COST VARIANCE INVESTIGATION POLICY BASED ON BAYES LIKELIHOOD-RATIO TEST

BY

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ABSTRACT

Since the beginning of the 1960's [Bierman, Fouraker, and Jaedicke(1961)], the studies of the decision making for cost variance investigation have brought about the reconsidering of a simple control chart which indicates whether or not a cost data is contained within a certain control limit. These are models to determine whether or not a process should be investigated with respect to the costs and benefits of control actions when a cost data is reported.

When control action is selected under considering opportunity costs, or operating costs and investigating costs, the previous literature can be classified into three types according to how control variables will be established. The first type is the Decision Theoretic Approach [BFJ(1961) and Dyckman(1969)], and the model in which the control variable is denoted as a Bayesian posterior probability with respect to opportunity costs. The second is the Bayesian Dynamic Programming Approach [Kaplan(1969) and Hughes(1975)] in which the control variable is denoted as a Bayesian posterior probability, relating whether or not to investigate the process with respect to operating costs and investigating costs. Adding to the two approaches above, another type is the Markovian Approach [Dittman and Prakash(1978)], a model used to decide whether or not to investigate the process with

respect to the operating costs and investigating costs, in which the control variable is denoted as a cost variance itself.

The purpose of this study is to develop another method (known as the "Normal Form of Bayesian analysis") of Decision Theoretic Approach employing Decision-Facilitating case defined by Demski and Kreps(1982), in which the control variable is denoted as a cost variance itself in place of Bayesian posterior probability used in the previous Decision Theoretic Approaches (Known as the "Extensive Form of Bayesian analysis"). The Normal Form starts by explicitly considering every possible decision rule instead of first determining the optimal act for every possible outcome. Thus, this approach used in this study finds an investigation region which minimizes the expected cost, based on Bayesian likelihood-ratio test, and then control actions are determined within the sample space. However, the Extensive Form of Bayesian analysis proceeds by working backwards from the end of the decision tree to the initial starting point, and then it has a condensed meaning because control actions are discussed within a [0,1] probability space.

This study does not discuss the comparison between the proposed model and the previous approaches, because the simulation results of Magee(1976) show that the differences between Dyckman's (1969) and Kaplan's(1969) models may have little effect on the incremental cost savings; and the Normal Form of this study and Extensive Form of Dyckman are mathematically equivalent and lead to identical results whether the pre-experimental viewpoint of the former or the post-experimental viewpoint of the latter is taken.

The proposed model assumes that the cost-generating-process is a two-states, discrete Markov process in which a cost report at the end of each period provides information about current state of the process.

Using the Normal Form of Bayesian analysis, different from the previous approaches, the proposed model simply requires the matching of a reported cost against a given investigation region depending on the prior probability; and shows how to determine the investigation region corresponding to the statistical variance ratio; and seeks the lower or upper bound of the prior probability, the critical value for determining when the process should be always or never investigated.

This study develops a Exploratory Investigation model as an extension of the Full Investigation model with respect to the Normal Form of Bayesian analysis, and simulates the relation between the Full and Exploratory Investigation cases in terms of the average total cost over 1000 simulated 12-month periods in a similar manner to Magee's(1976) study. The simulated results show that the smallest values of the average total costs are almost found in the Exploratory Investigation cases rather than in the Full Investigation cases.

This study also shows how the optimal set of processes to be investigated in any period can be selected in N-processes with each cost process being treated independently of the others. This differs from the model of N-processes by Dyckman(1969).

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CHAPTER ONE

INTRODUCTION

1) Motivation

All business planning, whether in terms of budgets or standards, are based on estimates of prices, volumes costs, etc., and any outcome can only be expected to approximate these estimates. However, outcomes will not necessarily equal the original estimates since some variance around the estimate or expected outcome is inevitable. Thus much of the decision making for variance generate the problem of control type.

When information for variance is reported, the manager should ask whether it represents a significant variance from the standard or whether it is simply a random fluctuation around the expected outcome. Thus the purpose of control action is, on the receipt of an accounting report for variance, to ascertain whether the agreed-upon actions were the ones taken, or whether the actual states of nature differed from the expectations and, finally, whether some corrective or adaptive action is appropriate.

The present work will concentrate on exploring how to assess an accounting report to determine whether or not the investigating action should be taken in a given cost variance system. The situation is probabilistic in that the disappointing performance does not necessarily mean that corrective action should be taken, since the disappointing performance may be caused by some uncontrollable and nonrecurring factor.

To better understand the nature of decision focus in the Cost Variance Investigation models, it is helpful to consider two polar cases [Demski and Kreps(1982)]. In some contexts, the accounting report is produced in order to better make a particular decision. In its purest form, such a report is obtained in a single-person setting or in a setting where the fact that the report is obtained does not change the opportunity set of the recipient. This is called Decision-Facilitating Case. In other contexts, the accounting report is produced in order to change the behavior of a second party, normally because the compensation received by the second party depends on the report. In its purest form, this sort of report would not be obtained until after the decision in question is taken, so that it can have no direct effect on the decision. This is called Decision-Influencing Case [Baiman and Demski(1980a;1980b) and Lambert(1985)]. The present study will be restricted within the Decision-Facilitating Case.

Since the beginning of the 1960's [Bierman, Fouraker and Jaedicke(1961)], the studies of the decision making for cost variance have brought about the reconsidering of a simple control chart which indicates whether or not a cost report is contained within a certain control limit. These are models to determine whether or not a Cost Process should be investigated with respect to the cost and benefit of control action when a cost report is submitted. When control action is selected under considering opportunity costs, or operating costs and investigation costs, the previous accounting literature can be classified into three types according to how the control variables are established. The first type is the Decision Theoretic Approach [BFJ(1961), Duvall(1967), Dyckman(1969)], and the model in which the control variable is denoted as a Bayesian posterior probability with respect to opportunity costs. The second is the Dynamic Programming Approach [Kaplan(1969;1975), Hughes (1975;1977), Buckman and Miller(1982)], and Bayesian-sequential analysis in which the control variable is denoted as a Bayesian posterior probability, relating whether or not to investigate the Cost Process with respect to operating costs and investigation costs. Adding to the two approaches above, another type is the Markovian Approach [Dittman and Prakash(1978;1979)], a model used to decide whether or not to investigate the Cost Process with respect to the operating costs and investigation costs, in which the control variable is denoted as a cost variance itself.

The present work, will concentrate on developing another Decision Theoretic Approach model in which the control variable is denoted as a cost variance itself, in place of Bayesian posterior probability used in the previous Decision Theoretic Approach models, and will be studied on the basis of assumption that the cost-generating-process is a two-state, discrete-Markov process in which a cost report at the end of each period provides information about current status of the process.

2) Purpose of the Study

In the case of Decision Facilitating, three approaches, that is, Decision Theoretic Approach, Bayesian Dynamic Programming Approach and Markovian Approach, have been introduced. However, these approaches may entail trade-offs with respect to the "best policy". As pointed out by Kaplan(1982), starting with complex models does not seem worthwhile when one considers the cost of the information requirements and processing algorithms and the small incremental benefits these models have yielded in simulation studies. In other words, due to the relative high costs of solving complex decision models, a more simple decision model in some cases have cost effective.

Thus the author was motivated to develop a new method, whereby even if the new method does not have cost savings against the other previous models it is more convenient and can obtain more technical advantages.

The purposes of the present work can be described, therefore, as follows:

1) to develop another method(known as the "Normal Form of Bayesian analysis"), with respect to the Decision Theoretic Approach, in which the control variable is

denoted as a cost variance itself in place of Bayesian posterior probability used in the previous Decision Theoretic Approach models(known as the "Extensive Form of Bayesian analysis").

2) to extend the proposed model by considering the Exploratory Investigation case.

3) to examine the proposed model as a problem in numerical analysis.

4) to develop the N-Cost Processes Case as the extension of One Cost Process problem.

5) comparison between the Full and Exploratory Investigation models proposed in this study by performing simulation.

3) Chapter Overview

In chapter II, the previous accounting literature about Cost Variance Investigation models, relevant other models and their comparisons are reviewed. However, control chart approaches used in industrial quality control techniques have been excluded, because they do not formally incorporate the expected costs and benefits from a variance investigation and/or the out-of-control state into the model.

Chapter III shows how development of the basic assumptions in each paper have been modified throughout all the previous models, and how the assumptions of the present work should be modified relatively to the previous.

Basic assumptions and the control procedure used in the present work are described in chapter IV. These assumptions can be classified into general assumptions similar to the previous models and additional assumptions used only in the present work.

A new form of Decision Theoretic Approach is proposed in chapter V. Furthermore, Investigation policies using statistical hypothesis are established, and the new form is shown as a Bayes test that turns out to be a Simple Likelihood-Ratio Test. In addition to the One Cost Process case proposed above, Cost Variance Investigation Policies in N-Cost Processes are also discussed. This case can be referred to as the Full Investigation due to the assumption that the investigation of an out-of-control process will always disclose its causes.

Chapter VI suggests a model that enables an Exploratory Investigation as opposed to the Full Investigation approach, by considering the probability that the cause of an out-of-control process will be discovered when it exists. This case being also discussed according to the proposed method above. Similarly to the previous chapter, the N-cost processes of the Exploratory Investigation cases are discussed.

Chapter VII shows how alternative values of parameters affect the proposed model by using alternative values of parameters similar to those used in Dittman and Prakash's(1979) study.

Furthermore, the relation between the Full and Exploratory Investigation cases is simulated with respect to the average total cost.

Chapter VIII, will summarize the results and contributions of the proposed model against the previous models, and will discuss the limitations and extensions of the proposed model.

CHAPTER TWO

LITERATURE REVIEW

1) Overview

The previous accounting literature in the area of cost variance analysis are reviewed in this chapter. However, the developing of basic assumptions for each approach will be studied in chapter three, because, in this chapter, connection between the previous literature throughout each approach is mainly reviewed.

The Decision Theoretic Approach models [Bierman, Fouraker and Jaedicke(1961), Duvall(1967) and Dyckman(1969)] are reviewed in section 2. The Bayesian Dynamic Programming Approach models [Kaplan(1969;1975), Hughes(1975;1977), Buckman and Miller(1982)], according to their relationship with the Decision Theoretic Approach models, are reviewed in section 3. A Markovian Approach model [Dittman and Prakash(1978)] is reviewed in section 4. Comparisons among three approach models [Magee(1976), Jacobs(1978), Dittman and Prakash(1979)] are reviewed with respect to expected costs, optimality, whether realistic or not, and some other criteria in section 5. Section 6 reviews the Exploratory (or Partial) Investigation models proposed by Dyckman's(1969) extensions, Kaplan(1975) and Kim(1983). In section 7, some other cost-variance analysis models [Demski(1970), Hannum(1970), Ozan and Dyckman(1971), Ronen(1974), Magee(1977) and Buckman and

Miller(1982)] with other viewpoints(e.g., multiorigin cost variances, grouping of sources and reporting time etc.) are reviewed.

2) Decision Theoretic Approach

The simple form of decision policy is the Control Chart procedure in which the process is investigated if the observed cost exceeds a critical value(or critical limit). However, this Control Chart Approach was criticized due to the fact that it did not consider the costs and benefits based on to investigation policy. Thus BFJ(1961) were the first to introduce the costs and benefits of an investigation into the investigation decision in the accounting literature. They tried to derive the investigation region by minimizing the expected cost, and proposed the cost-benefit matrix by action-state as follows:

actions / states	don't investigate	investigate	probability of state
in-control	0	C	P
out-of-control	L	C	1-P
expected costs of actions	(1-P)L	C	1.00

where: C= the cost of investigating the unfavorable variance
L= the present value of the costs that will be incurred in the future if an investigation is not made now

Figure 1: BFJ's Cost-Benefit Matrix by Action-State

When Pc is defined as P which setting the expected values of the two action equal yields C = (1-P)L, an investigation is signalled if P<Pc.

This model was criticized by Kaplan(1975) about the measure of L as follows:

> One problem with this formulation is the difficulty of estimating or even interpreting the benefit L. The benefits L depends upon future actions, and models which do not specially include the consequences of future actions will have a hard time defining, much less estimating, what is the benefit from current actions.(p.319)

This model also had an important defect which P[x|in-control] was used in order to determine P[in-control|x] with respect to forming Pc, so that Duvall(1967) proposed a new model.

Duvall suggested a procedure which showed how the distribution of controllable variance y and the relevant cost might be made a variables of the investigation policy in order to see whether an investigation was expected to be worthwhile. He assumed that an observed variance consisted of a non controllable component w (with $N(0,\sigma_w^2)$) and a controllable component y. The controllable component y was also assumed to be normally distributed and statistically independent of the noncontrollable component w. He developed a procedure which derived the distribution of y to be estimated from the observed variances, and derived the objective function associated with the expected profit resulting from an investigation by that procedure as follows:

the objective function $U(x) = \int_{-\infty}^{0} -(Ly+C)f(y|x)dy + \int_{0}^{\infty} (ky-C)f(y|x)dy$

where: x=each observed variance

y=a part due to controllable causes(out-of-control)

C=the average cost of an investigation

L=the present value of future savings resulting from discovering a controllable variance y that is proportional to the size of the variance

k=proportional constant

the profit resulting from an investigation, $P(y) = \begin{cases} ky-C & \text{if } y>0 \\ -(Ly+C) & \text{if } y \leq 0. \end{cases}$

By the objective function above, the investigation region is shown as in the following figure:

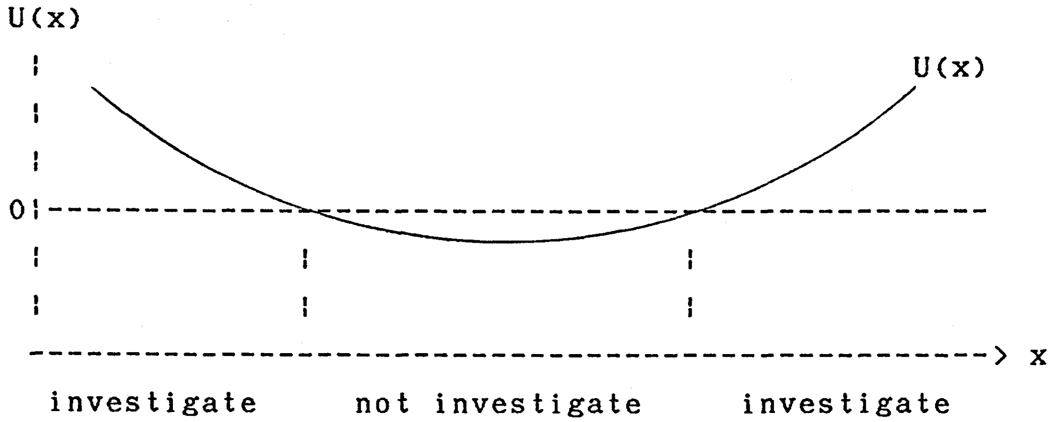


Figure 2: Duvall's Investigation Region

As a result, as compared with BFJ's model, there is a distingushing mark that he developed procedures which allegedly allowed the parameters of the distribution of y to be estimated from the observed variances.

However, this model also was criticized by Kaplan(1975) as in the following three sides:

> There is still a problem as to how to measure the savings from an investigation since this model does not incorporate the possibility of investigations in the future. The computational processes of the μ_y and σ_y treat each observation equally and symmetrically and this can only be done for a stationary process, one in which the parameters do not shift over the course of the observed period. Duvall's problems arise because he failed to specify the stochastic process which leads to changes in the distribution of y. (pp.328-330)

Dyckman(1969) extended the BFJ's and Duvall's model by including the posterior probability to consider the information from previous observations on the process and transition probability g from an in-control state to the in-control state just before a cost report was submitted. He suggested the following cost-benefit matrix by action-state in order to allow the investigation region:

states		in-control θ_1	out-of-control θ_2	expected costs of actions
transition probability		g	1-g	
action	don't investigate	0	L-C	$(L-C)P[\theta_2 \mid x]$
action	investigate	C	0	$C\ P[\theta_1 \mid x]$
posterior probability		$P[\theta_1 \mid x]$	$P[\theta_2 \mid x]$	

where: C= some constant investigating cost

L= the present value of the savings obtainable from an investigation when the activity is out of control

x= a random variable which represents the reported costs in a given period

P(=the transition probability between two states) $= \begin{bmatrix} g & 1-g \\ 0 & 1 \end{bmatrix}$

Figure 3: Dyckman's Cost-Benefit Matrix by Action-State

He suggested the decision rule that if $CP[\theta_1 | x]+(C-L)(1-P[\theta_1 | x]) \leq 0$, then investigating the process was better than not. He also suggested the decision problem of a continuum of states by revising Duvall's model(1967). He supposed the savings S to be different but proportional to the cost variances as follows:

$$S=\begin{cases} b_1(\theta_c-\theta)-C & \text{for } \theta \leq \theta_c \\ b_2(\theta-\theta_c)-C & \text{for } \theta \geq \theta_c \end{cases}$$

where θ_c is the expected cost when the process is in its optimal control state and b_1 need not equal b_2.

Therefore, following an observed cost of x, the expected saving from an investigation is given by

$$E[S]= \int_{-\infty}^{\theta c} [b_1(\theta_c-\theta)-C]f_1(\theta | x)d\theta + \int_{\theta c}^{\infty} [b_2(\theta-\theta_c)-C]f_1(\theta | x)d\theta$$

$$= [b_1+b_2][\sigma_1(\theta)][f_z(z_c)-z_cF_z(-z_c)]+b_2z_c\sigma_1(\theta)-C.$$

where $z_c= |E_1(\theta)-\theta_c|/\sigma_1(\theta)$

$f_z(z_c)$= standard normal density function

$F_z(z_c)$= standard normal cumulative distribution function

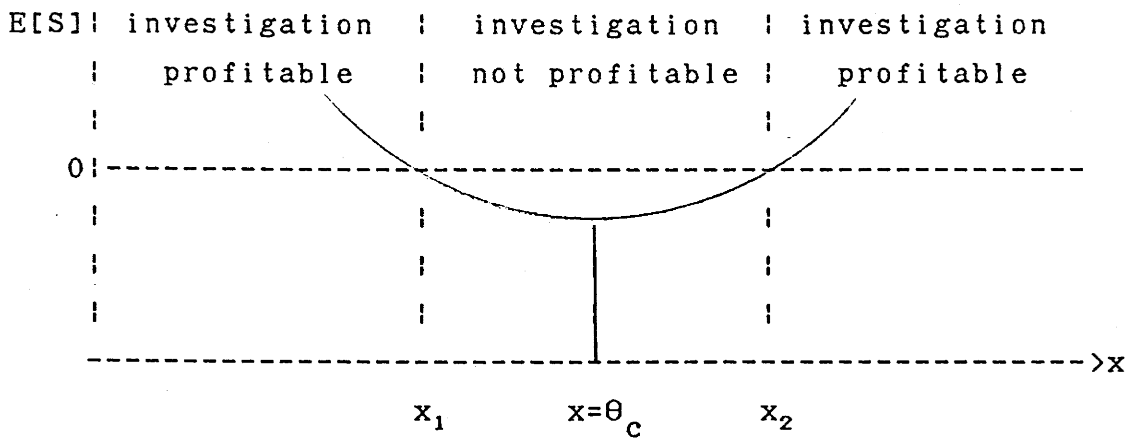


Figure 4: Expected Saving Conditional on a Single Cost Observation

Note that x_i (i=1,2) of the above figure cannot be sought because the normal cumulative function $F_z(-z_c)$ has not been known and then the inverse function of $F_z(-z_c)$ does not exist.

However, this model was criticized by Li(1970) about estimating L, while Duvall's model also had the similar faults, as follows:

> L cannot be estimated until the optimal decision rules are known; but the optimal decision rules are supposed to be output of the model.(p.283)

For examining and solving the above problem, Magee(1976) formulated the following equation for L and used it for his simulation model:

$$L_n = (\mu_2 - \mu_1)(\text{expected time until the process goes out of control})$$
$$= g^n n(\mu_2 - \mu_1) + \sum_{j=1}^{n-1} g^j (1-g) j (\mu_2 - \mu_1)$$

where: n= the number of periods left in the decision horizon

g= the probability that an in-control cost process will remain in control next period

μ_1= the expected operating cost per period when the process is under control

μ_2= the expected operating cost per period when the process is under out-of-control.

His simulation results are discussed in section 5.

3) Bayesian Dynamic Programming Approach

Kaplan(1969), used the discrete dynamic programming to find the optimal policy, and considered a two-states model used transition probabilities between the two states in successive periods for the first time.

The purpose of his model was to illustrate how the various costs and probability distributions that arised in this situation were able to be integrated to yield the optimal policy with respect to the minimum expected costs. This approach overcomed the difficulties of estimating L, namely, the benefits of investigation, by using the actual costs when operating in or out of control.

His model was developed on the basis of the same assumption with Decision Theoretic Approach. However, as opposed to Decision

Theoretic Approach, control actions are taken before the state transition is generated. Thus the control process of Kaplan's model is summarized as in the following process:

```
                         the end of period i
prior                    =the beginning of
probability               period i+1
=[q_{i-1},1-q_{i-1}]     [O---------->O]------------------------------------>
                          |           ↑          transition probability
                          ↓           |               [ g    1-g ]
          cost report x_i  control action             [ 0     1  ]

           the end of
           period i+1
--------------->O
                |
```

$$P_{i+1}[\theta_1 | x_i] = q_i | x = \frac{g f(x|\theta_1) q_{i-1}}{f(x|\theta_1) q_{i-1} + f(x|\theta_2)(1-q_{i-1})} = \tau_x q_{i-1}$$

Figure 5: Kaplan's Control Process

Given that after the process is investigated once, if the probability of being in control is one, the optimal equation is given by

$$C_n(q) = \text{Min}\ \{K + \int [x + \alpha C_{n-1}(\tau_x g)][g f(x|\theta_1) + (1-g) f(x|\theta_2)]dx;$$
$$\int [x + \alpha C_{n-1}(\tau_x q)[q f(x|\theta_1) + (1-q) f(x|\theta_2)]dx\}.$$

Here, K is assumed to be a cost of investigation and correction.

An extended version of Kaplan's(1969) model was treated by Ross(1971). Under supposing that the state of the process at time n is unknown and only becomes known when an item is sampled, he provided a framework for handling the problem due to three action, that is, produce without inspection; produce with inspection(to learn the current state); and revise the process to state 1. For his formulation, he was able to obtain the most generalized results.

Similarly to that of Ross(1971), Hughes(1975) extended Kaplan's(1969) optimal policy by redefining the actions available to the manager as in the three following case: 1)do nothing; 2)obtain a report of period costs; and 3)investigate the reported period costs and take a necessary-corrective action to place the process in control.

With reference to Kaplan's model, his validity was advocated as the following reasons:

> Kaplan assumes that cost variances are reported periodically. However, given that expenditures for preparing cost reports may oftentimes be nontrivial, these reports should only be prepared when, and if, they can be justified by the expected benefits to be derived from their use.(p.344)

He also defined the timing of revisions different from Kaplan(1969)(i.e., transition occurs during the operating period before costs are reported), making use of the following Bayesian operator:

$$\tau_x(q) = \frac{qgf(x|\theta_1)}{qgf(x|\theta_1)+(1-qg)f(x|\theta_2)} \quad .$$

Thus the optimal equation was given by

$$V_n(q) = \text{Min}\begin{cases} C(q)+\alpha V_{n-1}(qg) \\ C(q)+R+\alpha\int[qgf(x|\theta_1)+(1-qg)f(x|\theta_2)]V_{n-1}(\tau_x(q))dx \\ C(q)+K+\alpha V_{n-1}(1) \end{cases}$$

where $C(q) = qg\int xf(x|\theta_1)dx + (1-qg)\int xf(x|\theta_2)dx$

q_i = the probability that the process is in control at the start of the ith period

R = the cost of reporting

K = the costs of investigating and taking corrective actions.

