zero. Therefore,

$$Cov(x_h, y_h) = Cov(x_w, y_w) = 0. \tag{6}$$

Similarly, if any point is equally accessible from any other point in the city, it is likely that the covariance between the x coordinate of a home and the y coordinate of a workplace, and between the x coordinate of a workplace and the y coordinate of a home are zero. Therefore,

$$Cov(x_h, y_w) = Cov(x_w, y_h) = 0. \tag{7}$$

The most important covariances are those between the home and the workplace positions of an individual in two directions, given by $Cov(x_h, x_w)$ and $Cov(y_h, y_w)$. These variables indicate a worker's desire to live close to the workplace in x and y directions, respectively. Here we transform them to scale-free measures, that is, correlations given by

$$\rho_x = \frac{Cov(x_h, x_w)}{\sigma_{x_h}\sigma_{x_w}}, \tag{8}$$

and

$$\rho_y = \frac{Cov(y_h, y_w)}{\sigma_{y_h}\sigma_{y_w}}. \tag{9}$$

These variables satisfy

$$|\rho_x| < 1, \quad |\rho_y| < 1. \tag{10}$$

Moreover, if we assume circularly symmetric correlation, that is, no bias of development toward the x or y directions, the spread of homes and the spread of workplaces in two directions are assumed to take a common value.

$$\sigma_{x_h}^2 = \sigma_{y_h}^2 = \sigma_h^2 \tag{11}$$

$$\sigma_{x_w}^2 = \sigma_{y_w}^2 = \sigma_w^2 \tag{12}$$

Similarly, by assuming symmetric correlation, we have

$$\rho_x = \rho_y = \rho \tag{13}$$

so that the variance-covariance matrix is rewritten as

$$\mathbf{V} = \begin{pmatrix} \sigma_h^2 & \rho\sigma_h\sigma_w & 0 & 0 \\ \rho\sigma_h\sigma_w & \sigma_w^2 & 0 & 0 \\ 0 & 0 & \sigma_h^2 & \rho\sigma_h\sigma_w \\ 0 & 0 & \rho\sigma_h\sigma_w & \sigma_w^2 \end{pmatrix} \tag{14}$$

which has only three parameters. This is the third simplification. Hence, the joint distribution of homes and workplaces can also be rewritten as

$$f_{hw}(x_h, x_w, y_h, y_w) = \frac{P}{(2\pi)^2 \sigma_h^2 \sigma_w^2 (1-\rho^2)} \exp\left[-\frac{1}{2(1-\rho^2)} \left(\frac{x_h^2 + y_h^2}{\sigma_h^2} + \frac{x_w^2 + y_w^2}{\sigma_w^2} + \frac{2\rho(x_h x_w + y_h y_w)}{\sigma_h \sigma_w} \right) \right]. \quad (15)$$

This model represents the simplest model developed to date which takes account of the three basic factors that affect the distribution of commuting trips: spread of homes σ_h, spread of workplaces σ_w, and the correlation ρ between home and workplace locations. Evidences by Vaughan (1974) and Blumenfeld (1972) indicate that the simplification is verified to be reasonable to assume in the real world, if the city does not have a geographical peculiarity. Maximum likelihood estimates of three parameters are attempted by Blumenfeld (1972) and other researchers.

The marginal densities of homes and of workplaces derived from (15) are given by the following Sherratt's models.

$$f_h(x_h, y_h) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{hw}(x_h, x_w, y_h, y_w) \mathrm{d}x_w \mathrm{d}y_w = \frac{P}{2\pi\sigma_h^2} \exp\left(-\frac{1}{2} \frac{x_h^2 + y_h^2}{\sigma_h^2} \right) \quad (16)$$

$$f_w(x_w, y_w) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{hw}(x_h, x_w, y_h, y_w) \mathrm{d}x_h \mathrm{d}y_h = \frac{P}{2\pi\sigma_w^2} \exp\left(-\frac{1}{2} \frac{x_w^2 + y_w^2}{\sigma_w^2} \right) \quad (17)$$

Figure 1 shows a section of these marginal distributions. Wilkins' (1969) model, assuming that commuters select their homes without regard to their workplace locations, is a special case of the joint distribution, f_{hw}, in which homes and workplaces are independent, that is, $\rho = 0$. In this case, the joint distribution can be simply written as the product of the two marginal distributions divided by P.

After all, the following three variables are important: spread of homes σ_h, that of workplaces σ_w, and the spatial correlation ρ of homes and workplaces. Let us introduce a parameter $\alpha = \sigma_w / \sigma_h$ instead of σ_w. Since σ_h is usually greater than σ_w (homes are more spread out over the metropolis than workplaces), we can assume $0 \le \alpha \le 1$ tacitly.