He concluded in further remarks as follows:

> As long as the Markov property is fitting, there will be no benefit to obtaining cost information from any period other than the most recent when that action is taken.(p.348)

But in his correction paper(1977), he corrected his conclusion by referring that, although the above statement is obviously correct when cost information is perfect, regrettably, it is incorrect when that information is imperfect. Thus his additional correction was reconsidered by the following purpose:

> The purpose of this note is to show that there could be a benefit to cost information from other than the most recent period by presenting a numerical example using a model with a sampling rule calling for costs, in each period since the last time that the action to obtain a cost report or to restore the process was taken, to be reported.(p.313)

Therefore, he suggested a new process that revised the implicit sampling rule of receiving only the cost observation from the most recent period when a cost report was obtained, to receiving cost observations from each period since the last cost report was obtained or the process was investigated/corrected.

The model under K-periods sampling rule is expressed as follows:

$$V_n(q,k) = \operatorname{Min} \begin{cases} C(qg^k) + \alpha V_{n-1}(q,k+1) \\ C(qg^k) + R + \alpha \int \ldots \int f(x_1, x_2, \ldots, x_k) * \\ \qquad V_{n-1}(\tau_{x_1, x_2, \ldots, x_k}(q), L)\, dx_1, dx_2, \ldots, dx_k \\ C(qg^k) + K + \alpha V_{n-1}(1,1) \end{cases}$$

where

$$\tau_{x_1,\ldots,x_k}(q) = \frac{f(x_1,\ldots,x_k \mid \theta_1) qg^k}{f(x_1,\ldots,x_k \mid \theta_1) qg^k + f(x_1,\ldots,x_k \mid \theta_2)(1-qg^k)}$$

and where: $f(x_1,\ldots,x_k) = f(x_1,\ldots,x_k \mid \theta_1) qg^k + f(x_1,\ldots,x_k \mid \theta_2)(1-qg^k)$

$$f(x_1,\ldots,x_k \mid \theta_1) = \Pi f(x_m \mid \theta_1)$$

$$\begin{aligned} f(x_1,\ldots,x_k \mid \theta_2) = {} & \Pi f(x_m \mid \theta_2)(1-qg)/(1-qg^k) \\ & + \Pi f(x_1 \mid \theta_1) f(x_m \mid \theta_2)(qg-qg^2)/(1-qg^k) + \ldots \\ & + \Pi f(x_m \mid \theta_1) f(x_k \mid \theta_2)(qg^{k-1}-qg^k)/(1-qg^k). \end{aligned}$$

As a result, he identified the following three sampling rules as the dynamic-optimal-control problem:

1) to obtain a cost report containing one observation every period [Kaplan(1969)].
2) to obtain a cost report containing a cost observation from just the most recent period at times determined from an optimal policy [Hughes(1975)].
3) to obtain a cost report containing cost observations from all periods since the last time either a cost report was obtained or an investigation/correction was made at times determined under an optimal policy [Hughes(1977)].

Buckman and Miller(1982) suggested an N-Cost Processes system where each process was assumed to satisfy the assumptions of the Kaplan's(1969) model. Their cost processes were established by the assumption that correction action took place for all n processes or for none of them. The purpose of their model was to determine a decision rule which minimized the expected discounted cost over an infinite planning horizon. However, because the problem could not be solved by using the standard methods of dynamic programming for computational reasons, they proposed a Myopic Policy, namely, One Period Look Ahead Policy, which was optimal for certain problems. Their model can be also reduced into that of Kaplan(1969) in the case of One Cost Process except for using Optimal Stopping Rules. In section 6, their model is discussed, in detail, with respect to Multiple Cost Processes.

4) Markovian Approach

Magee(1976) conducted an empirical study for comparing the various Cost Variance Investigation models. Several simple Control Chart models with different control limits, Dyckman's(1969) Decision Theoretic model and Kaplan's(1969) Dynamic Programming model were considered as candidates for the best model, and were compared and evaluated.

The simulation results of Magee show that a control policy using a fixed cost control limit can capture much of the benefits obtainable from more complex models such as those of Kaplan or Dyckman. Hence, Dittman and Prakash(1978) suggested a Cost Variance Investigation model to determine the optimal cost control limit for the class of policies which used a fixed critical cost to signal a need for investigating the Cost Process.

They obtained an equation for the long-run average cost per period of controlled operation. They also used Markovian Process Transition Matrix corresponding to Kaplan(1969). But the process transition was supposed to have taken place before a cost report was generated.

They considered two control alternatives:
The first alternative is to regard the process as having gone out of control, and so to incur a fixed discretionary investigation cost T. If the process is found when being in out-of-control state, it is resetted into the in-control state with a constant correction cost K. But if the process is found when being in in-control state, it is left to operate as is, and then commits a Type-I error, $1-F_1(\bar{x})$. The second is to regard the process as being in-control state, allowing it to run without intervention for one more period but takeing the risk of committing a type-II error $F_2(\bar{x})$.

By two control alternatives above, the control process trasition matrix is shown as in the following figure:

$$
\begin{array}{ccc}
\text{state} & 1 & 2 \\
1 & g & 1-g \\
2 & 0 & 1
\end{array}
\qquad
\begin{array}{ccc}
 & 1 & 2 \\
1 & 1 & 0 \\
2 & 1-F_2(\bar{x}) & F_2(\bar{x})
\end{array}
$$

Process Transition Matrix, Control Transition Matrix

$$
\begin{array}{ccc}
\text{state} & 1 & 2 \\
1 & 1-(1-g)F_2(\bar{x}) & (1-g)F_2(\bar{x}) \\
2 & 1-F_2(\bar{x}) & F_2(\bar{x})
\end{array}
$$

Controlled Process Transition Matrix

where: $F_i(\bar{x}) = P[x \leq \bar{x} \mid i]$,

$i=1$: in-control state,

$i=2$: out-of-control state

x= observed cost

$\bar{x}$= critical value

Figure 6: Dittman and Prakash's Controlled Process Transition Matrix

They obtained the steady-state probabilities, $\pi_i(\bar{x})$ and $S_i(\bar{x})(i=1,2)$, respectively by using controlled process transition matrix as follows: $\pi_i(\bar{x})$ is the steady-state probability for the state in which the process finds itself at the end of the managerial control by $\pi_2(\bar{x}) = \pi_1(\bar{x})(1-g)F_2(\bar{x})+\pi_2(x)F_2(\bar{x})$ and $\pi_1(\bar{x})+\pi_2(\bar{x})=1$; $S_i(\bar{x})$ is the steady-state probability of the state before generating the cost reports by $S_1(\bar{x}) = g\pi_1(\bar{x})$ and $S_2(\bar{x}) = (1-g)\pi_1(\bar{x})+\pi_2(\bar{x})$.

The expected cost per period of operating the controlled process is obtained as the sum of the expected cost per period of 1) operating 2) investigating 3) correcting the process.

The sum of the expected cost per period was given by

$C(\bar{x}) = C_o(\bar{x}) + C_t(\bar{x}) + C_k(\bar{x}) = \mu_2 + \pi_1(\bar{x})\{a - bF_1(\bar{x})\}$

where:

$C_o(\bar{x})$ = operating costs= $\mu_1 S_1(\bar{x}) + \mu_2 S_2(\bar{x})$

$C_t(\bar{x})$ = investigating costs= $\{P[x>\bar{x} \mid i=1]S_1(\bar{x}) + P[x>\bar{x} \mid i=2]S_2(\bar{x})\}T$

$C_k(\bar{x})$ = correcting costs= $P[x>\bar{x} \mid i=2]S_2(\bar{x})K$

$a = (1-g)K + T - g(\mu_2 - \mu_1)$

$b = gT$.

Thus the fixed critical cost x^* to signal a need for investigating the Cost Process can be obtained by the computational result such that $C(x^*) = \text{Min } C(\bar{x})$.

But this proposed function has the problem that the existence of x^* is restricted within $0<a<b$, $F_1(x^*) > (a/b)$, and the total cost function is not easily differentiated.

They concluded about the properties of their Markovian Control Approach as follows:

> In summary, we offer the practicing accountant a method for determining cost control policy i)which does not require dynamic programming and ii)which does not require Bayesian updating of probabilities after each cost report. It simply requires the matching of a reported cost variance against a fixed critical limit. The critical limit is simple to caculate, and the resulting policy yields much of the benefits of the policies based upon more sophisticated cost control models.(p.25)

5) Comparisons of Investigation Policies

Three comparison studies of Cost Variance Investigation Policies were conducted by Magee(1976), Jacobs(1978) and Dittman and Prakash(1979).

In the study for what is the "best" policy among all possible control policies, Dittman and Prakash laid down a basis for the research design of studies on cost management. They insisted that the "best policy" depended upon the ranking criterion as the present value of expected cost or the long-run expected cost per period, or the other criterions, but the optimality with respect to one criterion might entail trade-offs with respect to some other criterions.

Magee simulated all possible control policies with ranking criterion used the average total cost of the first 12 periods, while Dittman and Prakash used long-run expected cost per period as the ranking criterion and obtained their data by using analytical(numerical) methods as opposed to simulation.

However, Jacobs carried out a field experiment from an actual firm to evaluate the effectiveness of variance investigation policies, rather than Magee's computer simulation and Dittman-Prakash's analytical method.

Firstly, Magee conducted an empirical study concerned with finding the relative superiority of various Cost Variance Investigation models, using the average total cost as a criterion for model evaluation. He examined three general types: The first is several Control Chart models based on the probability that a reported observation could have occurred when the process is in-control; the second is the statistical Decision Theory Approach employed by Dyckman(1969); and the third is Kaplan's(1969) Dynamic Programming model. He did not consider Dittman and Prakash's(1978) model because it had not been developed at that point. He performed a simulation study of the following seven investigation rules:

1. Investigate all unfavorable variances.
2. Investigate all variances that exceed the standard(μ_1) by 10 percent.
3. Investigate all cost observations that exceed the standard(μ_1) by at least one standard deviation σ_1.
4. Investigate all cost observations that exceed the standard(μ_1) by at least $2\sigma_1$.
5. Investigate if the updated probability that the process is in-control is less than a critical probability determined when using Dyckman's(1969) policy.
6. Investigate if the updated probability that the process is in-control is less than a critical probability determined when using Kaplan's(1969) policy.
7. The investigation decision based on perfect knowledge concerning the state of the process.

Each of the seven policies was performed for 200 simulated 12-month periods with respect to the average total cost.

The research can be summarized as follows:

1. In testing the hypothesis that the seven means are equal, method(6), the dynamic programming method, has the lowest expected cost if method(7) is disregarded as being

unrealistic.

2. The differences between the methods (5) and (6) are not very large, indicating that Li's(1970) criticism of Dyckman's(1969) approach for not considering future actions, while valid theoretically, may have little effect on the incremental cost savings, at least in the cases examined.
3. The best method in terms of average total cost under uncertainty appears to be method(6) which in all cases was closest to method(7). However, the differences between method(6) and method(4) are not terribly large in most cases.
4. There is no overwhelming evidence that a manager who uses a "native" method is making a poor model choice decision, and the opposite may be true.

Thus he concluded that a manager who was concerned with all costs (including operating costs, investigating costs, implementing costs and information costs) could very well prefer a less complex investigation rule, and that survey research should also determine the basis on which the decision-maker was evaluated and rewarded, since this would affect his or her model choice.

Secondly, Dittman and Prakash compared the long-run expected cost per period of the best Makovian control vis-a-vis the optimal control(the best Bayesian policy) for a wide range of cost situations. Using numerical analysis, they calculated the opportunity cost of using the best Markovian policy developed by Dittman and Prakash(1978), compared to an optimal policy based on Kaplan's(1969) dynamic programming technique.

They explained the reasons of caculating the opportunity cost as follows:

> It is a theorem of Arrow et al.(1949) that, with long-run expected cost per period as the ranking criterion, the optimal policy for any cost situation is equivalent to the best Bayesian policy for that situation. This means that a preference for Markovian control necessarily involves an opportunity cost."(p.359)

The results of their study are summarized as the following two basic findings:

1. The opportunity cost of the best Markovian control is a small fraction of the mean difference between the in-control and out-of-control costs unless the in-control cost has at least a moderately large coefficient of intrusion and a substantially greater dispersion than the out-of-control cost.
2. The opportunity cost of the Dittman-Prakash policy decreases as the level of the uncertainty of the out-of-control cost increases to a value somewhat higher than the level of uncertainty of the in-control cost; beyond this point the trend reverses.

As a result, Dittman and Prakash showed that the somewhat simpler Markovian Approach was able to produce total costs virtually identical to the optimal, but more complex, Dynamic Programming Approach.

Different from Magee's study, they also showed that critical cost range had to approximately be between $0.6\sigma_1$ and $2.7\sigma_1$, and then the classical rule, by which control limits are set at a distance of $2\sigma_1$ or $3\sigma_1$ from the mean in control cost, cannot be trusted to provide "good enough" control.

Finally, Jacobs presented the results of a field experiment in which six specific models had been evaluated as the models had been actually used to assess the significance of cost variances in a manufaturing firm. He classified models according to the taxonomy developed by Kaplan(1975) as follows:

	single observation	sequential of observation
cost and benefits of investigation not considered	Shewart x-control chart	Cumulative Sum Chart [Wald(1947)]
cost and benefits of investigation considered	Economic x-chart [Duncan(1956)]	Economic Cumulative Sum [Goel and Wu(1973)] Single-Period Bayesian Model[Dyckman(1969)] Multi-Period Bayesian Model [Kaplan(1969)]

Figure 7: Jacob's Taxonomy of Cost Variance Investigation Models

As shown in the taxonomy above, he used six decision policies on ten processes that considered physical usages and utility items. Two techniques were used to evaluate the relative effectiveness of the models. The first is an analysis of the relative frequencies of the type-I and type-II errors. The total cost of type-I and type-II errors committed by each model was calculated and compared with all other models to determine an effectiveness ranking. The second is a sensitivity analysis ranking technique. Model performances were examined with respect to:

1. all variables;

2. variable groups based on loss/investigation cost ratios;
3. variable groups based on normality of in-control distribution;
4. variable groups based on cost behavior patterns;
5. variable groups based on monitoring frequencies.

In addition, comparisons of the following model groups were made:

1. noneconomic versus economic;
2. single observation versus multi-observation.

By grouping the models, the empirical results can be supported by theoretical considerations in researching conclusions.

He concluded the relative cost effectiveness of six models as in the following results:

1. For all variables, the multi-observation models were generally more effective than the single-observation models.
2. for variables with low loss/investigation cost ratios(<6), the cumulative sum chart was consistently more effective than the other five models.
3. For variables with low loss/investigation cost ratios, the economic models were not consistently more(or less) effective than the noneconomic models.
4. For variables with high loss/investigation cost ratios(>17), the economic models were consistently more effective than the noneconomic models.
5. For variable groups based on the normality of the in-control distributions, the multi-observation models were consistently

more effective than the single-observation models.

6. For variable groups based on fixed behavior patterns and weekly monitoring, model performances were identical to those for the low-loss variables.

6) <u>Exploratory(or Partial) Investigation Policy</u>

Dyckman(1969), in his extensive chapter, extended the traditional two-action space (Investigate, Don't investigate) model by allowing for an Exploratory Investigation that the cost of such an investigation was assumed as C' (C'<C), and that h was assumed as the probability that the cause of an out-of-control process would be discovered when it existed.

He considered the following loss matrix due to the action such as the Exploratory Investigation:

	don't investigate	exploratory investigation
θ_1	0	C'
θ_2	Lh - C'	0

Figure 8: Loss Matrix by the Exploratory Investigation

Thus the investigation policy can be made by selecting the action with the smallest expected cost from the following alternatives:

alternative	action	expected cost
a	Don't Investigate	0
b	Exploratory Investigation	$C'f_n(\theta_1)+[C'-Lh][1-f_n(\theta_1)]$
c	Full Investigation	$Cf_n(\theta_1)+[C-L][1-f_n(\theta_1)]$

Figure 9: Expected cost from the Three Alternatives

Setting alternative b and c equal, two breakeven probabilities can be determined as $f_b(\theta_1)=(Lh-C')/Lh$ and $f_c(\theta_1)=1-[(C-C')/L(1-h)]$.

He also considered N-Cost Processes case as the extension of One Cost Process. He showed how the optimal set of processes to be investigated in any given period j was able to be selected in Multi-Cost Processes as in the following equation:

$$\max \sum_{i=1}^{k}\{y_i[(L_i-C_i)(1-f_{ij}(\theta_1|x_j))-C_if_{ij}(\theta_1|x_j)] \\ +y_i'[(L_ih-C_i')(1-f_{ij}(\theta_1|x_j))-C_i'f_{ij}(\theta_1|x_j)]\}$$

Subject to

$$\sum_{i=1}^{k}(Y_iC_i+Y_i'C_i')\leq M$$
$$y_i[(C_i-L_i)(1-f_{ij}(\theta_1|x_j))+C_if_{ij}(\theta_1|x_j)]\leq 0\,(i=1,\ldots k)$$
$$y_i'[(C_i'-L_ih)(1-f_{ij}(\theta_1|x_j))+C_i'f_{ij}(\theta_1|x_j)]\leq 0\,(i=1,\ldots k)$$
$$y_i+y_i'\leq 1$$
$$y_i,y_i'=0 \text{ or } 1$$

where M= a firm budgets.

Kaplan(1975) extended his Bayesian Dynamic Programming Model (1969) by allowing for an Exploratory Investigation like Dyckman(1969). If the probability that the system is in control is q, then the probability of finding it out of control with the Exploratory Investigation is (1-q)h. When the process is found to be out of control, it is reset and a new cycle starts. Conversely, the probability that the system is not found to be out of control by the Exploratory Investigation is 1-(1-q)h. If q was the probability of the system being in control before the unsuccessful Exploratory Investigation, then the posterior probability of being in control is obtained by $q/[1-(1-q)h] = q/[q+(1-q)(1-h)]$.

Formally, if K' is the cost of the Exploratory Investigation, the expected infinite horizon future cost from under taking the Exploratory Investigation is given by

$$K'+(1-q)h \int [x+\alpha C(\tau_x g)]f_g(x)dx+ [1-(1-q)h] \int [x+\alpha C(\tau_x q')]f_{q'}(x)dx$$

with $q'=q/[1-h(1-q)]$.

For a more general treatment, he defined the amount of money spent on an investigation as a continuous variable K' and a function h(k') as the probability of detecting an out-of-control situation when one existed. He also assumed that $h(0)=0$, $\lim_{K'\to\infty} h(k')=1$ and that h(k') was a nondecreasing function of K'. Therefore, the expected infinite horizon future cost from the equation above is revised as follows:

$$C(q)= \min_{K'\geq 0} \{K'+(1-q)h(k') \int [x+\alpha(\tau_x g)]f_g(x)dx + [1-(1-q)h(k')] \int [x+\alpha C(\tau_x q')]f_{q'}(x)dx \}$$

where q' is a function of K' as defined above and setting $K'=0$ with $h(0)=0$ and $q'(0)=q$ yields the expected cost when no investigation is undertaken.

He also criticized that Dyckman(1969) recognized that the probabilities of being in or out of control should be revised to reflect the Exploratory Investigation outcome but neglected it.

Kim(1983) attempted to extend the previous studies by allowing for a Partial (or Exploratory) Investigation. He considered three Partial Investigation cost function by using a function u(h) with u(0)=0 and u(1)=K, similarly to Kaplan's(1975) h(k'), as follows: 1) $K'=Kh$, 2) $K'=Kh^2$ and 3) $K'=Kh^3$. He insisted that the expected value of the opportunity loss resulted from the Partial Investigation, L', was denoted as L(1-h)(1-q) with $L=q^n n(\mu_2-\mu_1)+\sum_{j=1}^{n-1} jg^j(1-g)(\mu_2-\mu_1)$, then the optimal degree of investigation was obtained by minimizing K'+L' with respect to h.

7) The Other Cost Variance Analysis Models

Demski(1970) insisted that a search procedure was important because any given cost variance might be the result of a number of individual causes and each cause required different search activity. Thus he assumed that the relevant causes of cost variances were categorized by the following five separate variance sources:

i) implementation failure;

ii) estimation error of parameter;

iii) measurement error in measuring the actual cost;

iv) model error;

v) random variance.

He suggested a model to minimize the expected value of the total elapsed time from determination of a significant variance (i.e., the variance is to be searched out and the variance then corrected) to determination of the specific source and subsequent correction, with knowledge that a significant cost variance had occurred and that the source of variance was one of five possible sources.

His objective function for the expected search and correlation time consumed by a specific i,j,k,l,m search sequence is obtained by the following equation:

$$E[T(i,j,k,l,m)] = T_i P(i|D) + (T_i+T_j)P(j|D) + (T_i+T_j+T_k)P(k|D) + (T_i+T_j+T_k+T_l)P(l|D) + (T_i+T_j+T_k+T_l+T_m)P(m|D)$$

where: T_i = the known (and stable) time required to search out variance source i

$P(i|D)$ = the conditional probability that an observed variance was caused by a special source S_i.

As a result, the solution of equation E[T] is obtained by renumbering the five variance sources so that

$T_1/P(1|D) \leq T_2/P(2|D) \leq T_3/P(3|D) \leq T_4/P(4|D) \leq T_5/P(5|D)$.

But Demski's model was criticized by Ronen(1974) as follows:

> His model for optimizing the variance search and corrective activity is conditional on the knowledge that the process is now out of control. Basically, his approach consists of an adaptive formulation in which management assesses prior probability distribution for failures resulting from prespecified sources. This prior distribution may be revised in a Bayesian framework to optimize the search time. Reporting a variance and determining its significance are assumed given, when in fact both of these processes should be

made contingent on the net benefits to be derived from "optimal search."(p.51)

Ronen also pointed out that the previous approaches to the analysis of variances implicitly assumed independence of the reporting(accounting) decision and the control decision; all approaches dealt with the control of failures given that the decision on their reporting already had been made; and more specially, the problem that had by that time been dealt with in the literature centered on the evaluation of the benefits and costs of investigation given a repeated variance, and a finite subset of types of sources was selected for such evaluation.

However, the reporting decision and the investigating decision can be viewed both as parts of the total decision, and then the investigating decision to neglect the reporting decision may result in suboptimization.

Thus his interests was the problem that was transformed into that of determining the appropriate level of aggregation of variances which was resulted from individual sources.

His framework is consisted of as follows:

i) to identify the set of possible sources.
ii) to identify a subset of levels at which the sources can be aggregated for the purpose of reporting a variance.
iii) given the particular levels of aggregation identified in ii) above, to determine what actions may be taken given the resulting variance.
iv) to specify the probability distribution of net incremental savings to be gained from such actions.

v) to estimate the costs of each the aggregation levels specified.

vi) to select the level of aggregation which maximizes the expected net benefits.

Ozan and Dyckman(1971) considered a cost generating process that the total cost variance generated for each process was jointed by many resources. However, cost variance for a resource i was splitted into controllable in one process but noncontrollable in another.

In order to obtain the probability distributions of the controllable and noncontrollable parts of the total cost variance, he used Duvall's(1967) suggestion concerning the distribution between two parts of a cost variance. This distribution was used to obtain the present worth of the expected profit from an investigation. As opposed to the earlier papers, he developed a model of the cost relationships in a cost center that permitted the expected value of a noncontrollable cost variance to be nonzero and that formulated a linear program in which included budgetary and man power constraints.

His objective function is obtained by maximizing the expected utility with respect to investments with uncertain returns, and it is given by the following linear programming:

$$\text{Max } \sum_{i=1}^{m} (E[P_i]-kf(\sigma[P_i]))z_i$$

Subject to

$$\sum_{i=1}^{m} (M_i+L_i)z_i \leq B$$

$$\sum_{i=1}^{m} d_i z_i \leq D$$

$$0<z_i<1$$

where: z_i = the decision variable

d_i = the amount of manpower required to investigate the ith resource

B = the budgeted amount of money for the investigation of cost variances during the current period

D = the total manpower available to investigation all of the cost variances

P_i = present worth of the cost savings from correcting the ith resource cost variance

$f[\sigma(P_i)]$ = a general function of the standard deviation of the present worth of the cost savings

k = a certain constant coefficient.

m = total number of processes.

This model also was criticized by Kaplan(1975) as follows:

> a subsequent paper by Ozan and Dyckman expands on Dyckman's(1969) model by defining different types of controllable and noncontrollable variances. some guidance is offered as to how to estimate some of the many different probabilities this model requires, but the formulation is still in terms of using myopic decision rules which entails the difficulties already discussed in Dyckman's(1969) model. Ozan and Dyckman eventually derive a reward function similar to that used by Duvall(1967).(pp.327-328)

Magee(1977b) suggested a model which allowed for the use of information on cost process commonalities for the choice between a "simplified" cost variance analysis based on considering cost variances individually and a "complete" analysis based on the joint consideration of multi-cost variances.

He discussed an investigation decision model for two Cost Processes, by using Kaplan's(1969) model expanded to a Multiple-Cost Processes. He mainly considered an example that all factors were kept constant, except for the correlation between the two processes.

By analyzing the result of the above example, he summarized the effect of correlation between the two processes as follows:

1) The expected total costs decrease as the dependence increase, although the amount of decrease is relatively small.
2) The decision maker will consider the net benefits of the complete analysis since the expected cost of a simplified analysis will equal the expected cost with independent random cost distribution that correlation coefficient is 0.

Buckman and Miller(1982) modeled an M-Cost Processes system where each Cost Process evolved independently of the others, while Ozan and Dyckman(1971) assumed that the Cost Processes were statistically independent and Magee(1977b) assumed the case that the independence assumption was not made. Investigation and correction were assumed to be made for all M-Cost Processes at once, and the decision problem was to determine when investigation and correction should take place given the vector of probabilities that each cost process was "in-control."

Because the problem could not be solved using the standard methods of dynamic programming for computational reasons. They proposed a Myopic Policy(i.e., One Period Look Ahead Policy) which was optimal for certain problems and heuristic otherwise. In their model, each statistically independent Cost Process was assumed to satisfy the assumptions of the Kaplan's(1969) paper. They made use of the specialized theory in the dynamic programming, by considering the optimal stopping with the One-Period Look Ahead Policy and the regenerative optimal stopping.

They implemented their myopic rule by using the minimum expected discounted cost for an n-period problem,

$V_n(x) = \min\{k \mid C(x,1)-\lambda+\alpha k\}$, as follows:

continue if $C(x,1)-\lambda-k(1-\alpha)<0$

investigate if $C(x,1)-\lambda-k(1-\alpha)\geq 0$

where:

x= the vector of probabilities that each Cost Process is "in control"

$C(x,1) = \sum_{j=1}^{m} \mu_{j1} x_j + \sum_{j=1}^{m} \mu_{j2}(1-x_j)$

λ= a bonus to the decision to continue since that decision uses one time period

k= investigation and correction cost

α= a discount factor.

The myopic rule is optimal for the λ-Stopping Problem if the following monotone condition is satisfied[Ross(1983)]:
Let B represent the set of states for which stopping is at least as good as continuing for exactly one more period and then stopping. The monotone condition is that if $x \varepsilon B$, then the states which can be reached from x in one period have to also belong to B.

However, they implemented the regenerative stopping algorithm with heuristic method when the myopic rule was not optimal.

In their conclusion, they insisted that their model was the simplest meaningful N-Cost Processes system one could formulate, and that a main contribution of their paper would be the fact that it provided an example of a myopic procedure for this class of problems.

Magee(1977a) attempted to address the issue of parameter uncertainty in the cost control by considering two questions:

i) What effect do "mistakes" in assessing distributional parameters have on the control of costs ?

ii) How might uncertainty concerning the cost parameters affects the manager's investigation decisions ?

He discussed a cost system with the information about imperpect parameter knowledge(e.g., out-of-control state) by using Kaplan's(1969) Dynamic Programming Cost Variance Investigation model.

First, he considered the "robustness" of a Dynamic Programming Cost Variance Investigation model with respect to parameter "error". Parameter errors may result in large average costs, then it will indicate that a manager may be willing to expend resources to gain information about the true parameter values. Thus a method was suggested with which used the manager's prior beliefs to form estimates of the desired parameters. Prior beliefs were classified into prior beliefs f(θ) concerning their possible values of parameter and prior probability, P[in-control]=q, concerning the state of the Cost Process.

He suggested the Cost Variance Investigation model with parameter uncertainty that the expected cost of the cost system depended not only on the probability of being in control, but also f(θ) as follows:

$$C_n(q_n, f_{n-1}(\theta)) = \text{Min}\begin{cases} E[\text{future costs if investigate} \mid q_n, f_{n-1}(\theta)] \\ E[\text{future costs if don't investigate} \mid q_n, f_{n-1}(\theta)]. \end{cases}$$

This formulation is similar to the case that considers prior beliefs f(θ) concerning their possible values of parameter in Kaplan's model, but Kaplan's q_n is regarded as Magee's $q_n g$.

As opposed to the existing accounting literature, Hannum(1970) developed the model that determined the reporting schedules for ongoing managerial processes. His model specially discussed for setting reporting frequencies, for a single ongoing process which could be expected to undergo a significant shift(i.e., transformation into out-of-control) in the level of performance after some elapsed operating time.

The optimal reporting schedule is obtained by minimizing the sum of the long-run average opportunity (operating) loss per reporting cycle plus the long-run average cost per reporting cycle of obtaining information and taking necessary control action. His model's control strategy is to obtain information at distinct time points according to some schedule, and his objective function is given by

$$\text{Average cost per cycle} = \sum_{k=0}^{\infty} \int_{x_k}^{x_{k+1}} [C_1(k+1) + C_2(x_{k+1} - t) + C_3] f(t)dt$$
$$= C_1 n_x + C_2(m_x - \mu_t) + C_3$$

where: x_k = the elapsed time in "days" of process operation at which the kth report is to be obtained if the process has been found to be operating in control at the immediately previous(k-1th) report at time x_{k-1} $(k=1,2,\ldots)$

t = the process operating time at which a shift out of control occurs.

C_1 = direct cost of a single report

C_2 = direct cost per day of an undetected shift in the process

C_3 = direct cost of taking control action

$f(t)$ = probability density function of t

μ_t = mean of probability density function f(t)

n_x = average number of reports per cycle given x

X = the sequence $(x_1, x_2, \ldots)$ of reporting times

m_x = average length of cycle given x.

He also discussed how the optimal reporting schedule was obtained according to the character of function called failure rate, $r(t)= f(t)/(1-F(t))$.

He defined two general types of reporting schedules prior to obtain the optimal reporting schedules according to failure rate. The one is "regular" that cost reports are obtained at equal time intervals and the other is "irregular" that cost reports are obtained at unequal time intervals.

When the failure rate is constant, the optimal reporting schedule can be found by trial and error search. But when it is a strictly decreasing failure rate, no general statement can be made on the exact nature of the optimal reporting schedule. If a process has an increasing failure rate, although he mainly analyzed his model in the same case, he shows that a recursive analysis which would assure an optimal strategy can be used in place of a trial and error search.

As a result, the accounting literature discussed until now can be classified as in the following taxonomy:

		one cost process	N cost process	
		with two sources	two sources	N sources
full investigation	Decision Approach	BFJ(1961), Duvall(1967), Dyckman(1969)		Ozan and Dyckman (1971)
	Markovian Approach	Dittman and Prakash(1978)		
	Bayesian Dynamic Programming Approach	Kaplan(1969;1975), Hughes(1975;1977)	Buckman and Miller(1982), Magee(1977b)	
exploratory investigation	Decision Approach	Dyckman(1969), Kim(1983)		
	Bayesian Dynamic Programming Approach	Kaplan(1975)		
other studies		reporting schedule [Hannum(1970)], parameter uncertainty [Magee(1977a)] model comparisons [Magee(1976), Dittman and Prakash (1979), Jacobs (1978)]		aggregation of variance sources [Ronen(1974)] searching sources of variance [Demski (1970)]

Figure 10: Taxonomy of Cost Variance Models

CHAPTER THREE

SEARCH OF THE BASIC ASSUMPTIONS MODIFIED BY LITERATURE

1) Overview

This chapter shows how the development of the basic assumptions in each paper has been modified throughout all the previous accounting literature, and how assumptions of this paper should be modified relatively to the previous ones. In section 2, the author discusses the required information and their estimation, especially L which was mainly assumed in the Decision Theoretic Approach, and also about the definition of operating and investigation costs that were assumed in Bayesian Dynamic Programming and Markovian Approaches. The relation between the point generating a cost report and the point forming decision making is discussed in section 3. Section 4 shows how states of transition have been assumed and estimated, and section 5 covers the control variables in each paper.

2) Required Information and Their Estimation

BFJ(1961) developed a Cost Variance Investigation model to derive the investigation region associated with the required information, L and C. L and C are all the information required in their model, and are defined as follows:

C= the cost of investigating the unfavorable variance

L= the present value of the costs that will be incurred in the future if an investigation is not made now.

However, benefit L depends upon future action, and is therefore difficult to be estimated as discussed in chapter two.

Duvall's(1967) model required the information, μ_x, σ_x^2, σ_w^2, L and C, in order to formulate his model. He defined the cost C as the average cost of an investigation. He also defined the benefit L as the present value of future savings resulting from discovering a controllable variance y that is proportional to the size of the variance. However, the problem as to how to measure the savings from an investigation still exists, since this model does not incorporate the possibility of investigation in the future.

Dyckman(1969) suggested a model to require the information, μ_1, σ_1^2, μ_2, σ_2^2, g, L and C. He defined the cost C as the cost of an investigation, and the saving value L as the present value of the savings obtainable from an investigation when the activity is out of control. However, Li(1970) criticized this approach by saying, "L cannot be estimated until the optimal decision rules are known".

Kaplan(1969;1975) suggested a model that requires similar information with Dyckman's model but without using L. He used the actual costs when operating in or out of control to derive the optimal policies, by the reason that incremental costs of operating out-of-control arised directly from the higher costs that accrued when operating away from standard, and because of the difficulty about how to measure L. He also defined the cost K as the cost of investigation and correction. In his Extensions and Limitations section, the cost K was described as follows: If K_1 is the cost when actually in control and K_2 is the cost when actually out of control, then the expected investigation cost would be $qK_1+(1-q)K_2$; If K_1 and K_2 were themselves random variables, with means K_1' and K_2', then the expected investigation cost would be $qK_1'+(1-q)K_2'$.

Hughes'(1975;1977) model, when compared with kaplan's, requires additional information, that is, the cost of reporting,R.

Dittman and Prakash(1978) form their model by classifing the investigating cost K into the investigating cost I and correcting cost K, differently from Kaplan's K.

In the present study, the investigating cost C includes the value of the manager's or subordinate's time spent on the investigation and any cost of interrupting a production process as discussed by Kaplan(1982); the cost M is defined as the correcting cost, and it is assumed to be known before investigating the Cost Process.

On the other hand, Magee(1976) suggested a new method about estimating the opportunity cost L. He adjusted it to reflect the possibility of future out-of-control periods as follows:

$$L_n = (\mu_2-\mu_1)(\text{expected time until the process goes out of control})$$
$$= (\mu_2-\mu_1)E[N]$$

$= (\mu_2-\mu_1)[\sum_{j=1}^{n-1} jg^j(1-g)+ng^n]$.

He also simulated the relation between Dyckman's(1969) and Kaplan's(1969) models with respect to the average total cost of the first 12 periods by adjusting L. Then the differences between the above two models had little effect on the incremental cost savings.

Thus, the present study will form the Cost Variance Investigation model based on Magee's L_n with respect to Decision Theoretic Approach. Dyckman(1969), Kaplan(1975) and Kim(1983) also require additional information, h, representing the probability that the cause of an out-of-control process will be discovered when it exists, in the Exploratory Investigation cases of their extension parts. Therefore, the present study will seek the optimal policy by using the same information, such as models of Kaplan, and Dittman and Prakash in the Full Investigation case, and such as models of Dyckman, Kaplan and Kim of the Exploratory Investigation case.

3) <u>The Point Generating Cost Reports and Forming the Decision Making</u>

Kaplan(1969;1975) defined the posterior probability q_i in period i as the probability that the system was in a in-control state(i.e., state 1) during period i+1 given the most recent observation $X_i=x$ and the previous probability estimates$=q_{i-1}$. By application of Bayes' formula, it was obtained as $q_i=gf_1(x)q_{i-1}/[f_1(x)q_{i-1}+f_2(x)(1-q_{i-1})]$ where $f_i(x)=f(x|\theta_i)$. This means that transition occurs at the end of the period after costs were reported.

As opposed to Kaplan's model, Dyckman(1969) assumed that the cost observation was obtained posterior to the operation of the transition matrix, so that the posterior probability was obtained as $q_i= f_1(x)q_{i-1}g/[f_1(x)q_{i-1}g+f_2(x)(1-q_{i-1})g]$.

Dittman and Prakash(1978) assumed that the process transition had taken place before a cost was reported, similarly to Dyckman's model, in his Markovian Approach. However, the time difference among these models generates no problem with respect to seeking the optimal policy because it can be regarded as the methodological differences due to calculating the different prior probabilities corresponding to the different time levels.

As shown in chapter two, Hughes(1975) considered the case not to obtain a cost report similarly to Ross'(1971) model, and Hannum(1970) developed the model that determined the reporting schedules for setting the reporting frequencies for a single ongoing process which could be expected to undergo a significant shift(i.e., transformation into out-of-control) in the level of performance after some elapsed operating time.

However, the present study will consider the Cost Process in which the cost report is obtained every period, because the cost report is information that is periodically provided from production division into managerial division.

4) Transition of States and Estimation of Them

Duvall(1967) suggested a procedure which showed how the distribution of controllable variance y and the relevant costs were able to be made as a part of the investigating policies, in order to see if an investigation was expected to be worthwhile. However, it was criticized due to the fact that it was done only for a stationary process, because the μ_y and σ_y of controllable distribution were caculated by treating each observation equally and symmetrically.

Kaplan(1969) solved this problem by using the Markovian transition probability with a constant probability g. He suggested the model dealing with a two-state system(i.e., in-and out-of-control state) that, when in control, would remain in control during the reporting period with a constant probability g. He insisted that the parameter g was able to be estimated by noting that the mean of the number of periods before going out of control was $1/(1-g)$.

Moreover, Magee(1977a) discussed a Cost Process with information about imperfect parameter knowledge(e.g., out-of-control state) by using Kaplan's(1969) Dynamic Programming

Approach. Specially, he suggested a procedure using the manager's prior beliefs to form estimates of the desired parameters.

On the other hand, Kaplan(1975) discussed the process to expand the number of states to allow for varying degrees of out-of-controlness(e.g., S states with state 1 representing perfectly in control, state 2 representing slight deterioration and state s being well out-of-control); Ross(1971) suggested a general formulation by assuming that a process could be in a countable number of states; and Magee(1986) suggested a Markovian Approach with three states(i.e., state 1 representing out-of-control, state 2 representing in-control and state 3 also representing out-of-control).

But when the number of states increases, multi-state models may not obtain any benefit of realistic application from the complexity of formulating the model or the measuring of the conditional distributions given its states. Therefore, the number of states need to be formed by the realistic applicability and measurability of the conditional distributions given its states.

All the models discussed above also assume that the investigation of an out-of-control process will always disclose its causes. However, Dyckman(1969), Kaplan(1975) and Kim(1983) supplemented the Exploratory (or Partial) Investigation case in which the investigation of an out-of-control process did not always disclose its causes. Thus, the present study will deal with models with two-states of the Full and Exploratory Investigation cases.

5) Control Variables

BFJ(1961) and Dyckman(1969) used the posterior probability that denoted the possibility of being in control, when an observed cost variance was given, as a control variable. The control variable can be represented in the light of an action in the statistical Decision Theoretic Approach, in which the Cost Process will be investigated if the expected cost of investigating it is less than the expected cost of not-investigating it.

Kaplan(1969;1975), Hughes(1975;1977) and Buckman and Miller(1982) also used the posterior probability that denoted the possibility of being in control, when the most recent observation x was given, as the control variable with respect to the Bayesian Dynamic Programming Approach. However, Kaplan's control variable has a different value from the others because the point of generating transition probability g was defined differently from the others.

Duvall(1967) suggested a new model, in which the control variable was denoted as the cost variance x, that minimized the expected profit resulting from an investigation.

Dittman and Prakash(1978) defined the control variable with respect to the Markovian Approach, so that the cost variance x itself was used as the control variable and simply required the matching against a fixed critical limit.

Thus, by using the cost variance x as the control variable, the present study will establish a model to require only the matching of a reported cost variance to a fixed critical value depending on the prior probability, differently from the previous models in the Statistical Decision Theoretic Approach.

CHAPTER FOUR

ASSUMPTION AND CONTROL PROCESS

1) Overview

This chapter mainly discusses the basic assumptions and control process used in the Full Investigation case because the Exploratory Investigation case can be regarded as the extension of the Full Investigation case. Therefore, additional assumptions used in the Exploratory Investigation will be discussed at the starting point of chapter six.

Section 2 discusses general assumptions, similarly to those assumed in the previous accounting literature, that is required in forming a Cost Variance Investigation model proposed in this study. In section 3, additional assumptions are discussed for the Likelihood-Ratio Test Approach, which is the methodology of this study. Section 4 discusses the control process that will be considered in the present study. The control process can be described by using the same information, namely, μ_1, μ_2, σ_1^2, σ_2^2, g and C, such as Kaplan's and/or Dittman and Prakash's models.

2) <u>General Assumptions for Forming the Cost Variance Investigation Model</u>

The present study assumes a two-state situation with in- and out-of-control states that was discussed in chapter three. In-control state denotes the situation in which operating segment is operating in control. This indicates that, when costs are in control, there may be some variances but these are caused by short-term fluctuation and, there are no controllable inefficiencies that will continue unless corrected. Out-of-control state denotes the situation in which, when costs are out of control, there are controllable inefficiencies that will continue until corrected.

Let x denote a random sample which represents the reported costs in a given period. When the situation is in control, suppose that the reported cost x has a probability density function $f(x;\theta_1)$ which θ_1 is a vector with elements, μ_1 and σ_1^2. Similarly, when the situation is out of control, suppose that the probability density function for the reported costs x is given by $f(x;\theta_2)$, which θ_2 is a vector with elements, μ_2 and σ_2^2. Presumably, $f(x;\theta_1)$ is such that most of the probability is concentrated about low costs and $f(x;\theta_2)$ has most of its mass at high costs, and then μ_1 has a value less than μ_2. However, suppose that θ_1 and θ_2 have been known since standard costs were established, and that the probability density functions for parameters, θ_1 and θ_2, are normal density functions.

Let g denote a probability that will operate in control until the start of the next period, and (1-g) a probability that an inefficiency will develop at the start of the accounting period and will operate out of control for the period. The present study also assumes the case of there being no chance of self-correcting. Then the transition probability P between the two states is summarized as in the following transition matrix of the Markov chain:

$$P=\begin{bmatrix} g & 1-g \\ 0 & 1 \end{bmatrix}.$$

3) <u>Additional Assumptions for the Likelihood-Ratio Test Approach</u>

In order to define the control actions of the present model, let us set up a statistical hypothesis. When the cost report x is observed from normal distribution with parameter, $\theta_i (i=1,2)$, on the basis of which it is desired to test H_1: $\theta=\theta_1$ versus H_2: $\theta=\theta_2$, control action $a_i (i=1,2)$ is defined as the decision of deciding that $H_i (i=1,2)$ is correct. Let A denote the action space that consists of a_1 and a_2, and Φ the parameter space with θ_1 and θ_2 $(\mu_1<\mu_2)$. If actions are represented as the function of the outcome of the cost report x, $\delta(x)\varepsilon A$ can be defined as the decision rule which maps the sample space S into A.

In order to determine the investigating region due to testing H_1 versus H_2, let us partition the range of sample space S into

$\{x | \delta(x)=a_1\}$ and $\{x | \delta(x)=a_2\}$, define the former as S_1 and the latter as S_2. Then loss function for fixed numbers, θ_1 and θ_2, is given by the formulas,

$$\ell(a_1;\ \theta)=\begin{cases} 0 & \text{if } \theta=\theta_1 \\ L & \text{if } \theta=\theta_2 \end{cases} \quad \text{and} \quad \ell(a_2;\ \theta)=\begin{cases} C & \text{if } \theta=\theta_1 \\ C & \text{if } \theta=\theta_2. \end{cases}$$

Here, as discussed in section 2 of chapter three, C denotes the investigation cost used for investigating the Cost Process when control action a_2 is performed; M denotes the correcting cost after an investigation; and L denotes the value of the expected cost saving to reflect the possibility of future out-of-control periods over an infinite horizon. Then L can be derived similarly to Magee(1976) as follows:

$$\begin{aligned} L &= \lim_{n\to\infty} \Sigma_{j=1}^{n} g^{j-1}(1-g)(\Sigma_{i=1}^{j}\alpha^{i-1}(\mu_2-\mu_1)) \\ &= (1-g)(\mu_2-\mu_1)\lim_{n\to\infty} \Sigma_{j=1}^{n} g^{j-1}\Sigma_{i=1}^{j}\alpha^{i-1} \\ &= (1-g)(\mu_2-\mu_1)\lim_{n\to\infty} \Sigma_{j=1}^{n} g^{j-1}((1-\alpha^{j})/(1-\alpha)) \\ &= (\mu_2-\mu_1)/(1-\alpha g) \end{aligned}$$

where α is a discount factor and random variable N has a geometric distribution, $g^{n-1}(1-g)$.

Schematic representation of this procedure can be shown as in the following figure:

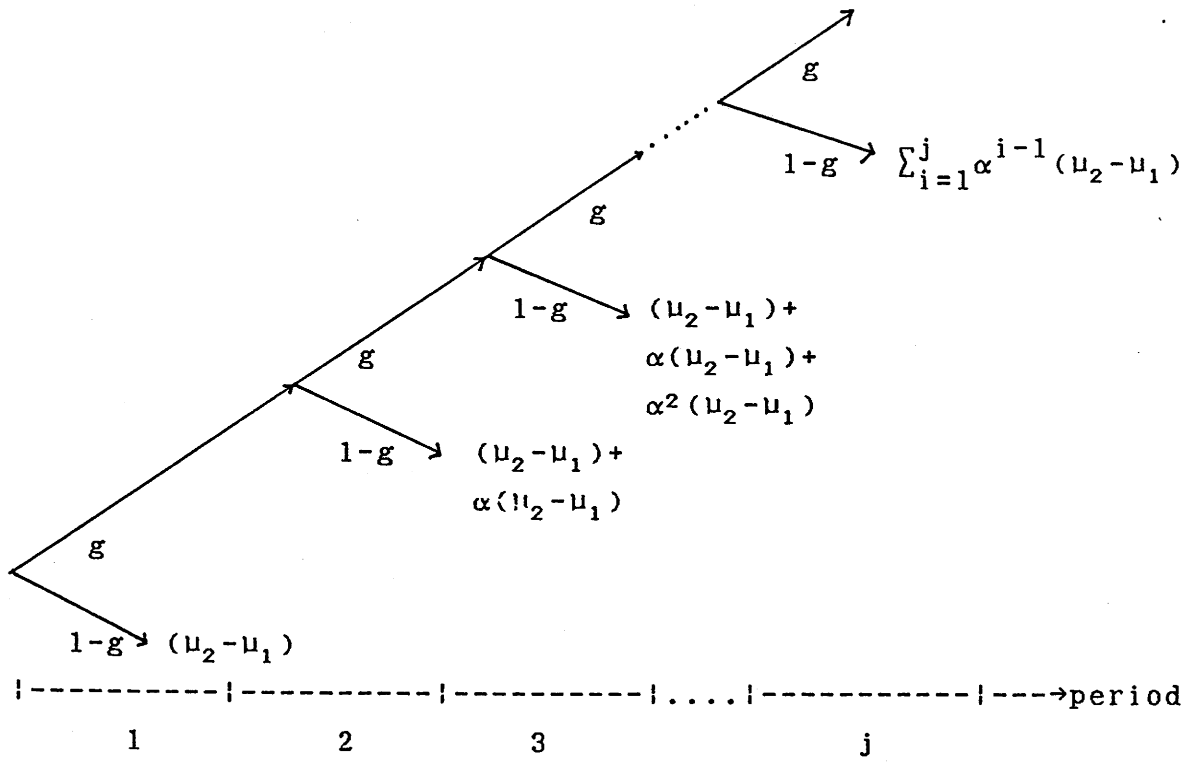


Figure 11: Branching Diagram of Expected Cost Savings from Investigation

As a result, loss function and probability of the investigating region are summarized as in the following figures:

Φ \ A	a_1	a_2
θ_1	0	C
θ_2	L	C+M

Figure 12: Loss Matrix

Φ \ S	$x \varepsilon S_1$	$x \varepsilon S_2$
θ_1	$P[x \varepsilon S_1 \mid \theta_1]$	$P[x \varepsilon S_2 \mid \theta_1]$
θ_2	$P[x \varepsilon S_1 \mid \theta_2]$	$P[x \varepsilon S_2 \mid \theta_2]$

Figure 13: Probability of Investigating Region

Additionally, note that the conditional probabilities of committing Type-I and Type-II errors are $P[x \varepsilon S_2 | \theta_1]$ and $P[x \varepsilon S_1 | \theta_2]$.