Figure 2 shows the change of the distribution of homes for those who work at x_w=1 with respect to the correlation ρ when $\alpha = \sigma_w / \sigma_h = 1/2$. Equation (15) can be rewritten as

$$\begin{aligned} f_{hw}(x_h, x_w, y_h, y_w) &= \frac{P}{2\pi\sigma_w^2} \exp\left[-\frac{1}{2} \frac{x_w^2 + y_w^2}{\sigma_w^2} \right] \\ &\cdot \frac{1}{2\pi\sigma_h^2(1-\rho^2)} \exp\left[-\frac{1}{2(1-\rho^2)} \left\{ \left(\frac{x_h}{\sigma_h} - \frac{\rho x_w}{\sigma_w} \right)^2 + \left(\frac{y_h}{\sigma_h} - \frac{\rho y_w}{\sigma_w} \right)^2 \right\} \right] \\ &= f_w(x_w, y_w) f_{h|w}(x_h, y_h \mid x_w, y_w) \end{aligned} \quad (18)$$

where $f_{h|w}(x_h, y_h| x_w, y_w)$ is the conditional distribution of homes for workers who work at (x_w, y_w). Thus a section of the distribution $f_{h|w}(x_h, y_h|1,0)$ is shown in Figure 2. If $\rho = 0$, that is, the Wilkins' model, the distribution of homes at a given workplace does not depend on the location of the workplace and coincide with the marginal distribution of homes, the peak of which is at x=0. We call the assignment of a home to a workplace as 'random assignment' for this case. If $\rho = 1$, the locations of homes and workplaces are perfectly correlated so that all workers who works at x_w=1 live at x_h=2. Here we name this assignment 'minisum assignment,' because, as proved later, the average commuting distance is consequently minimized with respect to ρ. For the general figure of ρ, the distribution of homes has a peak in the interval of $0<x<2$. Above all, if $\rho = \alpha$, the peak of the distribution of homes at a given workplace coincides with the workplace location. In the example of Figure 2, the distribution of homes for those who work at $x_w=1$ has a peak at $x=1$ for $\rho = \alpha = 1/2$. We call this assignment 'neighbor assignment.'

3.3 Average commuting distance

Instead of deriving the average commuting distance by calculating the mean direct trip length defined by

$$\begin{aligned}\bar{d} &= E[\sqrt{(x_h - x_w)^2 + (y_h - y_w)^2}] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sqrt{(x_h - x_w)^2 + (y_h - y_w)^2}\, f_{hw}(x_h, x_w, y_h, y_w)\mathrm{d}x_h \mathrm{d}x_w \mathrm{d}y_h \mathrm{d}y_w,\end{aligned} \quad (19)$$

let us obtain it according to Blumenfeld's (1977) approach by deriving the expected value of the commuting distance

$$d = \sqrt{(x_h - x_w)^2 + (y_h - y_w)^2}\ . \quad (20)$$

Since each of the four variables, x_h, x_w, y_h, and y_w, has a univariate normal distribution with the variances which are equivalent to spreads of homes or workplaces, we have

$$x_h, y_h \sim N(0, \sigma_h^2), \quad (21)$$

$$x_w, y_w \sim N(0, \sigma_w^2), \tag{22}$$

and also we have the following relation based on the discussion so far.

$$Cov(x_h, x_w) = Cov(y_h, y_w) = \rho\sigma_h\sigma_w \tag{23}$$

Therefore, the difference of the location of a home and a workplace is also distributed according to a normal distribution, that is,

$$x_h - x_w, y_h - y_w \sim N(0, \sigma_h^2 + \sigma_w^2 - 2\rho\sigma_h\sigma_w). \tag{24}$$

By standardizing variables, we obtain

$$\frac{x_h - x_w}{\sqrt{\sigma_h^2 + \sigma_w^2 - 2\rho\sigma_h\sigma_w}}, \frac{y_h - y_w}{\sqrt{\sigma_h^2 + \sigma_w^2 - 2\rho\sigma_h\sigma_w}} \sim N(0,1). \tag{25}$$

The square of d is the sum of the square of two variables. Then we have

$$\frac{d^2}{\sigma_h^2 + \sigma_w^2 - 2\rho\sigma_h\sigma_w} = \frac{(x_h - x_w)^2 + (y_h - y_w)^2}{\sigma_h^2 + \sigma_w^2 - 2\rho\sigma_h\sigma_w} \sim \chi^2(2). \tag{26}$$

The probability density function of x for the χ^2 distribution of two degrees of freedom is $\frac{1}{2}\exp(-\frac{x}{2})$. Therefore, the probability density function of $u = d^2$ is given by

$$\frac{1}{2(\sigma_h^2+\sigma_w^2-2\rho\sigma_h\sigma_w)}\exp\left(-\frac{u}{2(\sigma_h^2+\sigma_w^2-2\rho\sigma_h\sigma_w)}\right). \tag{27}$$