4) <u>Schematic Repesentation of Control Process</u>

Consider a Cost Process similar to the previous accounting literature(i.e., Dyckman(1969) and Hughes(1975;1977)). Then it can be shown as in the following figure:

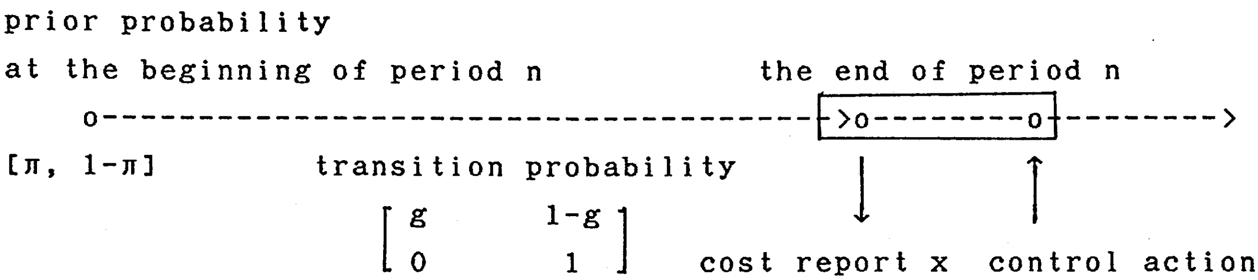


Figure 14: Cost Process

The state of Cost Process is unknown to the decision maker except immediately after an investigation, and it can only be inferred from the observed costs and the length of time since correction action. What is actually known is the probability that the Cost Process is in control. This probability is determined by Bayes formula. Thus π is defined as the prior probability of in-control state at the beginning of period n.

The π can be easily estimated by Bayes formula depending on the transition probability, because the Cost Process is reset and a new cycle starts when it is found to be out of control. When the observed costs are given, this prior probability can be specified by the generalized Bayes formula. This will be discussed in section seven of next chapter.

Suppose that the cost of obtaining a cost report is zero and the cost report is required every period, and that the state transition took place before a cost report is submitted as discussed in chapter three. Then the cost report provides information about the status of the process when the cost report is submitted at the end of a period. Additionally, suppose that the control action is carried out immediately after the cost report is submitted, and that the time taken to investigate and correct the Cost Process is small compared to the duration of an operating "period".

As a result, basic assumptions of the previous literature were examined in chapter three, and this chapter discusses the assumptions of the present study. However, the present study defines the basic assumptions similar to them of the previous literature. This is due to the reason why the author concentrates on developing another simple Decision Theoretic Approach that can capture much of the benefits obtainable from the other approaches, without requiring additional information (or assumptions).

CHAPTER FIVE

FULL INVESTIGATION BY BAYES LIKELIHOOD-RATIO TEST

1) Overview

A new procedure of the Decision Theoretical Approach with respect to the case of Full Investigation, by using the assumptions described in chapter four, is proposed in this chapter. However, the numerical analysis of this procedure will be discussed in chapter seven.

Section 2 describes two methods, the Extensive and Normal Forms, which minimize the expected cost. However, the results of two methods of Bayesian analysis are the same whether the pre-experimental or post-experimental view point is taken, so we can use whichever viewpoint is most convenient.

In section 3, the present study sets up the investigating policies by statistical hypothesis, and shows the new procedure in this chapter as a Bayes test that turns out to be a simple likelihood-ratio test. Properties of likelihood ratio $f(x|\theta_1)/f(x|\theta_2)$ are analyzed in section 4, so that the present study shows how to determine the investigating regions based on variance ratio σ_2^2/σ_1^2 in section 5.

Section 6 discusses the Cost Variance Investigation policies in N-Cost Processes, with each Cost Process being treated independently of the others, as the extension of One Cost Process problem. Section 7 shows that the two forms are mathematically equivalent and lead to identical results.

2) Normal and Extensive Forms of Analysis

a) The Normal and Extensive Forms

There are two basic methods of analysis which we can use to find the course of action which will minimize the expected cost: the Normal and Extensive Forms of Bayesian analysis(The names "Normal Form" and "Extensive Form" were first used by Raiffa and Schlaifer(1961)).

The Extensive Form of analysis proceeds by working backwards from the end of the decision tree to the initial starting point. The Bayes risk can be then described as follows:

Minimum Bayes Risk $r(a^{*}) = E_{x} \min_{a} E_{\theta|x} \ell(x, a, \theta)$.

This form can be shown as in the following figure:

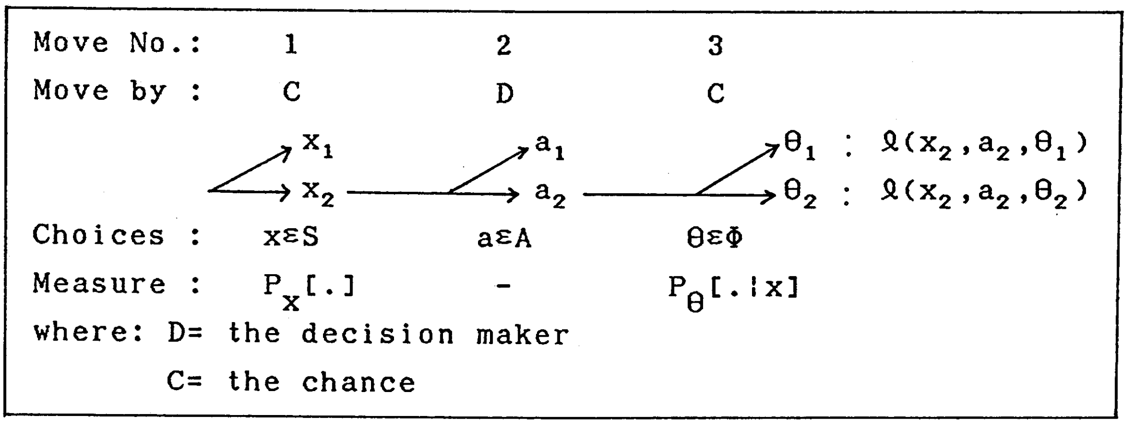


Figure 15: The Extensive Form of Analysis

However, the Normal Form of analysis starts by explicitly considering every possible decision rule instead of first determining the optimal act a for every possible outcome x, the Bayes risk being then described as follows:

Minimum Bayes Risk $r(\delta^*) = \min_{\delta} E_{\theta}E_{x|\theta}\ell(\delta(x),\theta)$.

This form can be also shown as in the following figure:

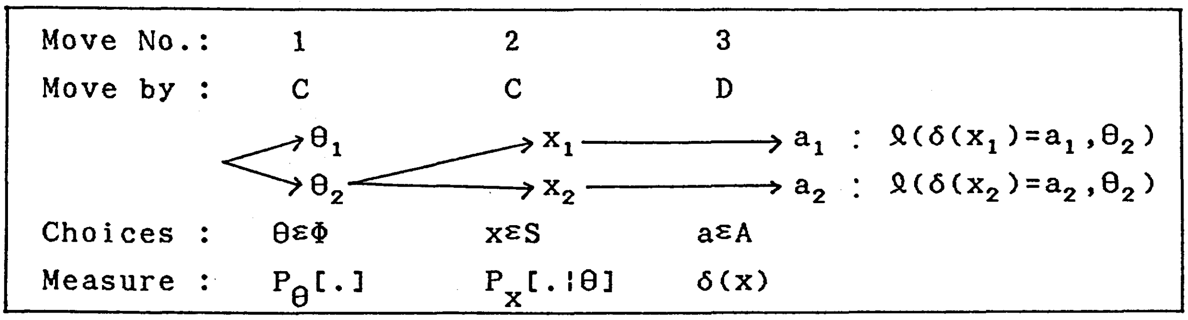


Figure 16: The Normal Form of Analysis

b) <u>Equivalence of the Normal and Extensive Forms</u>

Equivalence between the two forms can be proved as follows: The Normal and Extensive Forms of analysis will be equivalent if, and only if, they assign the same minimum Bayes risk, that is, if the formula $r(\delta^*) = \min_{\delta} E_{\theta}E_{x|\theta}\ell(\delta(x),\theta)$ by the Normal Form of analysis agrees with the formula $r(a^*) = E_x \min_{a} E_{\theta|x}\ \ell(x, a, \theta)$ by the Extensive Form.

Here, the operation $E_{\theta}E_{x|\theta}$ by the Normal Form is equivalent to expectation over the entire possibility space $\Phi x S$ and is therefore equivalent to $E_xE_{\theta|x}$. It follows that the Normal Form above can be written by $r(\delta^*) = \min_{\delta} E_xE_{\theta|x}\ell(\delta(x),\theta)$ and it is then obvious that the best δ will be the one which for every x minimizes $E_{\theta|x}\ell(\delta(x),\theta)$. This, however, is exactly the same thing as selecting for every x and a^* which satisfies $E_{\theta|x}\ \ell(x,\ a^*,\ \theta)= \min_a E_{\theta|x}\ \ell(x,\ a,\ \theta)$. We have thus proved that $\delta^*= a^*$ and formulas for $r(\delta^*)$ and for $r(a^*)$ are equivalent. Q.E.D.

Although the two forms are mathematically equivalent and lead to identical results, each form has technical advantages in certain situations. Thus the present study finds an investigating region which minimizes the expected cost with respect to the Normal Form of analysis.

3) <u>Investigation Policies by Statistical Hypothesis</u>

Let us set up the Bayes test T_π of H_1: $\theta=\theta_1$ versus H_2: $\theta=\theta_2$ with respect to a prior probability given by $\pi= P[\theta=\theta_1]$ according to the assumptions of chapter four. Note that θ_i is a vector such that the elements are $\mu_i\ (\mu_1<\mu_2)$ and $\sigma_i^2 (i=1,2)$, and that θ_i is a known value as discussed in chapter four.

Then the risk by loss function due to each state θ_i is obtained as in the following formulas:

$$R(\theta_1)= E_{x|\theta_1}[\ell(\delta(x);\theta_1)]$$
$$= \ell(a_1;\theta_1)P[x\varepsilon S_1|\theta_1]+\ell(a_2;\theta_1)P[x\varepsilon S_2|\theta_1]$$
$$= \ell(a_2;\theta_1)P[x\varepsilon S_2|\theta_1]$$
$$R(\theta_2)= E_{x|\theta_2}[\ell(\delta(x);\theta_2)]$$
$$= \ell(a_1;\theta_2)P[x\varepsilon S_1|\theta_2]+\ell(a_2;\theta_2)P[x\varepsilon S_2|\theta_2].$$

The prior probability of θ_1 just before obtaining a cost report is πg, then the expected cost $T(S_2)$ by prior probability $P[\theta=\theta_i](i=1,2)$ can be obtained as follows:

$$
\begin{aligned}
T(S_2) &= E_\theta\, E_{x|\theta}\, [\ell(\delta(x);\ \theta)] \\
&= \pi g R(\theta_1) + (1-\pi g) R(\theta_2) \\
&= \pi g \ell(a_2;\theta_1) P[x \varepsilon S_2 | \theta_1] + (1-\pi g)\{\ell(a_1;\theta_2) P[x \varepsilon S_1 | \theta_2] + \\
&\quad \ell(a_2;\theta_2) P[x \varepsilon S_2 | \theta_2]\} \\
&= (1-\pi g)\ell(a_1;\theta_2) + \int_{S_2} \{\pi g \ell(a_2;\theta_1) f(x|\theta_1) + (1-\pi g)[\ell(a_2;\theta_2) \\
&\quad -\ell(a_1;\theta_2)] f(x|\theta_2)\} dx.
\end{aligned}
$$

To find a Bayes test, we seek an investigating region S_2 that minimizes $T(S_2)$ as follows:

$$
\begin{aligned}
\inf_{S_2} T(S_2) &= \inf_{S_2} \{(1-\pi g)\ell(a_1;\theta_2) + \int_{S_2} [\pi g \ell(a_2;\theta_1) f(x|\theta_1) + \\
&\qquad (1-\pi g)[\ell(a_2;\theta_2) - \ell(a_1;\theta_2)] f(x|\theta_2)] dx\} \\
&= (1-\pi g)\ell(a_1;\theta_2) + \inf_{S_2} \int_{S_2} \{\pi g \ell(a_2;\theta_1) f(x|\theta_1) + \\
&\qquad (1-\pi g)[\ell(a_2;\theta_2) - \ell(a_1;\theta_2)] f(x|\theta_2)\} dx.
\end{aligned}
$$

We can minimize the expected cost $T(S_2)$ if S_2 is denoted to be a set of x for which the integrand of the above equation is negative, that is,

$$
\begin{aligned}
S_2 &= \{x | \pi g \ell(a_2;\theta_1) f(x|\theta_1) + (1-\pi g)[\ell(a_2;\theta_2) - \ell(a_1;\theta_2)] f(x|\theta_2) < 0\} \\
&= \{x | f(x|\theta_1)/f(x|\theta_2) < (1-\pi g)[\ell(a_1;\theta_2) - \ell(a_2;\theta_2)]/\pi g \ell(a_2;\theta_1)\} \\
&= \{x | f(x|\theta_1)/f(x|\theta_2) < (1-\pi g)(L-(C+M))/\pi g C\} \\
&= \{x | f(x|\theta_1)/f(x|\theta_2) < [(1-\pi g)/\pi g] * \\
&\quad [((\mu_2-\mu_1)/C(1-\alpha g)) - ((C+M)/C)]\} \text{ (by definition of L).}
\end{aligned}
$$

Note that a Bayes test turns out to be a simple likelihood-ratio test.

4) Properties of Likelihood Ratio

Let $\lambda(x)$ denote likelihood ratio, $f(x|\theta_1)/f(x|\theta_2)$. Then $\lambda(x)$ has a maximum or minimum value when $x=(\mu_1\sigma_2^2-\mu_2\sigma_1^2)/(\sigma_2^2-\sigma_1^2)$ $= \mu_1-|(\mu_1-\mu_2)\sigma_1^2/(\sigma_2^2-\sigma_1^2)| = \mu_2+|(\mu_1-\mu_2)\sigma_2^2/(\sigma_2^2-\sigma_1^2)|$ by the results calculated in Appendix A.

Thus the maximum or minimum value of $\lambda(x)$ is given by $(\sigma_2/\sigma_1)\exp[(\mu_1-\mu_2)^2/2(\sigma_2^2-\sigma_1^2)]$ when $x= (\mu_1\sigma_2^2-\mu_2\sigma_1^2)/(\sigma_2^2-\sigma_1^2)$.

Regarding properties of $\lambda(x)$ that depend on the relationship between σ_1^2 and σ_2^2, if σ_1^2 and σ_2^2 are the same values, the function $\lambda(x)$ with respect to x is a monotone decreasing function. If σ_1^2 has a lower value than σ_2^2, the function $\lambda(x)$ with respect to x is a concave function that has a maximum value $(\sigma_2/\sigma_1)\exp[(\mu_1-\mu_2)^2/2(\sigma_2^2-\sigma_1^2)]$ when $x= \mu_1-|\sigma_2^2(\mu_1-\mu_2)/(\sigma_2^2-\sigma_1^2)|$.
However, if σ_2^2 is less than σ_1^2, the function $\lambda(x)$ with respect to x is a convex function that has a minimum value $(\sigma_2/\sigma_1)\exp[-(\mu_1-\mu_2)^2/2(\sigma_1^2-\sigma_2^2)]$ when $x= \mu_2+|\sigma_2^2(\mu_1-\mu_2)/(\sigma_2^2-\sigma_1^2)|$.
This procedure is developed and provided, in detail, in Appendix A. The properties of $\lambda(x)$ can be shown as in the following figure:

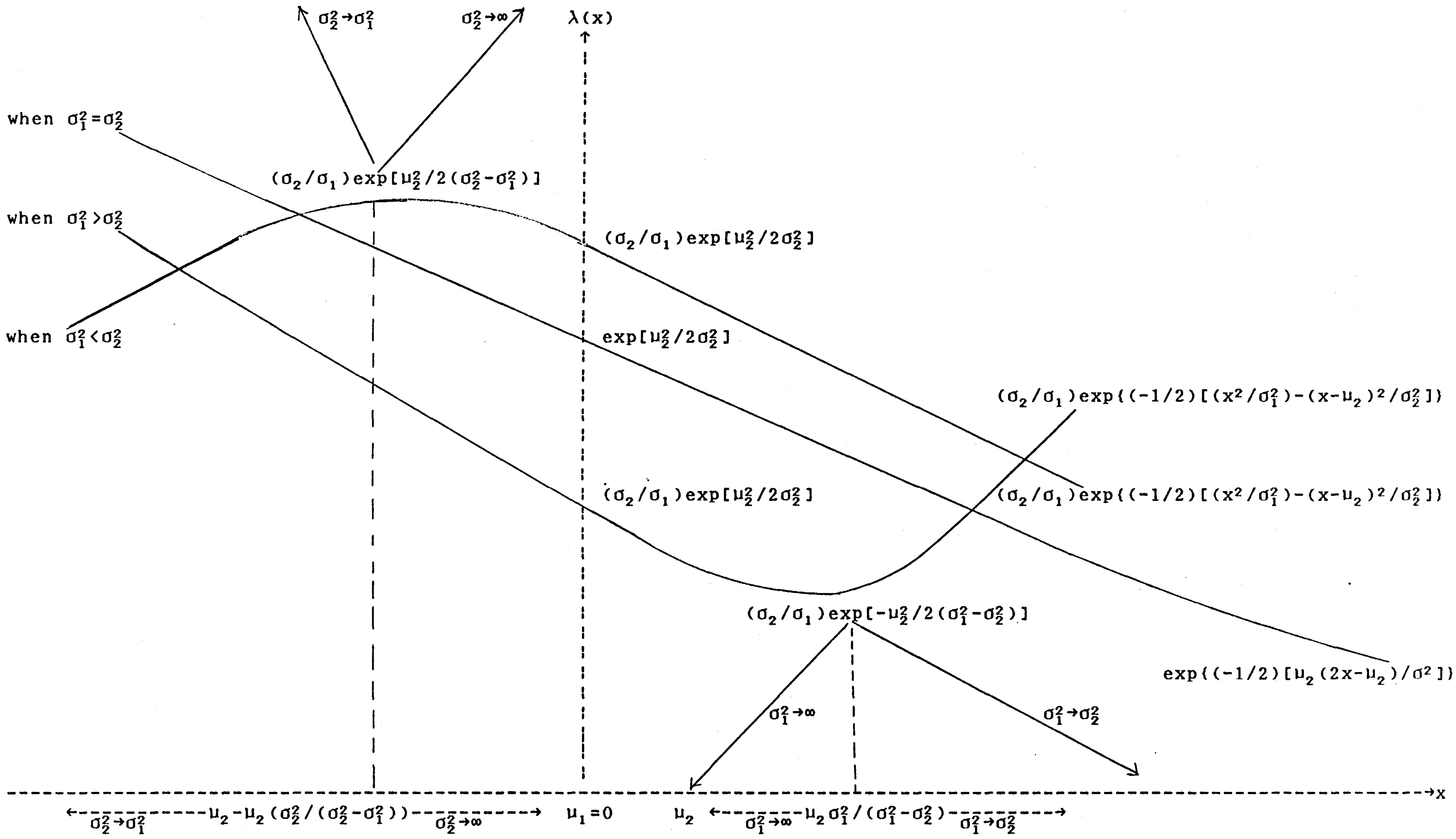


Figure 17: Properties of Likelihood Ratio $\lambda(x)$

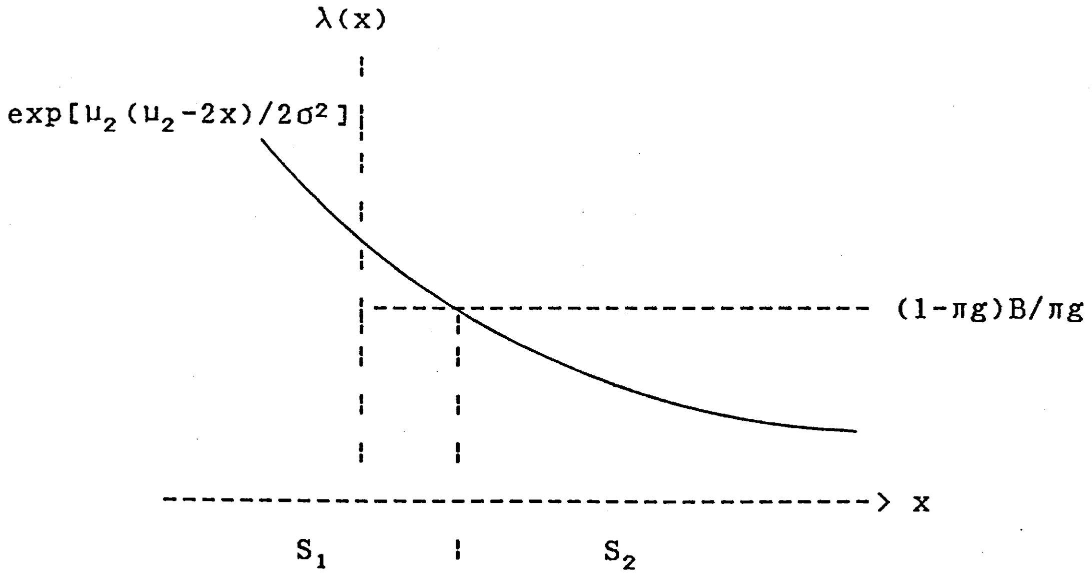
5) Determining the Investigating Regions Based on Variance Ratio σ_2^2/σ_1^2

The investigating region S_2 that minimized the expected cost in section 3 was given by $S_2=\{x \mid f(x \mid \theta_1)/f(x \mid \theta_2) < [(1-\pi g)/\pi g]*[((\mu_2-\mu_1)/C(1-\alpha g))-((C+M)/C)]\}$. Every term of the right hand side of the inequality in S_2 above(i.e.,$[(1-\pi g)/\pi g]*[((\mu_2-\mu_1)/C(1-\alpha g))-((C+M)/C)]$), while they are yet unknown, is constant, except for the prior probability π. Thus the right hand side can be described as a function of only π.

Note that if $\alpha=1$, then it becomes $[((\mu_2-\mu_1)/C(1-g))-((C+M)/C)][(1-\pi g)/\pi g]$, and that μ_1 will be regarded as 0 in the light of the establishment of a standard cost variance from here on.

a) Determining the investigating regions in the case of $\sigma_1^2=\sigma^2$

The investigating region, by the property of $\lambda(x)$ that was discussed in section 4, is shown as in the following figure:



where : S_1 = not-investigating region

S_2 = investigating region

$B = ((\mu_2-\mu_1)/C(1-\alpha g))-((C+M)/C)$

Figure 18: Investigating Region When $\sigma_i^2=\sigma^2$

Therefore, the investigating region S_2 based on inequality between $\lambda(x)$ and $(1-\pi g)B/\pi g$, is described as $\{x|\exp[\mu_2(\mu_2-2x)/2\sigma^2]\leq(1-\pi g)B/\pi g\}$. The investigating region S_2 to satisfy the above inequality is obtained by

$$\exp[\mu_2(\mu_2-2x)/2\sigma^2]\leq(1-\pi g)B/\pi g$$

$$\Leftrightarrow [\mu_2(\mu_2-2x)/2\sigma^2]\leq\log[(1-\pi g)B/\pi g]$$

$$\Leftrightarrow (\mu_2/2)-(\sigma^2/\mu_2)\log[(1-\pi g)B/\pi g]\leq x.$$

b) <u>Determining the investigating regions in the case of $\sigma_1^2>\sigma_2^2$</u>

In accordance with the property of $\lambda(x)$ described in section 4, the investigating region is shown as in the following figure:

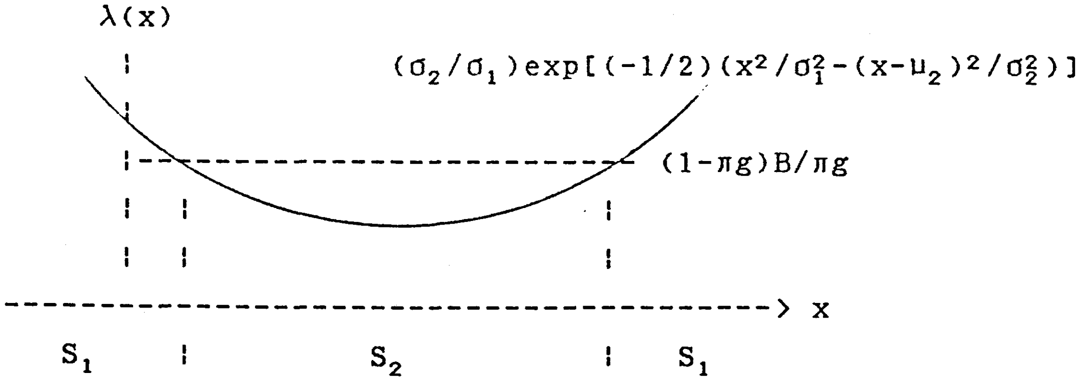


Figure 19: Investigating Region When $\sigma_1^2>\sigma_2^2$

As shown in the above figure, the investigating region can be specified by the relationship between $\lambda(x)$ and information π, g and B. Additionally, the region not requiring an investigation for all x can be described with respect to π so that it satisfies the following inequality:

$(1-\pi g)B/\pi g \leq (\sigma_2/\sigma_1)\exp[-\mu_2^2/2(\sigma_1^2-\sigma_2^2)]$, that is,

$\pi \geq B/[g[B+(\sigma_2/\sigma_1)\exp(-\mu_2^2/2(\sigma_1^2-\sigma_2^2))]]$.

c) <u>Determining the investigating regions in the case of $\sigma_1^2<\sigma_2^2$</u>

In accordance with the property of $\lambda(x)$ in section 4, the investigating region is shown as in the following figure:

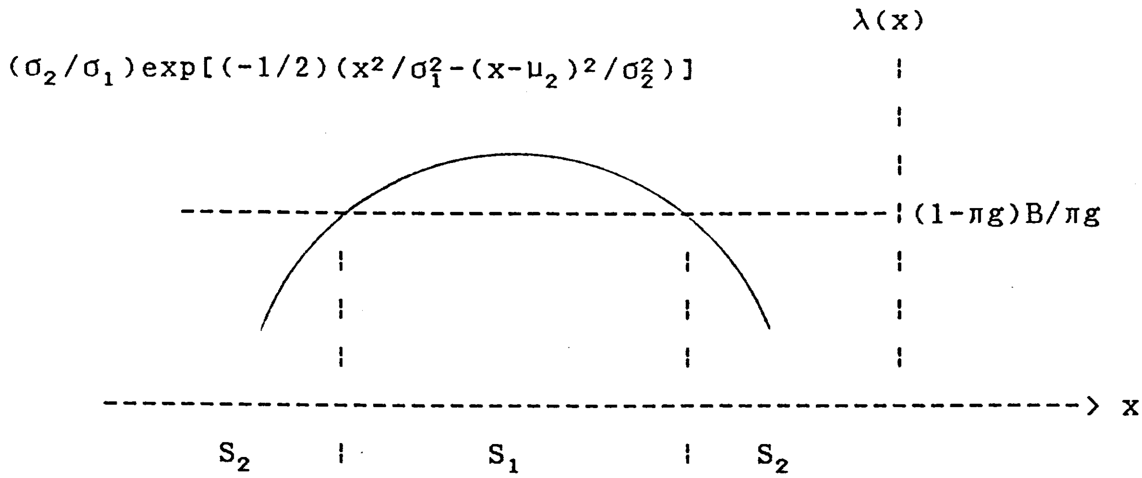


Figure 20: Investigating Region When $\sigma_1^2<\sigma_2^2$

As shown in the above figure, similarly to the way that was discussed in subsection (b), we can show the investigating region. Additionally, the region requiring an investigation for all x can be also described with respect to π so that it satisfies the following inequality:

$(1-\pi g)B/\pi g \geqq (\sigma_2/\sigma_1)\exp[\mu_2^2/2(\sigma_2^2-\sigma_1^2)]$, that is,

$\pi \leqq B/[g[B+(\sigma_2/\sigma_1)\exp[\mu_2^2/2(\sigma_2^2-\sigma_1^2)]]]$.

6) <u>Evaluating the Multiple Cost Processes Under Budget Constraint</u>

Let j denote Cost Process j (j=1,2,...,N) and θ_{ji} be a vector j with elements, μ_{ji} and σ_{ji}^2 (i=1,2), as discussed in section 3. Suppose that each Cost Process evolves independently of the others in a similar way to Ozan and Dyckman's(1971) and Buckman and Miller's(1982) models, and that x_j denotes a random sample that has a normal distribution $N_j(\mu_i, \sigma_i^2)$. Then Bayes test of $H_1: \theta_j = \theta_{j1}$ versus $H_2: \theta_j = \theta_{j2}$ can be set up with respect to the prior probability given by $\pi_j = P[\theta_j = \theta_{j1}]$, according to Bayes test discussed in section 3. Here, for simplicity, let us translate the loss function into the regret function, and suppose that the correcting cost M has the zero value.

Then the investigating region of Cost Process j is obtained, according to the result that was sought in section 3, as follows: $S_{j2} = \{x_j \mid f_j(x \mid \theta_1)/f_j(x \mid \theta_2) < [(1-\pi_j g_j)/\pi_j g_j][(L_j - C_j)/C_j]\}$. Let ρ be defined as the following set: $\rho = \{j \mid x_j \varepsilon S_{j2},\ j \varepsilon N\}$.

Thus the objective function that minimizes the total expected cost can be represented as follows:

$$\min_{y_j} \sum_{j=1}^{N} E[\ell(\delta(x_j, y_j); \theta_j)(1-y_j)]$$

Subject to

$$\sum_{j=1}^{N} C_j y_j \leq D$$

$$y_j \varepsilon \{0,1\}$$

where: $\delta(x_j, y_j) = a_1$ if $x_j \varepsilon S_{j1}$, or $x_j \varepsilon S_{j2}$ and $y_j = 0$

a_2 if $x_j \varepsilon S_{j2}$ and $y_j = 1$

D= the budget constraint of investigating Cost Processes.

This total expected cost is developed and revised, in detail, in Appendix B. According to the revised objective function of Appendix B, it is shown as follows:

$$\max_{y_j} \sum_{j\in\rho} \ell(a_1;\theta_{j2})P[x_j\in S_{j2}|\theta_{j2}]P[\theta_{j2}]y_j$$

Subject to

$$\sum_{j\in\rho} C_j y_j \leq D$$

$$y_j \in \{0,1\}.$$

The above objective function can be specified as follows:

$$\max_{y_j} \sum_{j\in\rho} [L_j - C_j]P[x_j\in S_{j2}|\theta_{j2}]P[\theta_{j2}]y_j$$

Subject to

$$\sum_{j\in\rho} C_j y_j \leq D$$

$$y_j \in \{0,1\}.$$

As a result, the original objective function that minimizes the total expected cost can be transformed into a 0-1 Knapsack problem as in the following proposition:

[PROPOSITION 1] the original objective function:

$$\min_{y_j} \sum_{j=1}^{N} E[\ell(\delta(x_j,y_j);\theta_j)(1-y_j)]$$

Subject to

$$\sum_{j=1}^{N} C_j y_j \leq D$$

$$y_j \in \{0,1\}$$

where: $$\delta(x_j,y_j) = \begin{cases} a_1 & \text{if } x_j\in S_{j1}, \text{ or } x_j\in S_{j2} \text{ and } y_j=0 \\ a_2 & \text{if } x_j\in S_{j2} \text{ and } y_j=1. \end{cases}$$

The following function is equivalent to the above function:

$$\underset{y_j}{\text{Max}} \sum_{j \in \rho} [L_j - C_j] P[x_j \in S_{j2} | \theta_{j2}] P[\theta_{j2}] y_j$$

Subject to

$$\sum_{j \in \rho} C_j y_j \leq D$$

$$y_j \in \{0,1\}.$$

7) <u>Equivalence of the Normal and Extensive Forms in This Study</u>

In accordance with the Normal Form, we sought an investigation region S_2 that minimized $T(S_2) = E_\theta E_{x|\theta} \ell(\delta(x);\ \theta)$ with respect to S_2. The investigating region S_2 was given by $S_2 = \{x | f(x|\theta_1)/f(x|\theta_2) < (1-\pi g)[\ell(a_1;\theta_2) - \ell(a_2;\theta_2)]/\pi g \ell(a_2;\theta_1)\}$. However, S_2 can be transformed as follows:

$$\begin{aligned} S_2 &= \{x | f(x|\theta_1)/f(x|\theta_2) < (1-\pi g)[\ell(a_1;\theta_2) - \ell(a_2;\theta_2)]/\pi g \ell(a_2;\theta_1)\} \\ &= \{x | f(x|\theta_1)\pi g / f(x|\theta_2)(1-\pi g) < [\ell(a_1;\theta_2) - \ell(a_2;\theta_2)]/\ell(a_2;\theta_1)\} \\ &= \{x | P[\theta_1|x]/P[\theta_2|x] < [\ell(a_1;\theta_2) - \ell(a_2;\theta_2)]/\ell(a_2;\theta_1)\} \\ &= \{x | P[\theta_1|x] < [\ell(a_1;\theta_2) - \ell(a_2;\theta_2)]/[\ell(a_2;\theta_1) + \ell(a_1;\theta_2) - \ell(a_2;\theta_2)]\}. \end{aligned}$$

This transformation explains the fact that the operation $E_\theta E_{x|\theta}$ is equivalent to expectation over the entire possibility space $\Phi x S$ and is therefore equivalent to $E_x E_{\theta|x}$.

Note that S_2 can be obtained by the generalized posterior probability $\pi(\theta_1|x_1,x_2,\ldots,x_n) = [1 + [f(x_n|\theta_2)(1-g)/f(x_n|\theta_1)g] + [f(x_n|\theta_2)f(x_{n-1}|\theta_2)(1-g)/f(x_n|\theta_1)f(x_{n-1}|\theta_1)g^2] + \ldots + [f(x_n|\theta_2)\ldots f(x_2|\theta_2)(1-g)/f(x_n|\theta_1)\ldots f(x_2|\theta_1)g^{n-1}] + [f(x_n|\theta_2)\ldots f(x_1|\theta_2)(1-\pi g)/f(x_n|\theta_1)\ldots f(x_1|\theta_1)\pi g^n]]^{-1}$ as follows:

$$S_2=\{x_n | f(x_n|\theta_1)/f(x_n|\theta_2)<(1-\pi(\theta_1|x_1,\ldots,x_{n-1})g)[\ell(a_1;\theta_2)-\ell(a_2;\theta_2)]$$
$$/\pi(\theta_1|x_1,\ldots,x_{n-1})g\ell(a_2;\theta_1)\}$$
$$= \{(x_1,\ldots,x_n)|\pi(\theta_1|x_1,\ldots,x_n)<[\ell(a_1;\theta_2)-\ell(a_2;\theta_2)]$$
$$/[\ell(a_2;\theta_1)+\ell(a_1;\theta_2)-\ell(a_2;\theta_2)]\}.$$

This generalized posterior probability $\pi(\theta_1|x_1,\ldots,x_n)$ is proved by mathematical induction, in detail, in Appendix C.

On the other hand, according to the Extensive Form described in section 2, given that the cost report x was observed, the optimal action can be obtained by
$\max_a E_{\theta|x}[\ell(x,a,\theta)]= \max\{ \ell(a_2;\theta_1)P[\theta_1|x]+\ell(a_2;\theta_2)P[\theta_2|x];$
$\ell(a_1;\theta_2)P[\theta_2|x]\}$. Therefore, when the cost report x was observed, the optimal action will be a_2 if, and only if,
$\{\ell(a_2;\theta_1)P[\theta_1|x]+\ell(a_2;\theta_2)P[\theta_2|x]\}<\ell(a_1;\theta_2)P[\theta_2|x]$
$\Leftrightarrow P[\theta_1|x]<[\ell(a_1;\theta_2)-\ell(a_2;\theta_2)]/[\ell(a_2;\theta_1)+\ell(a_1;\theta_2)-\ell(a_2;\theta_2)]$.

As a result, the results of two methods are the same whether the pre-experimental or post-experimental viewpoint is taken. Additionally, L was obtained with respect to geometric distribution $g^{n-1}(1-g)$, so that, according to the memoryless property of geometric distribution, determining S_2 is sought by $\pi(\theta_1|x_1,\ldots,x_n)$ with everything else the same for every period n.

CHAPTER SIX

EXPLORATORY INVESTIGATION BY BAYES LIKELIHOOD-RATIO TEST

1) Overview

This chapter discusses the Exploratory Investigation case, which is an extensive model of the Cost Variance Investigation (see chapter five). As opposed to the Full Investigation case, this case allows for the probability that the cause of an out-of-control process will be discovered when it exists.

Assumptions needed are almost equal to those of the Full Investigation case discussed in chapter five. Section 2 discusses additional assumptions used in the Exploratory Investigation case. Section 3 shows how to determine the investigating region by Bayes Likelihood-Ratio Test with respect to the Normal Form. In section 4, the present study shows how the investigating region can be determined with respect to variance ratio, σ_2^2/σ_1^2. Similarly to the way discussed in section 6 of chapter five, section 5 shows how to evaluate Multiple Cost Processes under budget constraint. In section 6, the present study shows that the Extensive and Normal Forms of Bayesian analysis are mathematically equivalent and lead to identical results.

2) Additional Assumptions for Likelihood-Ratio Test Approach

Adding to assumptions shown in chapter four, let w_1 denote the event that the cause of an out-of-control process is discovered and its probability denote h when it exists, and let w_2 denote the event that the cause of an out-of-control process is not discovered and its probability denote 1-h when it exists. If the Cost Process is found to be out of control after a cost report x is observed, it is reset and a new cycle starts. However, if the Cost Process is not found to be out of control by the exploratory investigation, then the prior probability of being in control after the unsuccessful exploratory investigation, $\pi'(\theta_1|x)$, can be denoted by $f(x|\theta_1)\pi g/[f(x|\theta_1)\pi g+f(x|\theta_2)(1-\pi g)(1-h)]$.

Similarly to the way shown in chapter four, on the basis of which it is desired to test H_1: $\theta=\theta_1$ versus H_2: $\theta=\theta_2$, let control action $a_i(i=1,2)$ be defined as the decision of deciding that $H_i(i=1,2)$ is correct. Then loss function for fixed numbers, θ_1 and θ_2, is given by the formulas,

$$\ell(a_1;\ \theta) = \begin{cases} 0 & \text{if } \theta=\theta_1 \\ L & \text{if } \theta=\theta_2 \end{cases}$$

and

$$\ell(a_2;\ w|\theta) = \begin{cases} C' & \text{if } \theta=\theta_1 \\ C'+M & \text{if } w=w_1 \text{ and } \theta=\theta_2 \\ C'+L & \text{if } w=w_2 \text{ and } \theta=\theta_2. \end{cases}$$

Here C' denotes the cost of the limited investigation for the exploratory investigation.

As a result, loss function and the probability of investigating region are summarized as in the following figures:

A		a_1	a_2
θ_1		0	C'
θ_2	w_1	L	C'+M
	w_2	L	C'+L

Figure 21: Loss Matrix of the Exploratory Investigation Case

Φ S	$x \varepsilon S_1$	$x \varepsilon S_2$
θ_1	$P[x \varepsilon S_1 \mid \theta_1]$	$P[x \varepsilon S_2 \mid \theta_1]$
θ_2	$P[x \varepsilon S_1 \mid \theta_2]$	$P[x \varepsilon S_2 \mid \theta_2]$

Figure 22: Probability of the Investigating Region of the Exploratory Investigation Case

3) Investigation Policies by Statistical Hypothesis

By the assumptions defined in chapter four and in section 2 of this chapter, the loss functions due to each action $a_i (i=1,2)$ and each state θ_i are obtained as follows:

$\ell(a_1;\theta_1)= 0$

$\ell(a_1;\theta_2)= L$

$\ell(a_2;\theta_1)= C'$

$\ell(a_2;\theta_2)=E_{w \mid \theta_2}[\ell(a_2;w \mid \theta_2)]=\ell(a_2;w_1 \mid \theta_2)P[w_1 \mid \theta_2]+\ell(a_2;w_2 \mid \theta_2)P[w_2 \mid \theta_2]$

$= (C'+M)h+(C'+L)(1-h)=C'+L+(M-L)h.$

Then the risk by loss function due to each state θ_i is obtained by

$R(\theta_1)= E_{x \mid \theta_1}[\ell(\delta(x);\theta_1)]$

$= \ell(a_1;\theta_1)P[x \varepsilon S_1 \mid \theta_1]+\ell(a_2;\theta_1)P[x \varepsilon S_2 \mid \theta_1]$

$$= \ell(a_2;\theta_1)P[x\varepsilon S_2|\theta_1]$$

$$= C'P[x\varepsilon S_2|\theta_1]$$

$$R(\theta_2) = E_{x|\theta_2}[\ell(\delta(x);\theta_2)]$$

$$= \ell(a_1;\theta_2)P[x\varepsilon S_1|\theta_2]+\ell(a_2;\theta_2)P[x\varepsilon S_2|\theta_2]$$

$$= LP[x\varepsilon S_1|\theta_2]+(C'+L+(M-L)h)P[x\varepsilon S_2|\theta_2].$$

Therefore, the Bayes risk $T(S_2)$ is obtained by

$$E_\theta[R(\theta)]$$

$$= E_\theta\ E_{x|\theta}\ \ell(\delta(x);\ \theta)$$

$$= \pi g R(\theta_1)+(1-\pi g)R(\theta_2)$$

$$= \pi g \ell(a_2;\theta_1)P[x\varepsilon S_2|\theta_1]+(1-\pi g)\{\ell(a_1;\theta_2)P[x\varepsilon S_1|\theta_2]+ \ell(a_2;\theta_2)P[x\varepsilon S_2|\theta_2]\}$$

$$= C'P[x\varepsilon S_2|\theta_1]\pi g+\{L+(C'+(M-L)h)P[x\varepsilon S_2|\theta_2]\}(1-\pi g).$$

To find a Bayes test, we seek an investigating region S_2 that minimizes $T(S_2)$ as follows:

$$\inf_{S_2} T(S_2) = \inf_{S_2}\{C'P[x\varepsilon S_2|\theta_1]\pi g+[L+(C'+(M-L)h)P[x\varepsilon S_2|\theta_2]](1-\pi g)\}$$

$$= L(1-\pi g)+\inf_{S_2}\int_{S_2}[C'\pi g f(x|\theta_1)+(C'+(M-L)h)(1-\pi g)f(x|\theta_2)]dx.$$

We can minimize the expected cost $T(S_2)$, if S_2 is denoted to be a set of x for which the integrand of the above equation is negative, that is,

$$S_2 = \{x|C'\pi g f(x|\theta_1)+(C'+(M-L)h)(1-\pi g)f(x|\theta_2)<0\}$$

$$= \{x|f(x|\theta_1)/f(x|\theta_2)<((L-M)h-C')(1-\pi g)/\pi g C'\}.$$

Note that if h=1 and $C'\to C$, then S_2 described above is equal to S_2 of the Full Investigation case.

4) Determining the Investigating Regions Based on Variance Ratio σ_2^2/σ_1^2

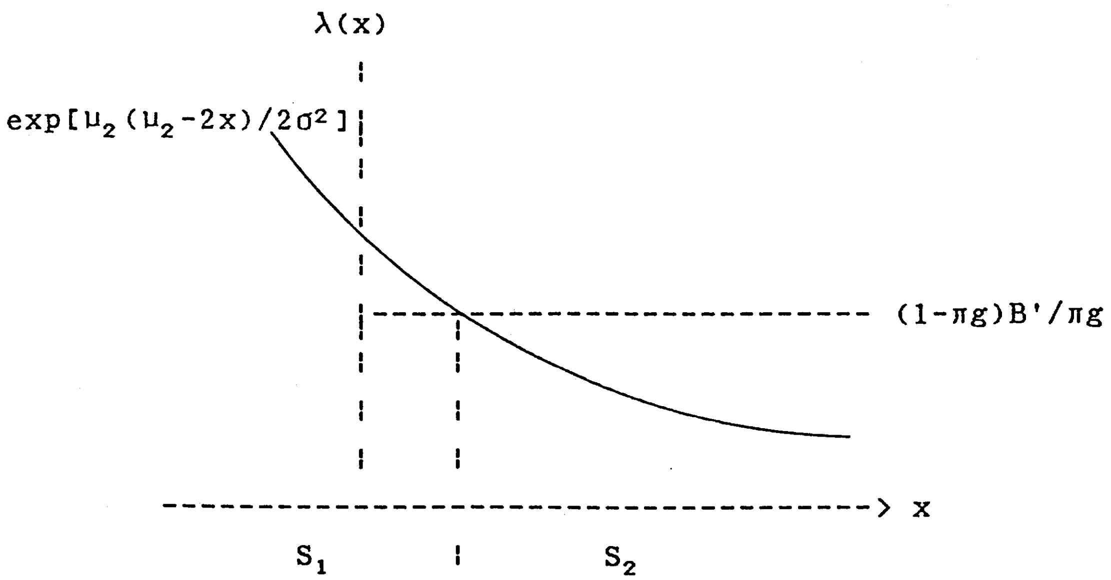
The investigating region S_2 that minimized the expected cost in section 3 was given by

$S_2 = \{x \mid f(x \mid \theta_1)/f(x \mid \theta_2) < ((L-M)h-C')(1-\pi g)/\pi g C'\}$.