After all, the mean of *d*, $\bar{d}$, is given by

$$\begin{aligned}\bar{d} &= E(\sqrt{d^2}) = \int_0^\infty \sqrt{u}\frac{1}{2(\sigma_h^2+\sigma_w^2-2\rho\sigma_h\sigma_w)}\exp\left(-\frac{u}{2(\sigma_h^2+\sigma_w^2-2\rho\sigma_h\sigma_w)}\right)du \\ &= \sqrt{2}\sqrt{\sigma_h^2+\sigma_w^2-2\rho\sigma_h\sigma_w}\,\Gamma(\tfrac{3}{2}) = \sqrt{\frac{\pi}{2}(\sigma_h^2+\sigma_w^2-2\rho\sigma_h\sigma_w)}\end{aligned} \tag{28}$$

which indicates that the average commuting distance is explained by the three parameters. Introducing $\alpha = \sigma_w/\sigma_h$ instead of σ_w, we can rewrite (28) as

$$\bar{d} = \sigma_h\sqrt{\frac{\pi}{2}(1+\alpha^2-2\rho\alpha)}\,. \tag{29}$$

3.4 Urban structure and average commuting distance

The parameter $\alpha = \sigma_w/\sigma_h$ represents the difference not only between spreads of homes and workplaces but also between densities of homes and workplaces at the city center, that is,

$$\frac{f_h(0,0)}{f_w(0,0)} = \frac{\sigma_w^2}{\sigma_h^2} = \alpha^2, \tag{30}$$

which can be represented by the term 'jobs-housing balance.' Usually the spread of homes is larger than that of workplaces, so we can assume $0 \le \alpha \le 1$. If α is close to

1, the spreads of homes and workplaces are almost the same and jobs and housing are balanced in an employment decentralized (dispersed) urban structure. On the contrary, if α becomes smaller, the difference of spreads turns to be larger and the city loses jobs-housing balance by employment concentration in the center. In this paper the sole parameter α is adopted to determine urban structure. Then Equation (29) implies that the average commuting distance $\bar{d}$ is determined by the three parameters: spread of homes σ_h, spatial correlation of homes and workplaces ρ, and the jobs-housing balance α. σ_h has a simply proportional effect on $\bar{d}$, so we are interested in the effects of the other two factors.

Figure 3 shows how the average commuting distance $\bar{d}$ varies with respect to jobs-housing balance α and correlation ρ. From these figures, we can observe the following:

i) When jobs are completely concentrated, i.e. $\alpha = 0$, the average distance is given by

$$\bar{d} = \sqrt{\frac{\pi}{2}}\sigma_h, \qquad (31)$$

and the same distance is achieved when $\alpha = 2\rho$. If we take it as a borderline, the average commuting distance is smaller than this criterion only when the relation $\alpha < 2\rho$ (intense concentration or strong correlation) is realized.

ii) Only when jobs are perfectly dispersed (implies $\alpha = 1$) and fully correlated (implies $\rho = 1$) do all locations of home and workplace perfectly coincide and commuting disappears, i.e. $\bar{d} = 0$.

iii) Given the correlation ρ, the parameter α which minimizes $\bar{d}$ is observed by solving the following first-order condition of the rooted term of Equation

(29).

$$\frac{\partial}{\partial \alpha}(1+\alpha^2-2\rho\alpha)=2(\alpha-\rho)=0 \tag{32}$$

Thus, we have

$$\alpha=\rho. \tag{33}$$

Suppose that ρ is given a priori and we can control only α by some land use policies. When $\alpha>\rho$, we obtain $\frac{\partial \bar{d}}{\partial \alpha}>0$, so that the average commuting distance increases by workplace decentralization. Inversely, when $\alpha<\rho$, we obtain $\frac{\partial \bar{d}}{\partial \alpha}<0$, which means that the average distance decreases by workplace decentralization.

3.5 Average distance under typical correlations of homes and workplaces

Recall the three typical types of home-workplace assignment defined in 3.2. Each of them is named random assignment, minisum assignment, and neighbor assignment. Let us calculate the average commuting distance for these assignments.

3.5.1 Random assignment

Random assignment is the case of $\rho=0$ at which workers choose their workplace without relating to the location of their homes. The average commuting distance is given by

$$\bar{d}_U=\sqrt{\frac{\pi}{2}(\sigma_h^2+\sigma_w^2)}=\sigma_h\sqrt{\frac{\pi}{2}(1+\alpha^2)}. \tag{34}$$

3.5.2 Minisum assignment

Full correlation, $\rho=1$, between homes and workplaces minimizes the total commuting distance with a constant value of α, and, hence, the average commuting distance $\bar{d}$, which is given by

$$\bar{d}_M = \sqrt{\frac{\pi}{2}}\left|\sigma_h - \sigma_w\right| = \sigma_h\sqrt{\frac{\pi}{2}}\left|1-\alpha\right|. \tag{35}$$

3.5.3 Neighbor assignment

In the case of $\alpha = \rho$, the average commuting distance is minimized under given ρ and obtained by

$$\bar{d}_N = \sqrt{\frac{\pi}{2}(\sigma_h^2 - \sigma_w^2)} = \sigma_h\sqrt{\frac{\pi}{2}(1-\alpha^2)}. \tag{36}$$

It is supposed to be closest to the real world commuting pattern, because the assignment pattern has a peak in the conditional distribution of homes at their workplaces.

Functional forms of each average commuting distance under three typical assignment patterns with respect to the change of α are shown in Figure 4. The average distance under random assignment increases as the jobs-housing relation turns to be balanced, whereas it decreases as the jobs are dispersed under minisum assignment. The average distance under neighbor assignment also decreases as the urban structure gets balanced, to an extent which is not so drastic, however.

3.6 Urban structure and excess commuting

Excess commuting is defined as the difference between actual and minimum average commuting distance or time, but usually presented by the ratio of excess (called 'excess' hereafter), ε, which is calculated by definition as the difference divided by the actual average commuting. Therefore,

$$\varepsilon = \frac{\bar{d} - \bar{d}_M}{\bar{d}} = 1 - \frac{\sigma_h - \sigma_w}{\sqrt{\sigma_h^2 + \sigma_w^2 - 2\rho\sigma_h\sigma_w}} = 1 - \frac{1-\alpha}{\sqrt{1+\alpha^2 - 2\rho\alpha}}, \quad (37)$$

which implies that the excess can be expressed by α and ρ (a scale variable σ_h has dropped out).

Figure 5 shows the relation among three variables: spatial correlation, jobs-housing balance, and excess commuting. Here we can observe the following:

i) When $\alpha = 0$, that is, employments are perfectly centralized, no excess exists. On the other extreme, when $\alpha = 1$, that is, the distributions of homes and workplaces are coincident, all commutes become excess.

ii) Excess decreases as jobs-housing balance, α, decreases. In other words, a highly job-centralized city has less excess commuting.

iii) Excess decreases as spatial correlation of homes and workplaces, ρ, increases. In other words, a city in which jobs and housing are highly correlated and sectorially structured has less excess commuting.

We cannot say what the actual assignment pattern of homes and workplaces is because it is practically complicated. However, taking a macroscopic view, we can regard the neighbor assignment as the actual assignment pattern, since, for almost all cities, inner commute is dominant and the number of workers closer to the center tends to be larger than that farther. Therefore, we take the neighbor assignment as the actual assignment. By considering $\rho = \alpha$ as being realized, then, the excess is expressed as

$$\varepsilon = \frac{\bar{d}_N - \bar{d}_M}{\bar{d}_N} = 1 - \frac{1-\alpha}{\sqrt{1-\alpha^2}} \tag{38}$$

in which α is the only variable. Excess commuting under the neighborhood assignment is explained by jobs-housing balance as shown in Figure 6. This graph shows us that the more jobs are dispersed the more the excess is. This fact can be the explanation of the difference in excess between US and Japanese metropolises.

4. Discussion

We found that the decentralization of workplaces can increase or decrease average commuting distance depending on spatial correlation. van Ommeren, Rietveld and Nijkamp (1996) focused that residence and workplace location are jointly determined and their mobility are also mutually dependent. They analyzed it by a bivariate duration model of residential and labor market mobility, motivated by simultaneous search model on the labor and housing market taking commuting costs into account. Their empirical results show that residential and labor mobility depend positively on one another. Thus it is natural that we consider the locations of home and workplace as interrelated.

If we assume that the actual home-workplace assignment is neighbor assignment, we can decrease commuting distance by decentralization of workplaces from the discussion in 3.5. However, the extent of decrease is not so drastic. In this case, if we can control ρ by some policies such as activation of housing market, an attempt to encourage minisum assignment should be an effective way to reduce commuting time.

How can we interpret the decentralization policy of workplace such as that promoted by the Japanese Government for Tokyo region? In our context, if employments are already to some extent dispersed ($\alpha > \rho$), commuting distance will increase by the decentralization policy. On the contrary, if employments are too concentrated ($\alpha < \rho$), commuting distance will decrease. Urban structure in large metropolises corresponds to the latter. This is why decentralization policy is supported.