For a more general treatment, let the amount of money spent on an invcstigation be a continuous variable C', and define a function h(C') as the probability that the cause of an out-of-control situation is discovered when it exists. Presumably h(0)=0, $\lim_{C' \to C} h(C')=1$ and h(C') is a nondecreasing function of C'.

a) Determining the investigating regions in the case of $\sigma_1^2=\sigma^2$

The investigating region by the property of $\lambda(x)$, as discussed in section 4 of chapter five, is shown as in the following figure:



where : S_1 = not investigating region

S_2 = investigating region

$B' = ((L-M)h(C')-C')/C'$

Figure 23: Exploratory Investigating Region When $\sigma_i^2=\sigma^2$

Therefore, the investigating region S_2 with respect to π, based on the inequality between $\lambda(x)$ and $(1-\pi g)B'/\pi g$, can be described as $\{x \mid \exp[\mu_2(\mu_2-2x)/2\sigma^2] \leq (1-\pi g)B'/\pi g\}$. Thus the investigating region S_2 to satisfy the above inequality can be obtained by

$$\exp[\mu_2(\mu_2-2x)/2\sigma^2] \leq (1-\pi g)B'/\pi g$$

$$\Leftrightarrow [\mu_2(\mu_2-2x)/2\sigma^2] \leq \log[(1-\pi g)B'/\pi g]$$

$$\Leftrightarrow (\mu_2/2)-(\sigma^2/\mu_2)\log[(1-\pi g)B'/\pi g] \leq x.$$

However, h is a function of C', and B' is a function of h or C'. Therefore, the investigating region S_2 can be determined only in the case where either h or C' is given.

b) <u>Determining the investigating regions in the case of $\sigma_1^2 > \sigma_2^2$</u>

In accordance with the property of $\lambda(x)$, the investigating region is shown as in the following figure:

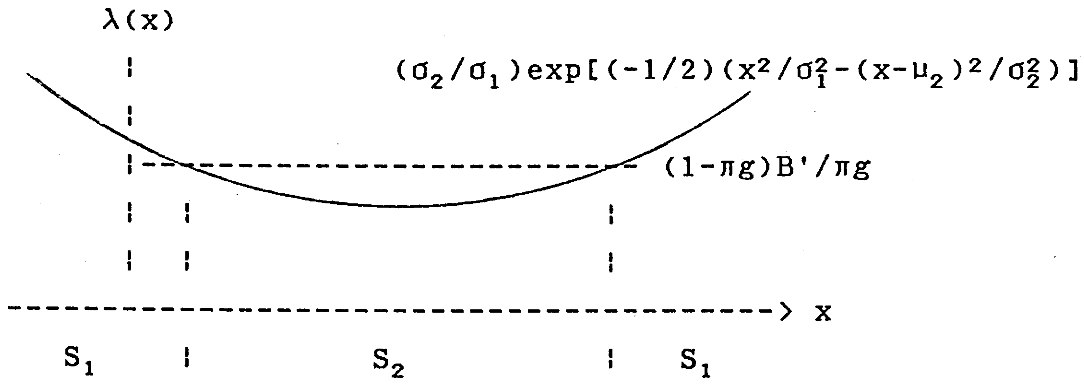


Figure 24: Exploratory Investigating Region When $\sigma_1^2 > \sigma_2^2$

As shown in the above figure, the investigating region can be specified by the relationship between $\lambda(x)$ and information π, g and B', as in the Full Investigation case. However, this investigating region can be determined under any given h(C'), different from that of the Full Investigation.

c) <u>Determining the investigating regions in the case of $\sigma_1^2 < \sigma_2^2$</u>

In accordance with the property of $\lambda(x)$, the investigating region is shown as in the following figure:

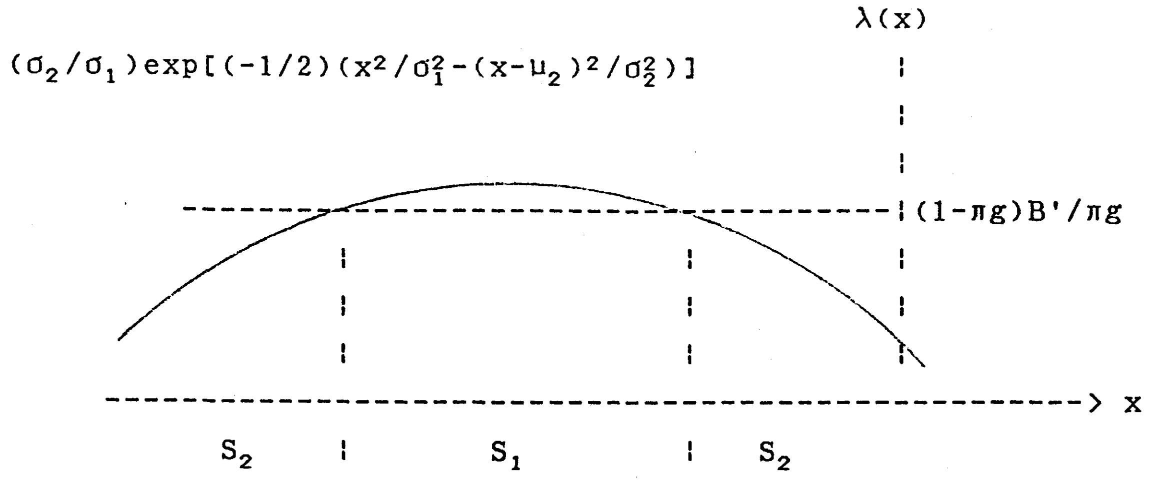


Figure 25: Exploratory Investigating Region When $\sigma_1^2 < \sigma_2^2$

As shown in the above figure, the investigating region is determined by the method discussed in the Full Investigation under any given h(C'). Thus the region requiring an investigation for all x can be also described with respect to π as in the following inequality:

$(1-\pi g)B'/\pi g \geq (\sigma_2/\sigma_1)\exp[\mu_2^2/2(\sigma_2^2-\sigma_1^2)]$, that is,

$\pi \leq B'/g[B'+(\sigma_2/\sigma_1)\exp[\mu_2^2/2(\sigma_2^2-\sigma_1^2)]]$.

On the other hand, when π was known by the posterior probability of the last period n-1, the region requiring an investigation for all x can be also determined with respect to h(C').

5) Evaluating the Multiple Cost Processes Under Budget Constraint

Similarly to the assumptions discussed in section 6 of chapter five, let j denote Cost Process j (j=1,2,...,N). Suppose that each Cost Process evolves independently of the others in a similar way to Ozan and Dyckman's(1971) and Buckman and Miller's(1982) models, and that x_j denotes a random sample that has a normal distribution $N_j(\mu_i,\sigma_i^2)$. Then this case can set up the Bayes test of $H_1:\theta_j=\theta_{j1}$ versus $H_2:\theta_j=\theta_{j2}$ with respect to the prior probability given by $\pi_j=P[\theta_j=\theta_{j1}]$ according to Bayes test of section 3.

Here, for simplicity, let us translate the loss function into the regret function, and suppose that the correcting cost M has the zero value. Then the investigating region of Cost Process j is obtained, according to the result that was sought in section 3, as follows: $S_{j2}=\{x_j \mid f_j(x\mid\theta_1)/f_j(x\mid\theta_2)<(L_j h_j-C_j')(1-\pi_j g_j)/\pi_j g_j C_j'\}$.
Let ρ be defined as the following set: $\rho=\{j \mid x_j \varepsilon S_{j2},\ j\varepsilon N\}$.

Thus the objective function that minimizes the total expected cost can be represented as follows:

$$\operatorname*{Min}_{y_j} \sum_{j=1}^{N} E[\ell(\delta(x_j,y_j);w\mid\theta_j)(1-y_j)]$$

Subject to

$$\sum_{j=1}^{N} C'_j y_j \leqq D$$

$$y_j \varepsilon \{0,1\}$$

where: $\delta(x_j,y_j) = \begin{cases} a_1 & \text{if } x_j \varepsilon S_{j1}, \text{ or } x_j \varepsilon S_{j2} \text{ and } y_j=0 \\ a_2 & \text{if } x_j \varepsilon S_{j2} \text{ and } y_j=1 \end{cases}$

D= the budget constraint of investigating Cost Processes.

This total expected cost is developed and revised, in detail, in Appendix D. In accordance with the revised objective function of Appendix D, it is shown as follows:

$$\underset{y_j}{\text{Max}} \sum_{j \varepsilon \rho} \ell(a_1;\theta_{j2}) P[x_j \varepsilon S_{j2} | \theta_{j2}] P[\theta_{j2}] y_j$$

Subject to

$$\sum_{j \varepsilon \rho} C'_j y_j \leq D$$

$$y_j \varepsilon \{0,1\}.$$

The above objective function can be specified as follows:

$$\underset{y_j}{\text{Max}} \sum_{j \varepsilon \rho} (L_j h_j - C'_j) P[x_j \varepsilon S_{j2} | \theta_{j2}] (1-\pi_j g_j) y_j$$

Subject to

$$\sum_{j \varepsilon \rho} C'_j y_j \leq D$$

$$y_j \varepsilon \{0,1\}.$$

As a result, the original objective function that minimizes the total expected cost can be transformed into a 0-1 Knapsack problem as in the following proposition:

[PROPOSITION 2] the original objective function:

$$\underset{y_j}{\text{Min}} \sum_{j=1}^{N} E[\ell(\delta(x_j, y_j); w|\theta_j)(1-y_j)]$$

Subject to

$$\sum_{j=1}^{N} C'_j y_j \leq D$$

$$y_j \varepsilon \{0,1\}$$

$$\text{where: } \delta(x_j, y_j) = \begin{cases} a_1 & \text{if } x_j \varepsilon S_{j1}, \text{ or } x_j \varepsilon S_{j2} \text{ and } y_j = 0 \\ a_2 & \text{if } x_j \varepsilon S_{j2} \text{ and } y_j = 1. \end{cases}$$

The following function is equivalent to the above function:

$$\underset{y_j}{\text{Max}} \sum_{j \varepsilon \rho} (L_j h_j - C'_j) P[x_j \varepsilon S_{j2} | \theta_{j2}](1 - \pi_j g_j) y_j$$

Subject to

$$\sum_{j \varepsilon \rho} C'_j y_j \leq D$$

$$y_j \varepsilon \{0,1\}.$$

6) <u>Equivalence of the Normal and Extensive Forms in This Study</u>

In accordance with the Normal Form, it was possible to seek an investigating region S_2 that minimized $T(S_2) = E_\theta E_{x|\theta} \ell(\delta(x);\theta)$ with respect to S_2. The investigating region S_2 was given by

$$S_2 = \{x | f(x|\theta_1)/f(x|\theta_2) < [\ell(a_1;\theta_2) - (\ell(a_2;w_1|\theta_2)P[w_1|\theta_2] + \ell(a_2;w_2|\theta_2)P[w_2|\theta_2])](1-\pi g)/\pi g \ell(a_2;\theta_1)\}$$

$$= \{x | f(x|\theta_1)\pi g / f(x|\theta_2)(1-\pi g) < [\ell(a_1;\theta_2) - (\ell(a_2;w_1|\theta_2)P[w_1|\theta_2] + \ell(a_2;w_2|\theta_2)P[w_2|\theta_2])]/\ell(a_2;\theta_1)\}$$

$$= \{x | P[\theta_1|x]/P[\theta_2|x] < [\ell(a_1;\theta_2) - (\ell(a_2;w_1|\theta_2)P[w_1|\theta_2] + \ell(a_2;w_2|\theta_2)P[w_2|\theta_2])]/\ell(a_2;\theta_1)\}$$

$$= \{x | P[\theta_1|x] < [\ell(a_1;\theta_2) - (\ell(a_2;w_1|\theta_2)P[w_1|\theta_2] + \ell(a_2;w_2|\theta_2)P[w_2|\theta_2])]/$$

$[\ell(a_1;\theta_2)+\ell(a_2;\theta_1)-(\ell(a_2;w_1|\theta_2)P[w_1|\theta_2]+\ell(a_2;w_2|\theta_2)P[w_2|\theta_2])]\}$

This transformation explains the fact that the operation $E_\theta E_{x|\theta}$ is equivalent to expectation over the entire possibility space $\Phi x S$ and is therefore equivalent to $E_x E_{\theta|x}$.

Note that S_2 can be obtained by the generalized prior probability of period n+1, $\pi'(\theta_1|x_1,x_2,...,x_n) = [1+[f(x_n|\theta_2)(1-g)(1-h)/f(x_n|\theta_1)g]+[f(x_n|\theta_2)f(x_{n-1}|\theta_2)(1-g)(1-h)^2/f(x_n|\theta_1)f(x_{n-1}|\theta_1)g^2]+ \quad ...+[f(x_n|\theta_2)...f(x_2|\theta_2)(1-g)(1-h)^{n-1}/f(x_n|\theta_1)...f(x_2|\theta_1)g^{n-1}]$
$+[f(x_n|\theta_2)...f(x_1|\theta_2)(1-\pi g)(1-h)^n/f(x_n|\theta_1)...f(x_1|\theta_1)\pi g^n]]^{-1}$
as follows:

$$S_2=\{x_n|f(x_n|\theta_1)/f(x_n|\theta_2)<[\ell(a_1;\theta_2)-(\ell(a_2;w_1|\theta_2)h + \ell(a_2;w_2|\theta_2)(1-h))](1-\pi'(\theta_1|x_1,...,x_{n-1})g) /\pi'(\theta_1|x_1,...,x_{n-1})g\ell(a_2;\theta_1)\}.$$

This generalized prior probability $\pi'(\theta_1|x_1,...,x_n)$ is proved by mathematical induction, in detail, in Appendix E.

On the other hand, according to the Extensive Form described in section 2 of chapter five, given that the cost report x was observed, the optimal action can be obtained by

$$\min_a E_{\theta|x}[\ell(x,a,\theta)]$$
$$= \min \{\ell(a_2;\theta_1)P[\theta_1|x]+(\ell(a_2;w_1|\theta_2)P[w_1|\theta_2]+ \ell(a_2;w_2|\theta_2)P[w_2|\theta_2])P[\theta_2|x];\ \ell(a_1;\theta_2)P[\theta_2|x]\}.$$

Therefore, when the cost report x was observed, the optimal action will be a_2 if, and only if,

$$\ell(a_1;\theta_2)P[\theta_2|x] > \ell(a_2;\theta_1)P[\theta_1|x]+(\ell(a_2;w_1|\theta_2)P[w_1|\theta_2]+ \ell(a_2;w_2|\theta_2)P[w_2|\theta_2])P[\theta_2|x]$$

$$\Leftrightarrow P[\theta_1|x]<[\ell(a_1;\theta_2)-(\ell(a_2;w_1|\theta_2)P[w_1|\theta_2]+\ell(a_2;w_2|\theta_2)P[w_2|\theta_2])]/ [\ell(a_2;\theta_1)+\ell(a_1;\theta_2)-(\ell(a_2;w_1|\theta_2)P[w_1|\theta_2]+\ell(a_2;w_2|\theta_2)P[w_2|\theta_2])].$$

As a result, the results of the two methods are the same whether the pre-experimental or post-experimental viewpoint is taken.

Additionally, as discussed in section 7 of chapter five, determining S_2 depends only upon $\pi'(\theta_1|x_1,...,x_{n-1})$ for every period n by the definition of L.

7) <u>Managerial Implication in Accounting Practice</u>

When a manager receives a cost variance report, it is necessary to understand what the variance was caused by, and what to do with the variance. Let us say that the variance is a variance for a direct cost, such as materials. There are a variety of events that could have caused that variance. It could have resulted from external events, for example, temporary price fluctuations, or higher prices paid for a special rush order. Or it may be that the variance was caused by internal events. These internal events could have resulted from two factors. The first factor is uncontrollable, for example, short-term fluctuation that have no controllable inefficiencies, and the second factor is controllable, for example, purchasing substandard quality material at a lower than standard unit price, or some inefficiencies of material usage brought about by poorly maintained machinary. If the variance is due to external events, it is extremely unlikely that the manager can take any action. Thus the present study assumes that the variance due to external events is negligible.

Accordingly, the present study considers only the internal events for the variance. However, among the controllable factors, the purchasing substandard quality materials may incur more waste. This means that there will be a trade-off between a favorable price variance(materials bought at less than standard unit price) and an unfavorable quantity variance affecting quality. But the present study does not consider the variance due to substandard quality material, because line managers must be responsible for both the price and the quality of materials when purchasing.

As a result, the present study considers only the controllable and uncontrollable factors in terms of machinary condition.

On the other hand, as shown in chapter five and six, the investigating region S_2 based on the Normal Form was classified into five cases as follows:

a) $x_1 \leqq S_2 \leqq x_2$, $x_1, x_2 \varepsilon S$ when $\sigma_1 > \sigma_2$

b) $S_2 \leqq x_1', x_2' \leqq S_2$, $x_1', x_2' \varepsilon S$ when $\sigma_1 < \sigma_2$

c) $x_3 \leqq S_2$, $x_3 \varepsilon S$ when $\sigma_1 = \sigma_2$

d) $S_2 = \Phi$ (the special case of (a))

e) $S_2=\{x|x\varepsilon S\}$ (the special case of (b)).

First, the case (a), may rarely exist in accounting practice, so that we have a difficulty in finding an appropriate instance.

However, the cases (b) and (c), are usually regarded as extensions of a simple control chart. The cost variance brought about incurring more waste than provided for in the standards, can be contained within $x_2' \leqq S_2$ of (b) above and within $x_3 \leqq S_2$ of (c) above. This variance may result from more material waste due to a poorly maintained machinary. But when the actual quantity of raw materials used was less than the quantity range allowed for the units produced, the cost variance can be contained within $S_2 \leqq x_1'$ of (b) above. This variance may also result from less material usage due to the poor maintained machinary. The above cost variances do not always mean that they are due to the poor maintained machinary, and in these cases we must investigate their causes. Furthermore, the case (d) reflects the fact that the Cost Process need not be investigated, because the investigation cost has a larger value in contrast with the opportunity cost generated when the Cost Process is not investigated. But the case (e) is in opposition to that of the case (d). The cases (d) and (e), are discussed, in detail, in the next chapter.

CHAPTER SEVEN

NUMERICAL ANALYSIS

1) Overview

This chapter, by using sensitivity and simulation analysis, shows how alternative values of parameters affect the proposed model. This analysis was conducted by using the PASCAL compuer-programming language.

Section 2 shows graphs of log $\lambda(x)$(i.e., the log function of likelihood-ratio) according to alternative parameter values used in Dittman and Prakash's(1979) study of sensitivity analysis.

Section 3 shows what values the lower or upper bound of prior probability, the critical value of the prior probability for determining when a Cost Process should be never or always investigated, must have according to alternative parameter values. The critical values above are classified according to the Full and Exploratory Investigations, therefore, the relation between these Investigations is also discussed.

Section 4 shows how the investigating region S_2 is changed according to alternative values of parameters in the Full and Exploratory Investigation cases, and simulates the relation between the two cases above with respect to the average total cost over 1000 simulated 12-month periods, in a similar manner to Magee's(1976) study.

2) <u>Graphs of Likelihood-Ratio $\lambda(x)$</u>

Consider the following parameter values of likelihood-ratio $\lambda(x)$ as given by Dittman and Prakash(1979):

parameter	alternative values
μ_1	0
μ_2	20
σ_1	5, 10, 15, 20, 25, 30
σ_2	5, 10, 15, 20, 25, 30

Note that Magee(1976) considered alternative values of the required information as follows: μ_1=100, μ_2=120, 150, σ_i=20(i=1,2), C=10, 30, 60 and g=0.5, 0.7, 0.9.

We consider the case that the value x in the graph of likelihood-ratio $\lambda(x)$ is ranged from -30 to 50 for all combinations (σ_1,σ_2) above. These graphs are drawn on semi-logarithmic graph paper by computer program as listed in Appendix F. Figures below give a visual picture of the variety of cost situations for all combinations (σ_1,σ_2) above.

i) The combinations that $\sigma_1=5$ and $\sigma_2=5,10,...,30$

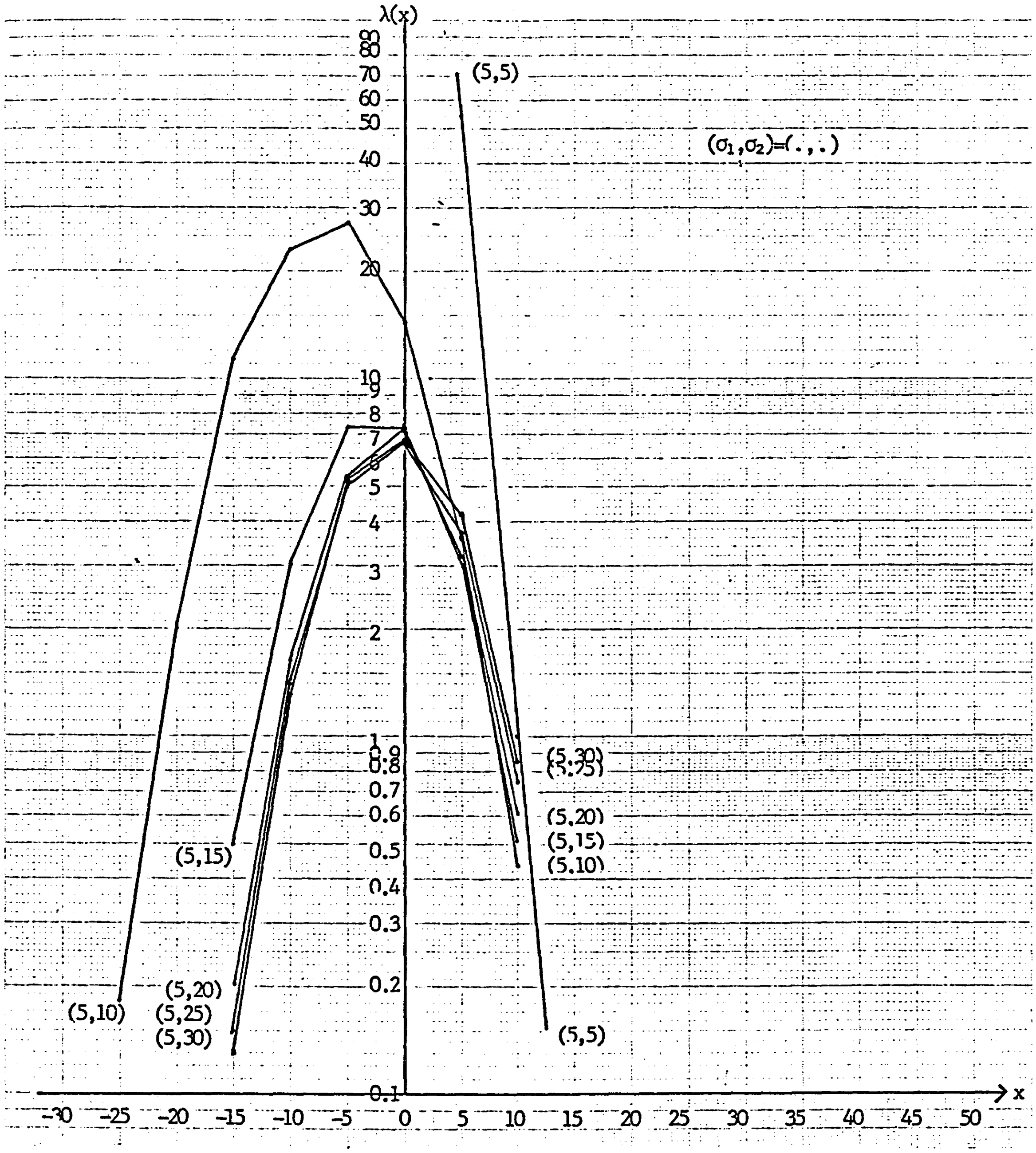


Figure 26: The Graphs of $\lambda(x)$ in the Combinations That $\sigma_1=5$ and $\sigma_2=5,10,...,30$

ii) The combinations that $\sigma_1=10$ and $\sigma_2=5,10,...,30$

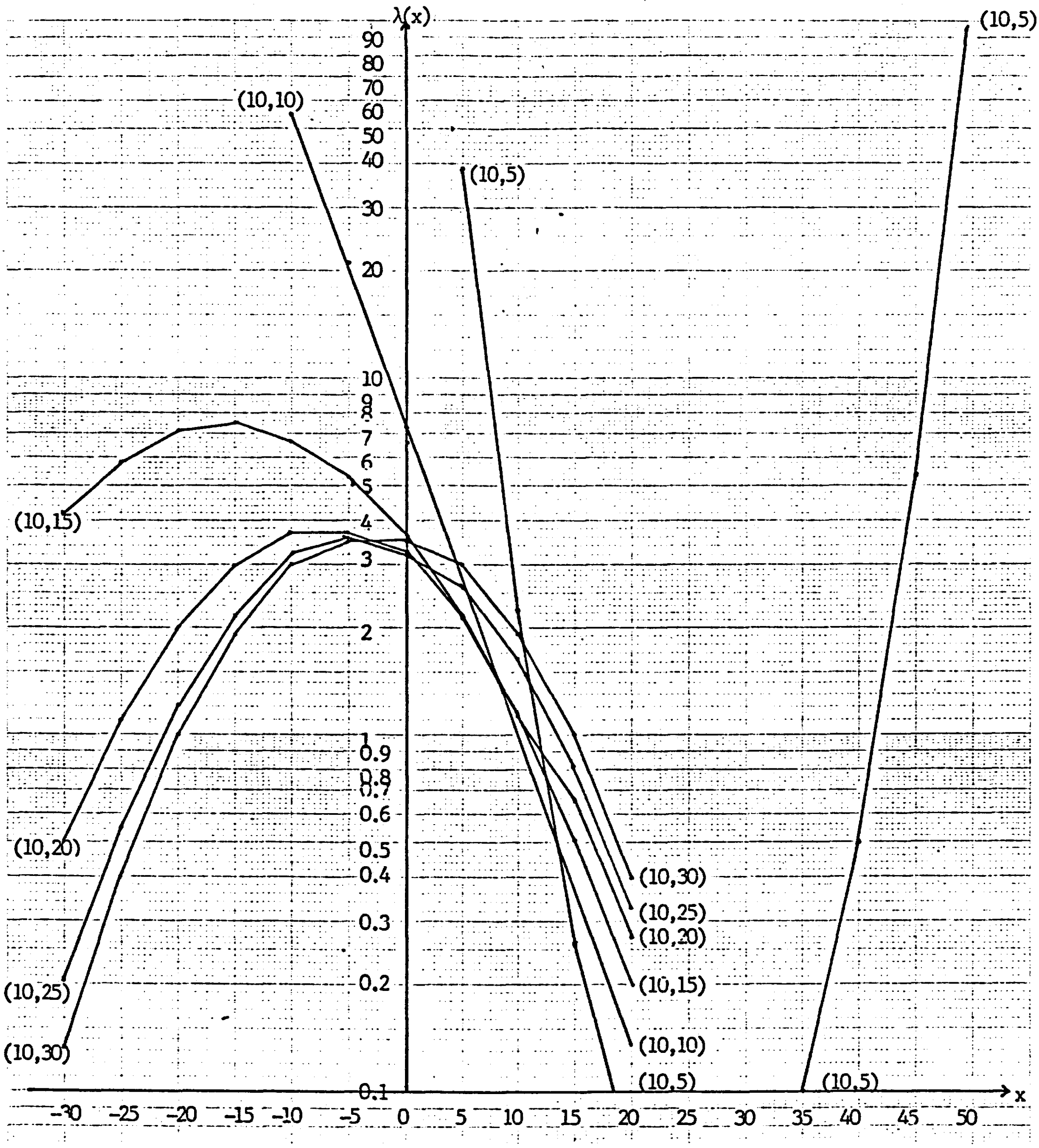


Figure 27: The Graphs of $\lambda(x)$ in the Combinations That $\sigma_1=10$ and $\sigma_2=5,10,...,30$