Excess commuting is also an important measure of appropriateness of urban structure. We found that if the actual home-workplace assignment is close to neighbor assignment, excess commuting increases as decentralization of workplaces is promoted. This fact implies that the possibility of decreasing distance can be larger if we promote a decentralization policy. Combination with a policy for promoting home-workplace matching seems to be a powerful commuting shortening solution.

It could be expected that the correlation ρ between a worker's home and his/her workplace location would increase with city size, since it is expected that in a small city every home is almost equally accessible from every workplace. In a large city, however, a workplace on the side of the city where the worker's home is will be more accessible than those on the other side. Vaughan (1987) presented estimated values of α and ρ for British and Australian cities: $\alpha = 0.6$ and $\rho = 0.2$ in small cities, and $\alpha = \rho = 0.8$ in large cities.

Though we do not have the exact estimate, Tokyo metropolitan area seems to have much smaller α and much greater ρ. If this is true, our theory predicts that decentralization decreases commuting distance and that excess takes smaller values. This is consistent with the facts observed in Chapter 2 of this paper or in Merriman, Ohkawara and Suzuki (1995).

Giuliano and Small (1993) concluded that 'attempts to alter the metropolitan-wide structure of urban land use via policy intervention are likely to have disappointing impacts on commuting patterns, even if successful in changing the degree of jobs-housing balance.' Figure 3 and Figure 4 show that average commuting distance would not be affected by the change of jobs-housing balance α or the correlation ρ of homes and workplaces. Generally, since the growth of a metropolis brings about an increase in spread of homes and it countervails the decrease of $\bar{d}$, consequently the average commuting distance does not change so drastically. Though we cannot conclude illogically, it can be said that the model can explain it to some extent and support Giuliano and Small (1993).

5. Conclusions

According to our results, we first conclude that the decentralization of workplaces can increase or decrease average commuting distance depending on spatial correlation. We can decrease commuting distance if neighbor assignment is assumed, to an extent which is not so drastic, however. In this case, an attempt to encourage minisum assignment should be an effective way to reduce commuting time. Second, we found that excess commuting increases as decentralization of workplaces is promoted. This means that the possibility of decreasing distance can be larger if we promote a decentralization policy. Third, we found that the extent of excess is larger for US cities because they generally have more balanced urban structure than Japanese cities.

This paper showed that average commuting distance and excess commuting could be analyzed theoretically. If the relative discrepancy between the distributions of homes and workplaces is larger, then average commuting distance becomes larger and excess becomes smaller. If the spatial correlation between the locations of home and workplace is stronger, then average commuting distance becomes shorter and excess becomes smaller. Jobs decentralization in smaller cities tends to cause an increase in commuting distance, because jobs are usually dispersed relatively in those cities. However, in larger cities, jobs decentralization tends to cause a decrease in commuting distance, because jobs are usually concentrated relatively. If the actual commuting is realized in such a way as neighbor assignment, more dispersed jobs bring about larger excess. The measurement of the three key parameters from actual commuting flow data, the effects of job decentralization policies, and modeling multicentric continuous distribution remain to be studied for future research.

Acknowledgement

Thanks are due to Professors Takeshi Koshizuka, Tatsuo Hatta, Yoshitsugu Kanemoto, Takatoshi Tabuchi, Kazuyuki Konagaya for their valuable comments. Professor David Merriman's assistance in data utilization is gratefully acknowledged. The presentation at the Sixth World Congress of the Regional Science Association International held in Lugano, Switzerland is supported by the Obayashi Foundation. This support is also gratefully acknowledged.

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Table 1. Average and minimum commuting distances and times, excess commuting in US and Japanese metropolises (as of 1990 except Chicago data in 1980).

Metropolis	# of zones	# of total commuters	Average commuting distance (km)		Average commuting time (minutes)		Excess (percent)	
			Actual	Minimum	Actual	Minimum	Distance	Time
Tokyo	280	16,686,226	10.81	6.90	51.40	43.20	36.2	16.0
Nagoya	150	4,693,606	8.27	5.22	41.90	—	36.8	—
Osaka	194	8,626,997	9.00	5.57	49.02	—	38.2	—
Chicago	159	2,950,295	14.72	8.33	28.39	18.92	43.4	33.4
San Francisco Bay Area	149	3,057,035	—	—	25.15	16.34	—	35.0

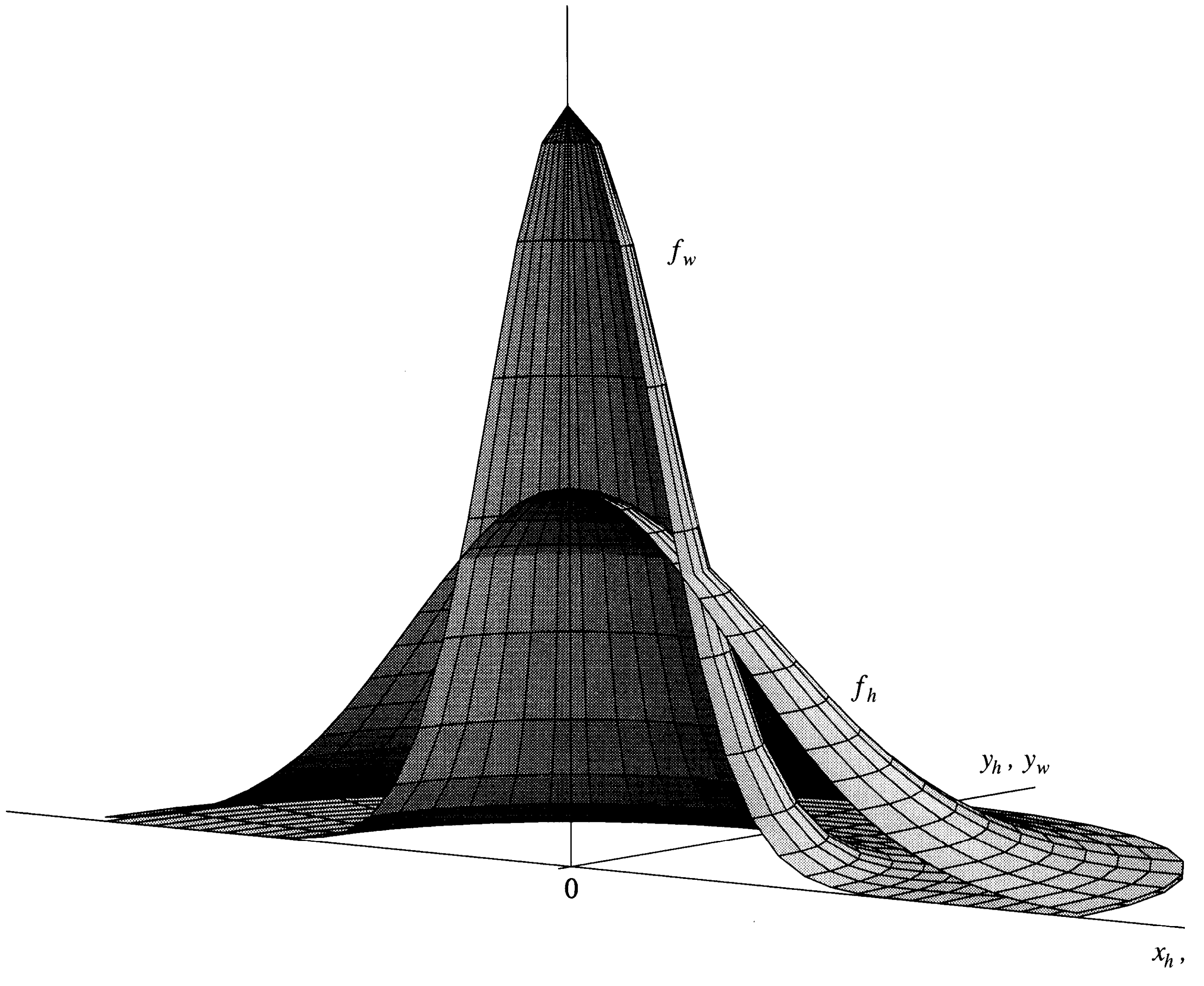


Figure 1. Marginal distributions of homes and workplaces of the quadrivariate joint distribution model.