iii) The combinations that $\sigma_1=15$ and $\sigma_2=5,10,...,30$

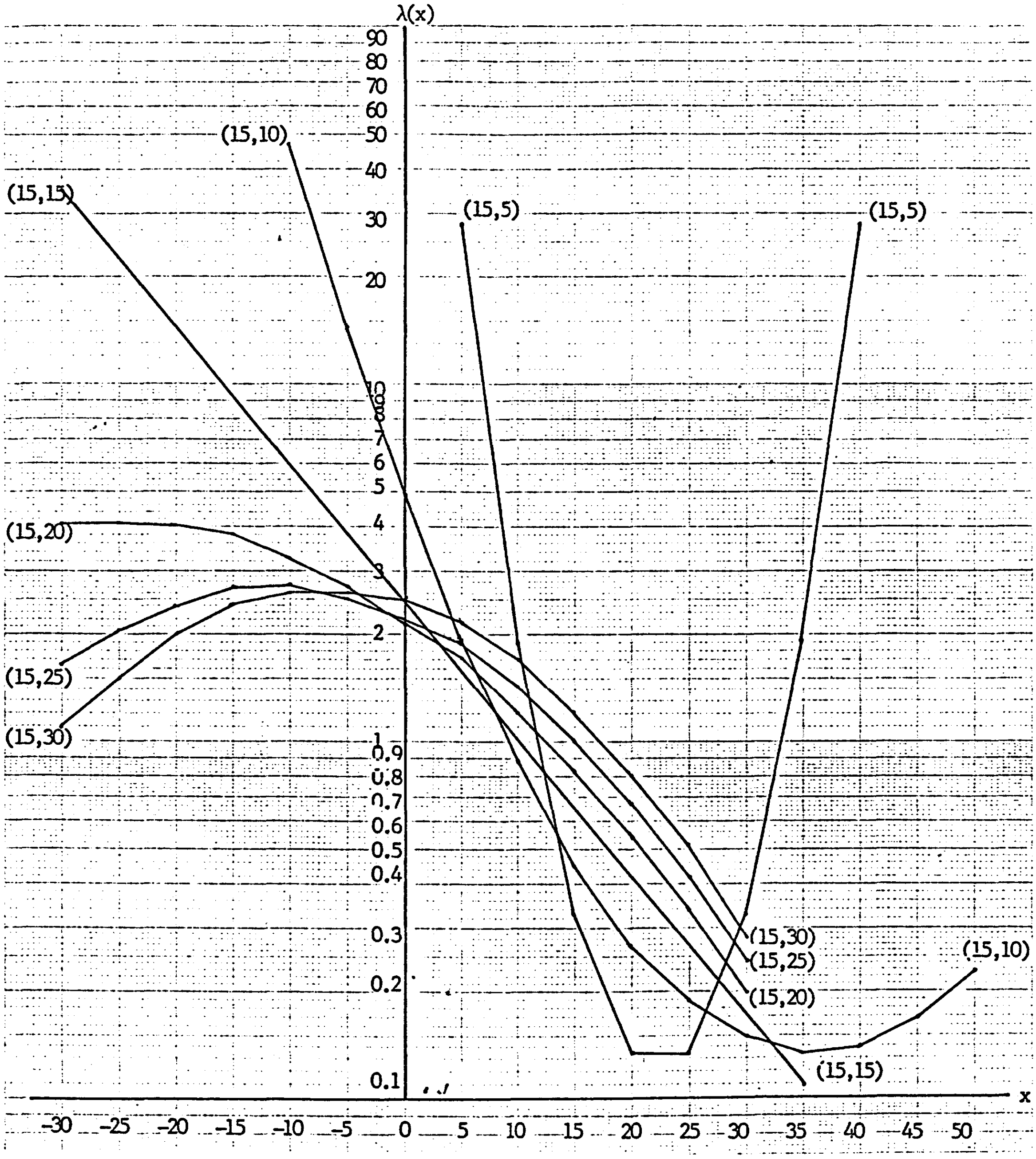


Figure 28: The Graphs of λ(x) in the Combinations That $\sigma_1=15$ and $\sigma_2=5,10,...,30$

iv) The combinations that $\sigma_1=20$ and $\sigma_2=5,10,...,30$

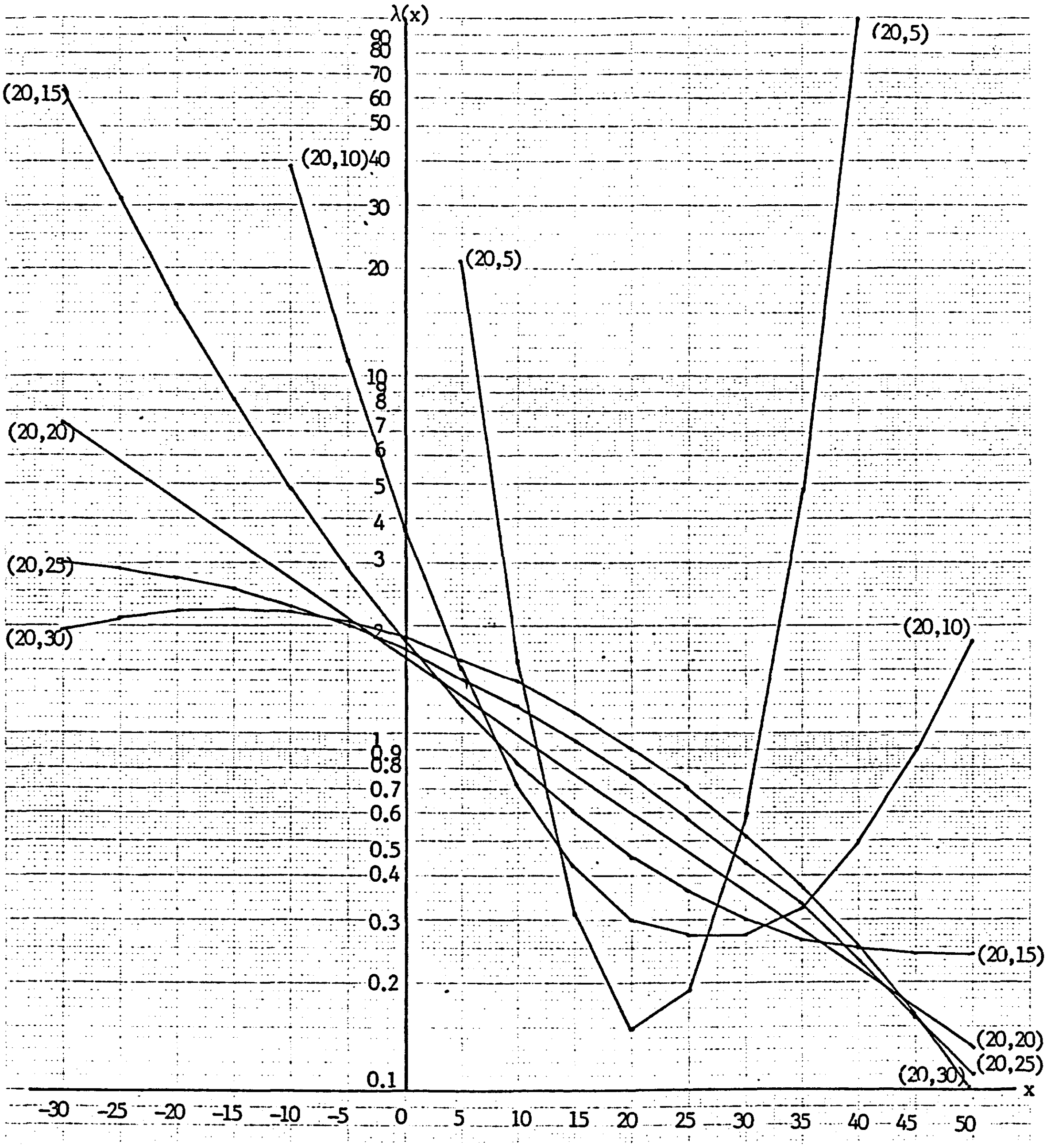


Figure 29: The Graphs of $\lambda(x)$ in the Combinations That $\sigma_1=20$ and $\sigma_2=5,10,...,30$

v) The combinations that $\sigma_1=25$ and $\sigma_2=5,10,\ldots,30$

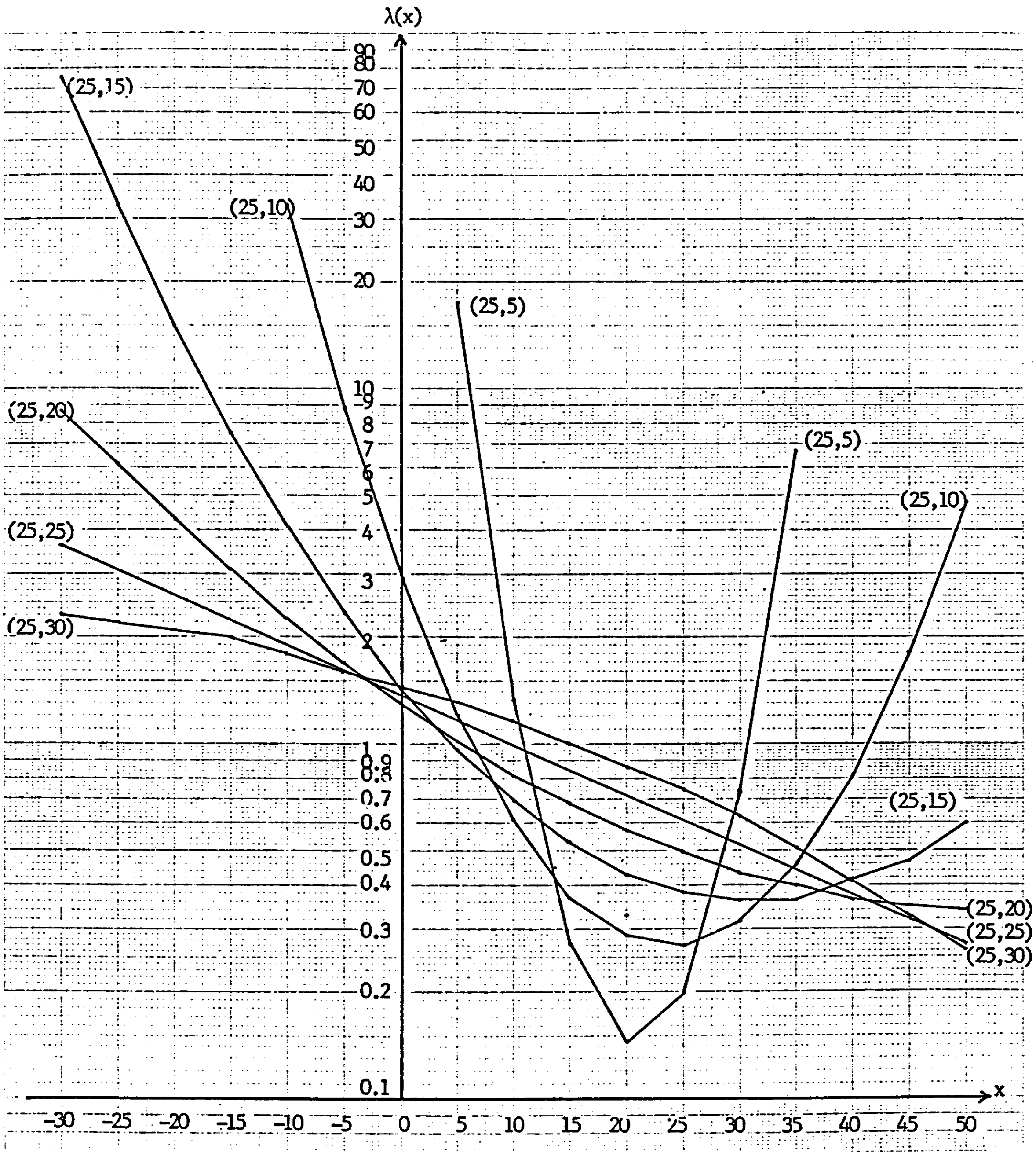


Figure 30: The Graphs of $\lambda(x)$ in the Combinations That $\sigma_1=25$ and $\sigma_2=5,10,\ldots,30$

vi) The combinations that $\sigma_1=30$ and $\sigma_2=5,10,...,30$

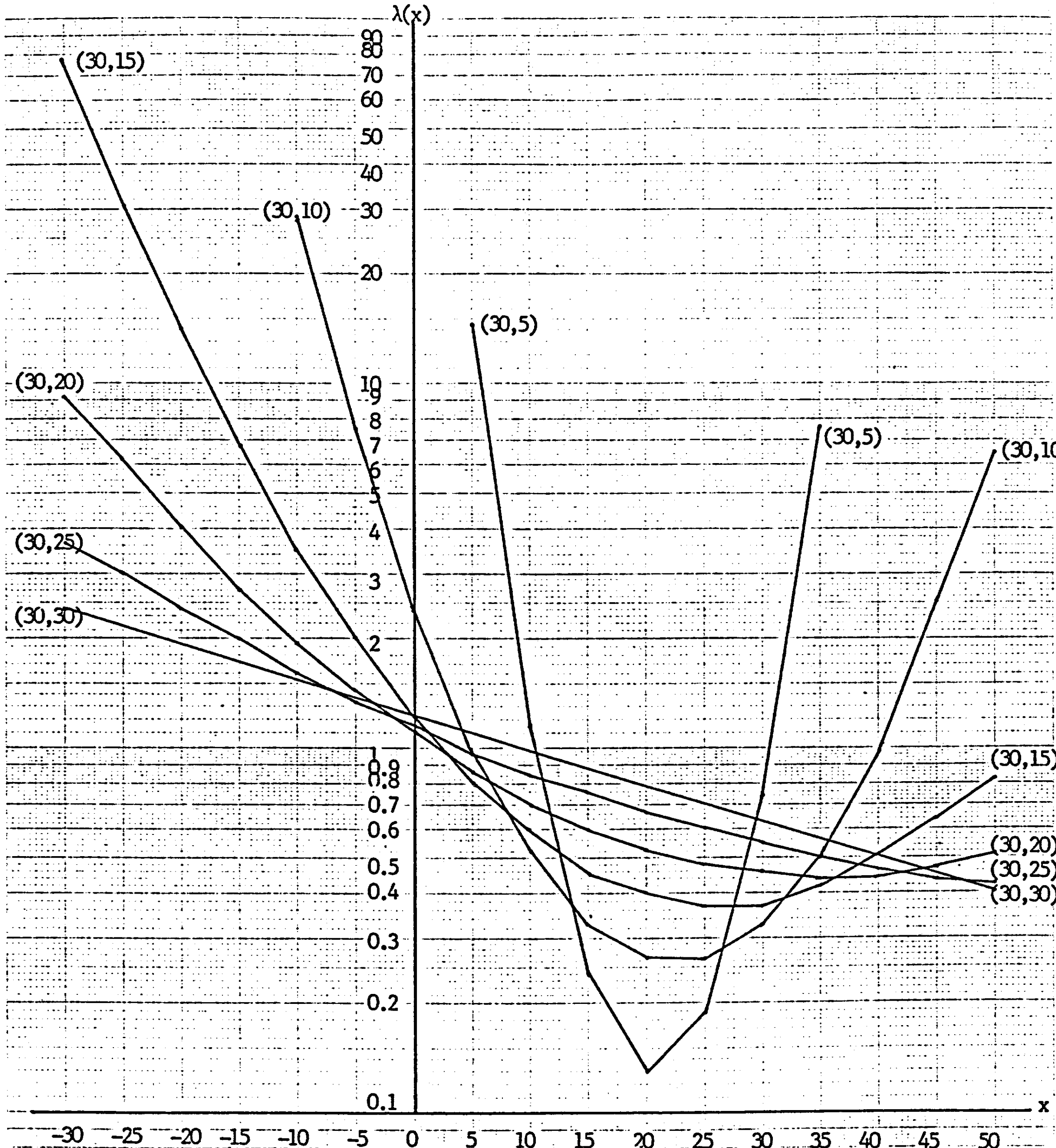


Figure 31: The Graphs of λ(x) in the Combinations That $\sigma_1=30$ and $\sigma_2=5,10,...,30$

3) <u>Determining the Lower or Upper Bound of the Prior Probability due to Opportunity Costs</u>

In order to seek the lower or upper bound of the prior probability, the critical value of the prior probability for determining when a Cost Process should be never or always investigated, consider the Full and Exploratory Investigations according to alternative parameter values similar to those in Dittman and Prakash(1979). In this numerical analysis, the correcting cost M will be assumed to have the zero value.

a) <u>Full Investigation case</u>

Additionally to the parameters discussed in section 2, consider the following alternative values of parameters:

parameter	alternative values
α	0.98
C	10, 20, 30, 40, 50, 60
g	0.5, 0.7, 0.9

The results performed according to these alternative values were conducted by the computer program listed in Appendix G. They can be classified into eighteen cases. As all eighteen cases bring about similar results, only the six cases where g=0.7 are listed below, as follows:

i) The case that C=10 and g=0.7

constant: $\mu_1=0, \mu_2=20, \alpha=0.98, C=10, g=0.7$

$\sigma_1 \backslash \sigma_2$	5	10	15	20	25	30
5		0.22	0.56	0.63	0.62	0.59
10	*		0.60	0.83	0.85	0.83
15	*	*		0.80	0.95	0.95
20	*	*	*		0.91	**
25	*	*	*	*		0.98
30	*	*	*	*	*	

where * = the case that the lower bound of prior probability does not exist(i.e., $(1-\pi g)B/\pi g$ is larger than the minimum value of $\lambda(x)$ for all π)

**= the case that always investigates a Cost Process for all x regardless of π

Table 1: The Lower or Upper Bounds of the Prior Probability in the Case That C=10 and g=0.7

ii) The case that C=20 and g=0.7

constant: $\mu_1=0, \mu_2=20, \alpha=0.98, C=20, g=0.7$

$\sigma_1 \backslash \sigma_2$	5	10	15	20	25	30
5		0.10	0.30	0.35	0.34	0.32
10	*		0.32	0.51	0.53	0.52
15	*	*		0.49	0.63	0.64
20	*	*	*		0.60	0.71
25	*	*	*	*		0.67
30	*	*	*	*	*	

Table 2: The Lower or Upper Bounds of the Prior Probability in the Case That C=20 and g=0.7

iii) The case that C=30 and g=0.7

constant: $\mu_1=0, \mu_2=20, \alpha=0.98, C=30, g=0.7$

$\sigma_1 \backslash \sigma_2$	5	10	15	20	25	30
5		0.05	0.17	0.20	0.20	0.19
10	*		0.19	0.32	0.34	0.32
15	*	*		0.30	0.41	0.42
20	*	*	*		0.39	0.48
25	*	*	*	*		0.44
30	*	*	*	*	*	

Table 3: The Lower or Upper Bounds of the Prior Probability in the Case That C=30 and g=0.7

iv) The case that C=40 and g=0.7

constant: $\mu_1=0, \mu_2=20, \alpha=0.98, C=40, g=0.7$

$\sigma_1 \backslash \sigma_2$	5	10	15	20	25	30
5		0.03	0.10	0.11	0.11	0.10
10	*		0.11	0.19	0.20	0.19
15	*	*		0.18	0.25	0.26
20	*	*	*		0.23	0.30
25	*	0.98	0.88	0.92		0.28
30	*	0.99	0.88	0.81	0.85	

Table 4: The Lower or Upper Bounds of the Prior Probability in the Case That C=40 and g=0.7

v) The case that C=50 and g=0.7

constant: $\mu_1=0, \mu_2=20, \alpha=0.98, C=50, g=0.7$

$\sigma_1 \backslash \sigma_2$	5	10	15	20	25	30
5		0.01	0.05	0.06	0.05	0.05
10	*		0.05	0.09	0.10	0.09
15	0.99	0.96		0.09	0.13	0.13
20	0.93	0.74	0.76		0.12	0.16
25	0.94	0.72	0.61	0.65		0.14
30	0.96	0.73	0.61	0.54	0.58	

Table 5: The Lower or Upper Bounds of the Prior Probability in the Case That C=50 and g=0.7

vi) The case that C=60 and g=0.7

constant: $\mu_1=0, \mu_2=20, \alpha=0.98, C=60, g=0.7$

$\sigma_1 \backslash \sigma_2$	5	10	15	20	25	30
5		*	0.01	0.01	0.01	0.01
10	0.91		0.01	0.02	0.02	0.02
15	0.48	0.45		0.02	0.03	0.03
20	0.42	0.28	0.29		0.03	0.04
25	0.43	0.26	0.21	0.23		0.03
30	0.45	0.27	0.20	0.17	0.19	

Table 6: The Lower or Upper Bounds of the Prior Probability in the Case That C=60 and g=0.7

As discussed in section 5 of chapter five, the investigating region S_2 is determined by $S_2=\{x|\lambda(x)<(1-\pi g)B/\pi g\}$ satisfying that $B>0$ and $B=(\mu_2/C(1-\alpha g))-1$, so that the value of $(1-\pi g)B/\pi g$ decreases when the prior probability increases. Therefore, in accordance with the property of $\lambda(x)$, the Cost Process can always be investigated for all x if the prior probability π is smaller than the upper bound of the prior probability when $\sigma_1^2<\sigma_2^2$. However, when $\sigma_1^2>\sigma_2^2$, the Cost Process can never be investigated for all x if the prior probability π is larger than the lower bound of the prior probability.

As shown in the results tabulated above, in the cases that C=10,20 and 30, the lower bounds of the prior probability do not exist when $\sigma_1^2>\sigma_2^2$. This reflects the fact that $(1-\pi g)B/\pi g$ has a large enough value to form the investigating region S_2 for every prior probability π, because the investigation cost C has a smaller value in contrast with the opportunity cost L generated when the Cost Process is not investigated. However, when $\sigma_1^2<\sigma_2^2$, there exists an upper bound of the prior probability π, the critical value that the Cost Process can be always investigated for all x, because the investigation cost C has a smaller value in contrast with the opportunity cost L generated when the Cost Process is not investigated.