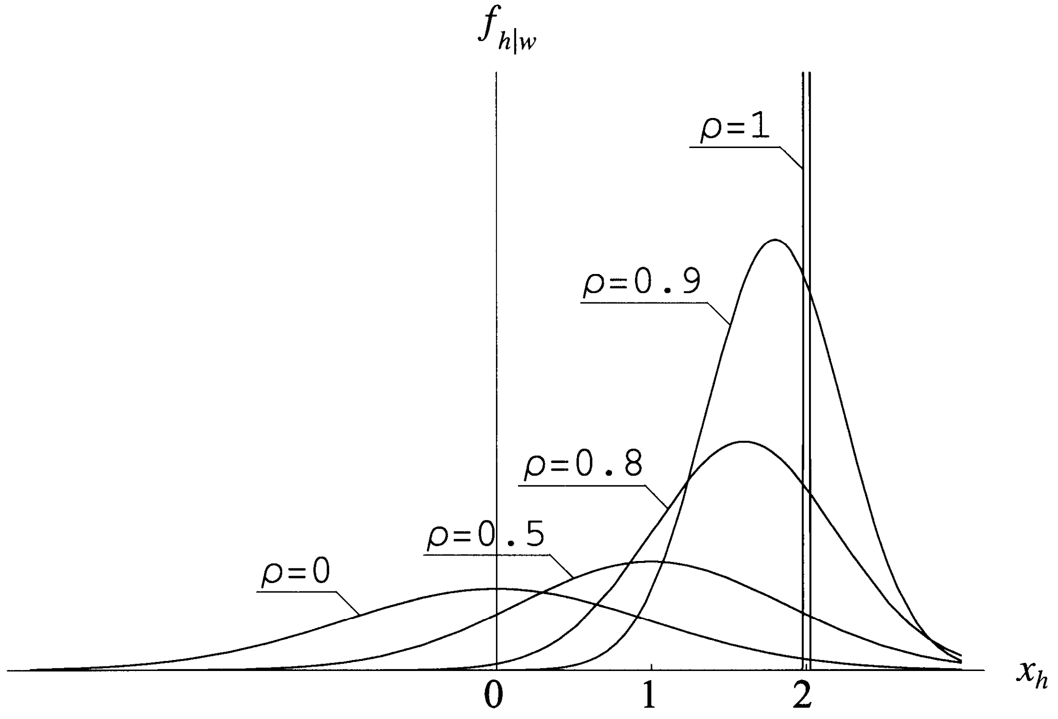


Figure 2. Spatial correlation, ρ , and the conditional distributions of homes for those who work at x_w=1 (in the case of $\alpha = \sigma_w / \sigma_h$=1/2).

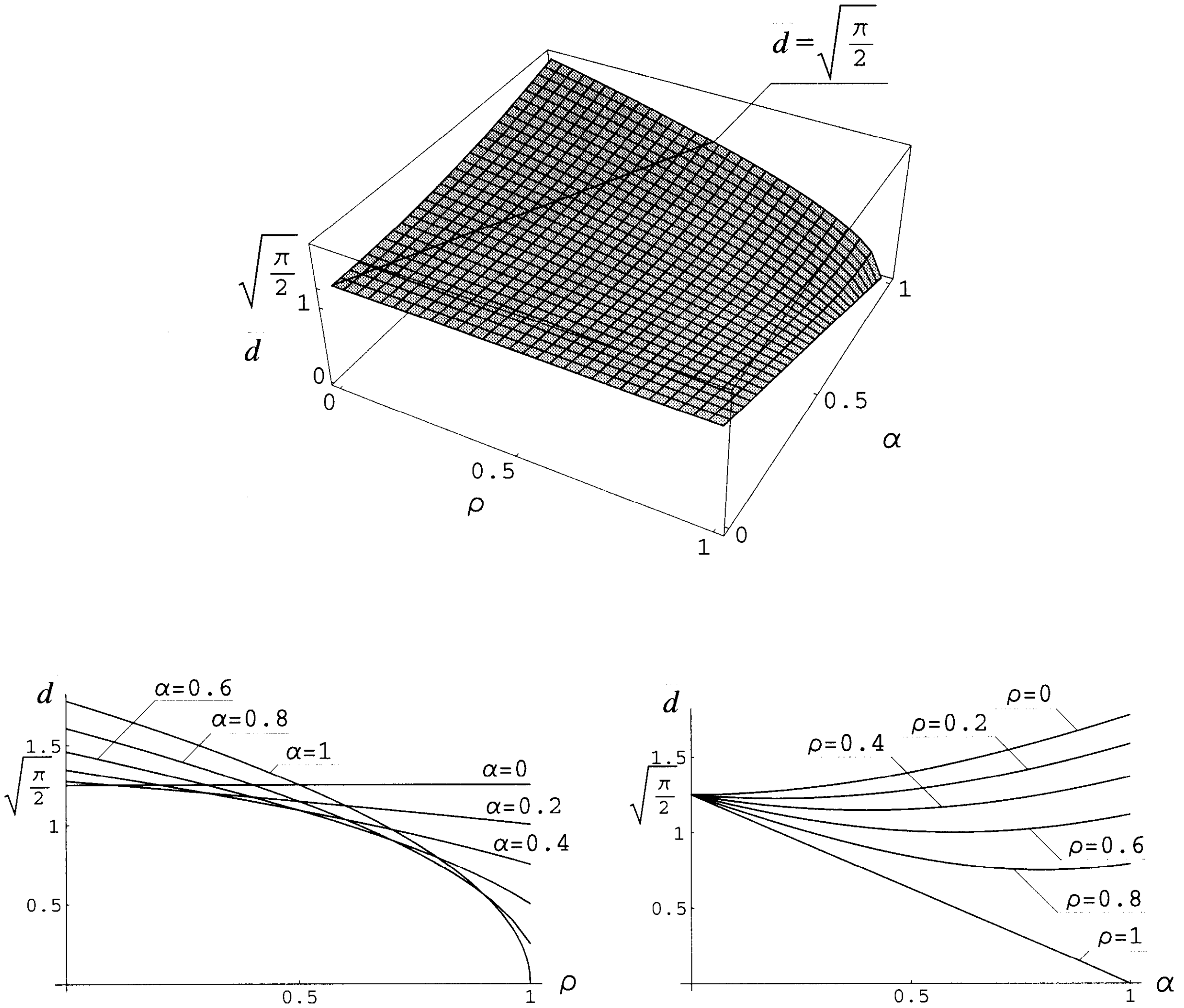


Figure 3. Spatial correlation, jobs-housing balance, and average commuting distance.

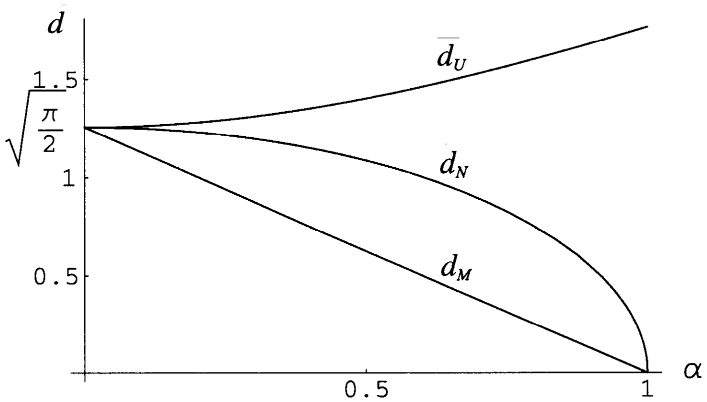
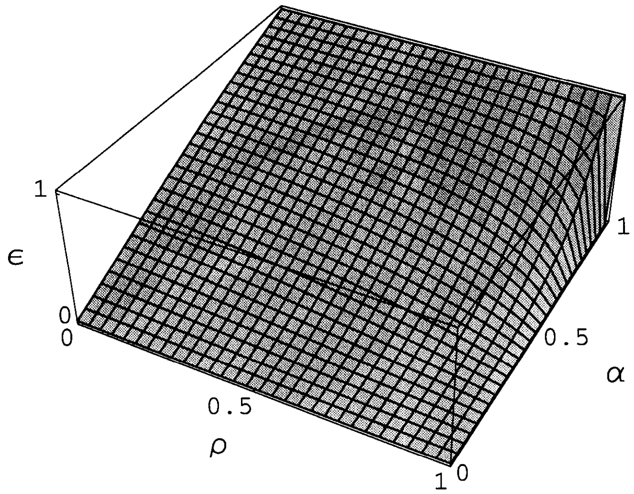


Figure 4. Average commuting distance under three typical assignment patterns.



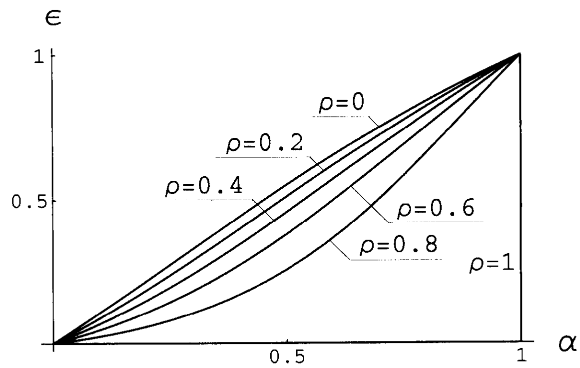


Figure 5. Spatial correlation, jobs-housing balance, and excess commuting.

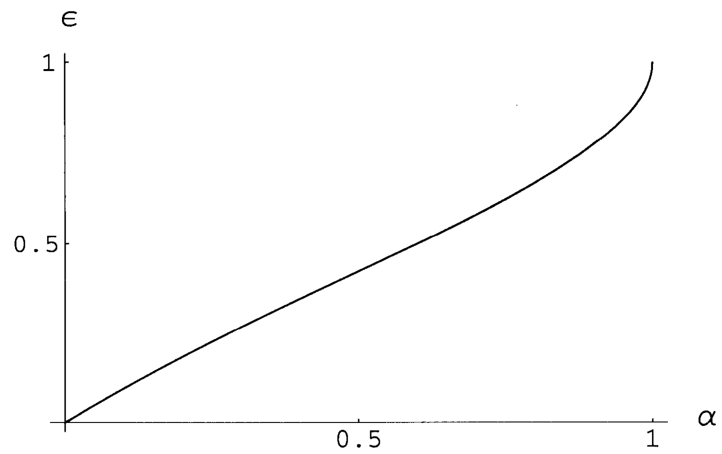


Figure 6. Excess commuting under the neighbour assignment.