On the other hand, in the case that C=40, the lower bound of the prior probability partially exists when $\sigma_1>\sigma_2$. However, in the case that C=50 and 60, there is a reverse trend compared to the case when C=10,20 and 30. This reflects the fact that the lower bound of the prior probability can be selected, because $(1-\pi g)B/\pi g$ has relatively a small value in this case than in the case of C=10,20 and 30, according to the decreasing of B by increasing C.

b) <u>Exploratory Investigation case</u>

In addition to the parameters discussed in section 2 and the parameters of the Full Investigation case, consider the following alternative values of the parameter h where h is taken to be a linear function and a nonlinear function of C':

C'	parameter	alternative values
C'=Ch	h	0.5, 0.7, 0.9
$C'=Ch^2$	h	0.5, 0.7, 0.9

On the other hand, as discussed earlier, we determined the investigating regions by using $S_2 = \{x \mid f(x \mid \theta_1)/f(x \mid \theta_2) < (1-\pi g)B/\pi g\}$ (B=(L-C)/C) in the Full Investigation case and $S_2 = \{x \mid f(x \mid \theta_1)/f(x \mid \theta_2) < (1-\pi g)B'/\pi g\}$ (B'=(Lh-C')/C') in the Exploratory Investigation case. However, when C'=Ch, the Exploratory Investigation case has the same results with the Full Investigation case due to the fact that B'=(Lh-Ch)/Ch=(L-C)/C=B. Therefore, from here, we discuss only the case where $C'=Ch^2$ in the Exploratory Investigation.

The results obtained according to alternative values of parameters described earlier, were sought by implementing the computer program listed in Appendix H. As all 54 cases bring about similar results, only the six cases where g=0.7 and h=0.7 are listed below, as follows:

i) The case that C=10, g=0.7 and h=0.7 when $C'=Ch^2$

constant: $\mu_1=0, \mu_2=20, \alpha=0.98, C=10, g=0.7, h=0.7 (C'=4.9)$

$\sigma_1 \backslash \sigma_2$	5	10	15	20	25	30
5		0.31	0.71	0.78	0.77	0.74
10	*		0.75	0.96	0.98	0.97
15	*	*		0.94	**	**
20	*	*	*		**	**
25	*	*	*	*		**
30	*	*	*	*	*	

Table 7: The Lower or Upper Bounds of the Prior Probability in the Case That C=10, g=0.7 and h=0.7 when $C'=Ch^2$

ii) The case that C=20, g=0.7 and h=0.7 when $C'=Ch^2$

constant: $\mu_1=0, \mu_2=20, \alpha=0.98, C=20, g=0.7, h=0.7 (C'=9.8)$

$\sigma_1 \backslash \sigma_2$	5	10	15	20	25	30
5		0.16	0.43	0.49	0.48	0.46
10	*		0.46	0.68	0.70	0.67
15	*	*		0.66	0.81	0.81
20	*	*	*		0.77	0.88
25	*	*	*	*		0.84
30	*	*	*	*	*	

Table 8: The Lower or Upper Bounds of the Prior Probability in the Case That C=20, g=0.7 and h=0.7 when $C'=Ch^2$

iii) The case that C=30, g=0.7 and h=0.7 when $C'=Ch^2$

constant: $\mu_1=0, \mu_2=20, \alpha=0.98$, C=30, g=0.7, h=0.7(C'=14.7)

$\sigma_1 \backslash \sigma_2$	5	10	15	20	25	30
5		0.09	0.29	0.33	0.32	0.30
10	*		0.31	0.49	0.51	0.49
15	*	*		0.47	0.61	0.62
20	*	*	*		0.57	0.68
25	*	*	*	*		0.64
30	*	*	*	*	*	

Table 9: The Lower or Upper Bounds of the Prior Probability in the Case That C=30, g=0.7 and h=0.7 when $C'=Ch^2$

iv) The case that C=40, g=0.7 and h=0.7 when $C'=Ch^2$

constant: $\mu_1=0, \mu_2=20, \alpha=0.98$, C=40, g=0.7, h=0.7(C'=19.6)

$\sigma_1 \backslash \sigma_2$	5	10	15	20	25	30
5		0.06	0.19	0.23	0.22	0.21
10	*		0.21	0.35	0.37	0.36
15	*	*		0.33	0.45	0.46
20	*	*	*		0.42	0.52
25	*	*	*	*		0.48
30	*	*	*	*	*	

Table 10: The Lower or Upper Bounds of the Prior Probability in the Case That C=40, g=0.7 and h=0.7 when $C'=Ch^2$

v) The case that C=50, g=0.7 and h=0.7 when $C'=Ch^2$

constant: $\mu_1=0, \mu_2=20, \alpha=0.98$, C=5 ,g=0.7, h=0.7(C'=24.5)

$\sigma_1 \backslash \sigma_2$	5	10	15	20	25	30
5		0.04	0.13	0.15	0.15	0.14
10	*		0.14	0.25	0.26	0.25
15	*	*		0.23	0.33	0.33
20	*	*	*		0.30	0.38
25	*	*	0.99	*		0.35
30	*	*	0.98	0.92	0.96	

Table 11: The Lower or Upper Bounds of the Prior Probability in the Case That C=50, g=0.7 and h=0.7 when $C'=Ch^2$

vi) The case that C=60, g=0.7 and h=0.7 when $C'=Ch^2$

constant: $\mu_1=0, \mu_2=20, \alpha=0.98$, C=60, g=0.7, h=0.7(C'=29.4)

$\sigma_1 \backslash \sigma_2$	5	10	15	20	25	30
5		0.03	0.09	0.10	0.10	0.09
10	*		0.09	0.17	0.18	0.17
15	*	*		0.16	0.23	0.23
20	*	0.95	0.98		0.21	0.27
25	*	0.93	0.84	0.87		0.25
30	*	0.95	0.83	0.77	0.80	

Table 12: The Lower or Upper Bounds of the Prior Probability in the Case That C=60, g=0.7 and h=0.7 when $C'=Ch^2$

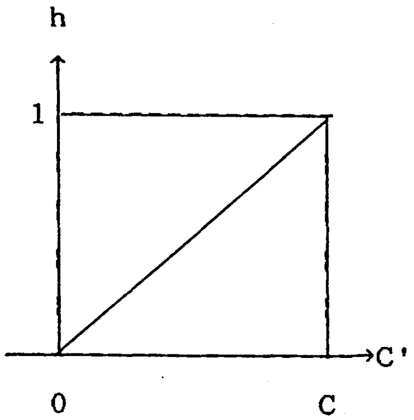
As discussed in the Full Investigation case earlier, the investigating region S_2 is determined by $S_2=\{x|\lambda(x)<(1-\pi g)B'/\pi g\}$ satisfying that $B'=(Lh-C')/C'$ and $B'>0$. This can be analyzed in a similar manner to the Full Investigation case. That is, the function B' is written as $(L-Ch)/Ch=[(L/h)-C]/C$ by using $C'=Ch$, so that it has a larger value than the B of the Full Investigation case independent on alternative values of h. Accordingly, the critical probabilities of the Exploratory Investigation case are a little larger than those of the Full Investigation case.

As a result, the prior probability π, by the Normal Form, plays an important part for determining the critical prior probability, base that a Cost Process always or never be investigated, while the Extensive Form cannot determine the critical prior probability. Furthermore, the results of the numerical analysis show the fact that the control policy, due to the prior probability, can be sufficiently implemented depending on alternative parameter values when $\sigma_1<\sigma_2$.

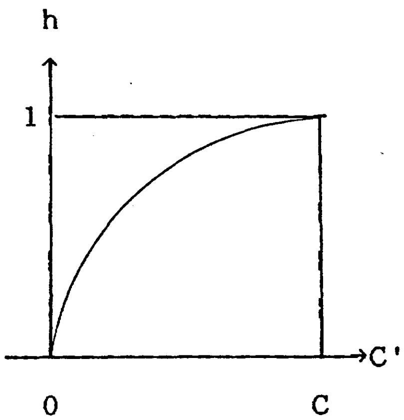
4) Determining the Investigating Region S_2 according to the Alternative Values of Parameters

a) Determining the investigating region S_2

This study classified the investigation policies into the Full and Exploratory Investigation cases, and the latter was divided into two cases that C'=Ch and $C'=Ch^2$. However, note that due to the reason described earlier, when C'=Ch, the investigation region of the Exploratory Investigation case results in that of the Full Investigation case. The investigating region S_2, according to alternative parameter values of each case, can be obtained by implementing computer programs listed in Appendix I and Appendix J. Additionally, the exploratory investigation costs can be shown as in the following figures:



The Graph of C'=Ch



The Graph of $C'=Ch^2$

b) <u>The relation between the Full and Exploratory Investigation cases</u>

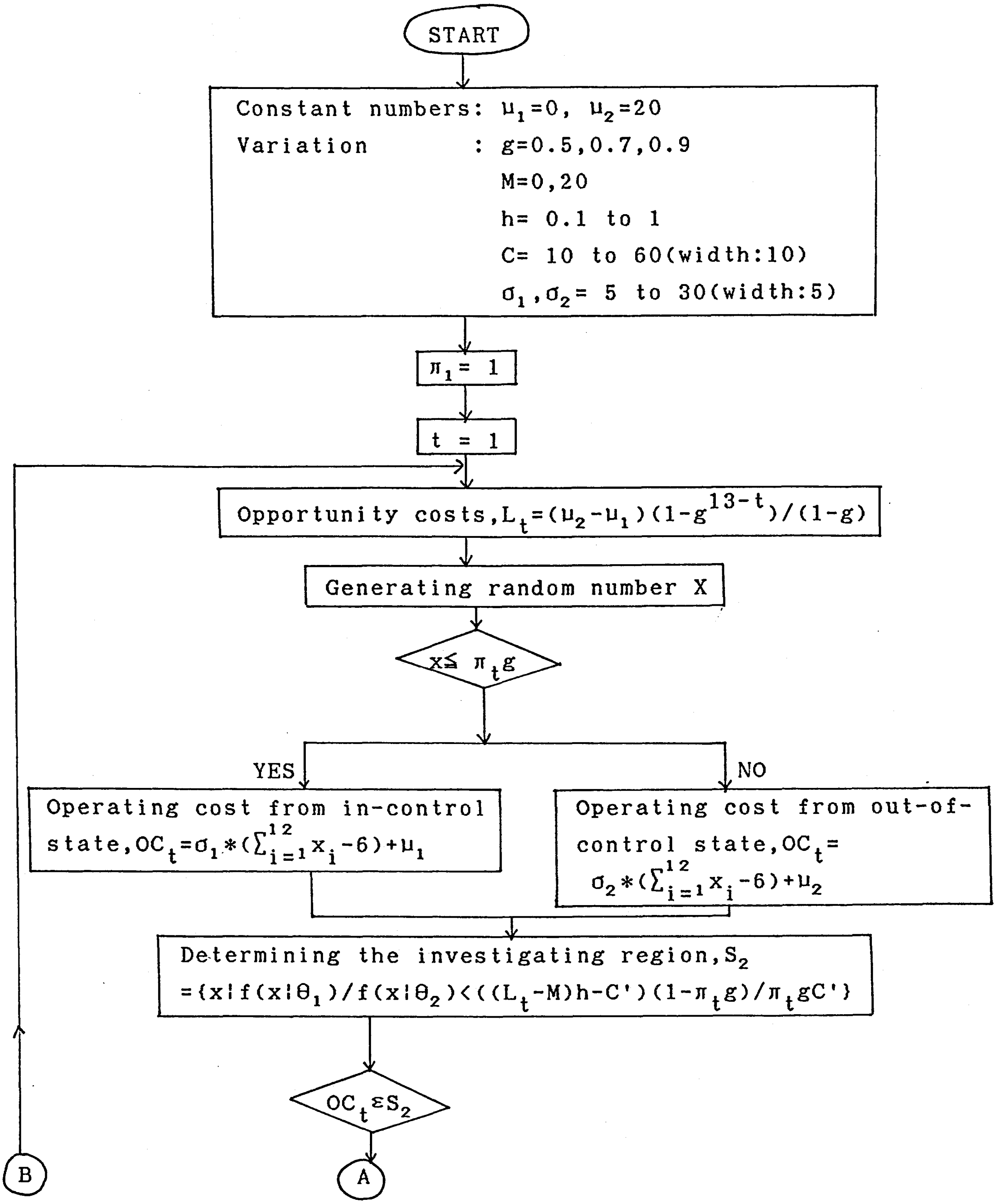
As discussed earlier, the investigating regions are given by $S_2=\{x|f(x|\theta_1)/f(x|\theta_2)<(1-\pi g)B/\pi g\}$ $(B=(L-C)/C)$ in the Full Investigation case and $S_2'=\{x|f(x|\theta_1)/f(x|\theta_2)<(1-\pi g)B'/\pi g\}$ $(B'=(Lh-C')/C')$ in the Exploratory Investigation case. Here, the relation between the two cases is influenced by the trend of deviation between B and B'. Therefore, we need to investigate their trend with respect to h.

Firstly, when $C'=Ch$, the deviation between B and B' has a zero value for all h due to the fact that $B'-B= [(Lh-C')/C']-[(L-C)/C] = [(Lh-Ch)/Ch]-[(L-C)/C]=0$. This indicates that a decision maker had better implement the Exploratory Investigation for all h because the same result can be derived with less cost than the full investigation cost. However, as discussed in section 2 of chapter six, the prior probability of being in control after the unsuccessful Exploratory Investigation, $\pi'(\theta_1|x)$, can be obtained by $f(x|\theta_1)\pi g/[f(x|\theta_1)\pi g+f(x|\theta_2)(1-\pi g)(1-h)]$. This indicates that the Exploratory Investigation, which is carried out at costs lower than that of the full-investigation cost, makes the prior probability of being out of control after unsuccessful Exploratory Investigation higher. Thus a decision maker cannot determine which of the above two policies is best.

On the other hand, when $C'=Ch^2$, the deviation between B and B' is a monotone decreasing function with respect to h due to the fact that $B'-B=[(Lh-C')/C']-[(L-C)/C]= [(Lh-Ch^2)/Ch^2]-[(L-C)/C]= L(1-h)/Ch$. Thus this case also cannot be analyzed with the analytical method by the same reason discussed in the case when $C'=Ch$.

Here, the relation between the Full and Exploratory Investigation cases is carried out over 1000 simulated 12-month periods with respect to the average total costs using a method similar to that used by Magee(1976). This total cost includes operating costs plus the costs of investigations, and the operating costs are regarded as the random numbers and hence are randomized in the present simulation. The computer program for this relation is listed in Appendix K.

Before showing the simulated results, let us show the flow chart indicating the relation between the two cases with respect to the average total costs as follows:


START
Constant numbers: μ1=0, μ2=20
Variation : g=0.5,0.7,0.9
M=0,20
h= 0.1 to 1
C= 10 to 60(width:10)
σ1,σ2= 5 to 30(width:5)
π1= 1
t = 1
Opportunity costs,Lt=(μ2-μ1)(1-g^(13-t))/(1-g)
Generating random number X
x≤ πtg
YES
NO
Operating cost from in-control state,OCt=σ1*(Σ12i=1 xi-6)+μ1
Operating cost from out-of-control state,OCt= σ2*(Σ12i=1 xi-6)+μ2
Determining the investigating region,S2 ={x|f(x|θ1)/f(x|θ2)<((Lt-M)h-C')(1-πtg)/πtgC'}
OCt ε S2
B
A

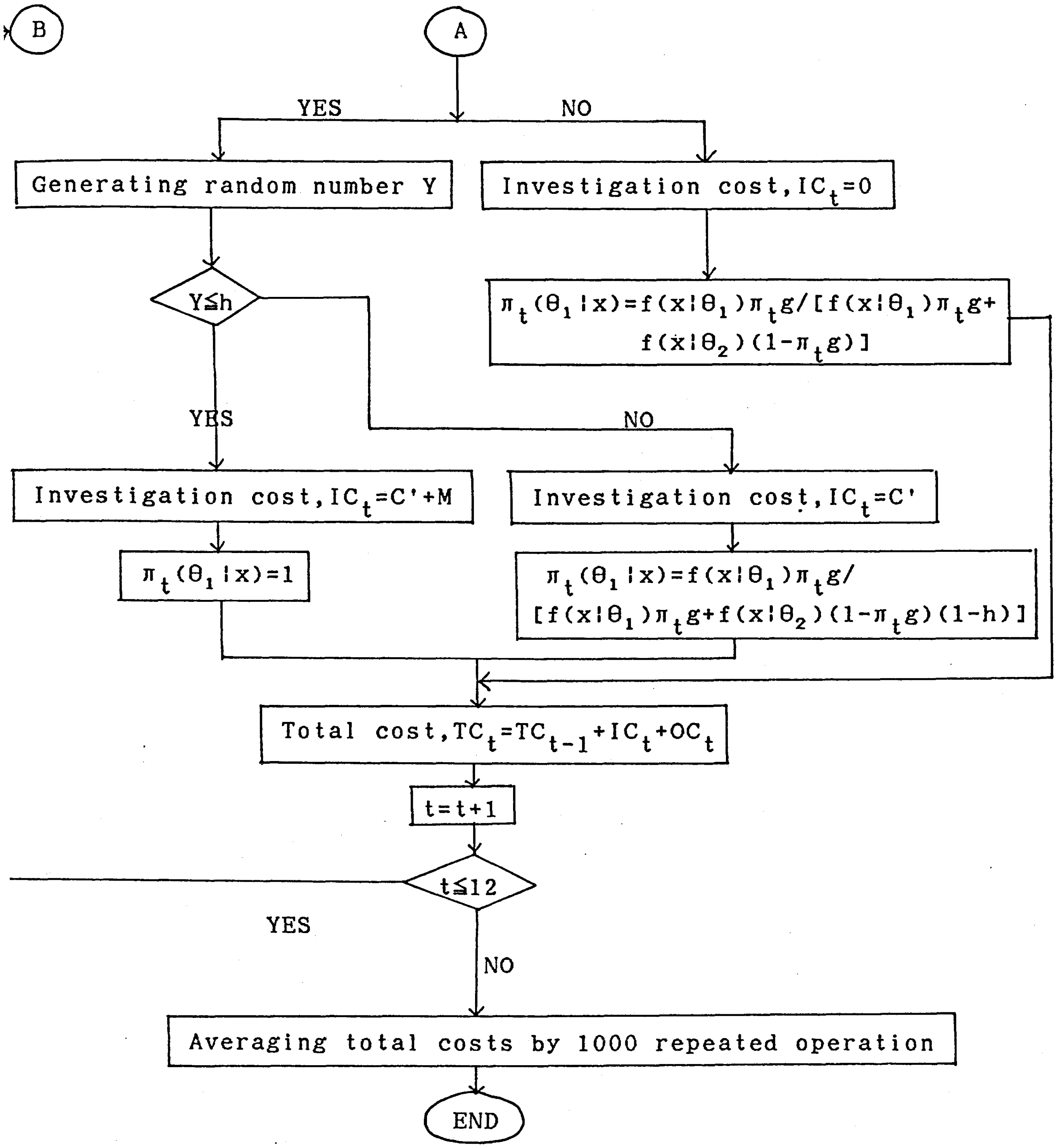


Figure 32: Flow Chart for the Average Total Costs

On the basis of the flow chart shown above, the simulated sults according to the alternative parameter values are shown as the following tables:

constant: $\mu_1=0, \mu_2=20, g=0.7$ and $M=0$

σ_2	C		h									
			0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5	10	OC	166.5	145.5	130.3	117.7	104.5	97.8	91.2	82.9	76.7	73.1
		IC	8.5	14.9	20.0	24.2	26.8	30.5	33.0	34.8	36.3	38.6
		TC	175.0	160.4	150.3	141.9	131.3	124.2	124.0	117.7	113.0	111.7*
	30	OC	167.6	146.3	127.5	115.9	108.4	95.2	88.6	83.5	78.7	72.8
		IC	22.7	40.0	52.4	63.2	73.2	77.7	84.6	89.8	95.5	98.6
		TC	190.3	186.3	179.9	179.1	181.6	172.9	173.2	173.3	174.2	171.4*
	50	OC	171.3	154.5	137.8	126.0	121.6	107.5	102.7	96.3	92.6	87.1
		IC	29.2	52.5	67.9	82.7	98.5	103.3	109.4	117.4	126.1	131.0
		TC	200.5*	207.0	205.7	208.7	220.1	210.8	212.1	213.7	218.7	218.1
20	10	OC	165.7	147.5	131.6	118.4	109.6	104.1	92.2	90.5	85.2	78.4
		IC	8.7	15.6	20.9	25.3	29.3	33.4	35.5	39.2	41.5	44.0
		TC	174.4	163.1	152.5	143.7	138.9	137.5	127.7	129.7	126.7	122.4*
	30	OC	170.4	154.8	133.0	123.6	115.5	105.6	101.3	95.0	92.4	87.2
		IC	21.4	38.3	49.4	59.7	67.2	72.6	77.9	83.0	87.3	93.4
		TC	191.8	193.1	182.4	183.3	182.7	178.2	179.2	178.0*	179.7	180.6
	50	OC	170.1	159.0	143.2	135.5	123.5	119.0	114.1	110.4	105.5	102.6
		IC	27.2	48.0	61.0	74.1	83.7	91.7	99.6	104.0	108.9	113.9
		TC	197.3*	207.0	204.2	209.6	207.2	210.7	213.7	214.4	214.4	216.5
5	10	OC	164.7	142.2	122.4	113.7	92.9	89.1	84.8	77.1	74.2	75.7
		IC	12.0	24.0	36.0	48.0	60.0	72.0	84.0	96.0	108.0	120.0
		TC	176.7	166.2	158.4	161.7	152.9*	161.1	168.8	173.1	182.2	195.7
	30	OC	171.1	141.2	123.7	112.9	101.6	90.4	80.6	78.8	76.9	72.3
		IC	32.3	63.5	94.0	124.1	153.6	183.3	212.2	240.9	270.0	300.0
		TC	203.4*	204.7	217.7	237.0	255.2	273.7	292.8	319.7	346.9	372.3
	50	OC	175.2	160.2	143.0	141.0	134.6	134.4	131.4	120.4	122.2	116.5
		IC	27.8	48.2	63.2	76.0	86.4	95.5	104.2	108.0	114.8	119.4
		TC	203.0*	208.4	206.2	217.0	221.0	229.9	235.6	228.4	237.0	235.9

where: OC= The operating costs

IC= The investigating costs

TC= The total costs

*= Minimum total costs for each case

Table 13: Average Total Cost over 1000 12-Month Periods with respect to C'=Ch Where M=0

constant: $\mu_1=0, \mu_2=20, g=0.7$ and $M=20$

1	σ_2	C		h									
				0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5	5	10	OC	164.5	146.2	127.9	116.7	105.8	95.0	92.0	81.5	79.5	73.3
			IC	22.1	41.0	54.1	65.6	73.7	80.4	87.8	92.5	101.1	103.7
			TC	186.6	187.2	182.0	182.3	179.5	175.4	179.8	174.0*	180.6	177.0
		30	OC	172.4	154.7	135.0	125.9	116.5	109.9	99.7	96.4	90.1	87.0
			IC	29.6	52.3	68.3	83.7	94.2	105.7	111.8	119.5	124.5	127.2
			TC	202.0*	207.0	203.3	209.6	210.6	215.6	211.5	215.9	214.6	214.2
		50#	OC	193.5	195.5	193.6	194.3	196.3	192.0	195.1	195.0	191.5	193.3
			IC	0	0	0	0	0	0	0	0	0	0
			TC	193.5	195.5	193.6	194.3	196.3	192.0	195.1	195.0	191.5	193.3
5	20	10	OC	163.6	144.2	131.7	120.3	112.0	107.9	96.8	88.7	88.0	80.5
			IC	22.6	38.2	53.0	63.5	73.5	82.0	88.4	93.0	100.3	107.2
			TC	186.2	182.4	184.7	183.7	185.5	189.9	185.2	181.7*	188.3	187.7
		30	OC	172.9	156.2	141.9	136.1	124.7	114.6	115.3	108.9	105.3	100.3
			IC	27.3	48.3	63.0	76.7	84.7	92.9	99.0	107.1	111.6	115.5
			TC	200.2*	204.5	204.9	212.8	209.4	207.5	214.3	216.0	216.9	215.8
		50#	OC	192.2	189.8	197.1	194.8	195.1	193.5	194.2	190.8	195.7	195.8
			IC	0	0	0	0	0	0	0	0	0	0
			TC	192.2	189.8	197.1	194.8	195.1	193.5	194.2	190.8	195.7	195.8
0	5	10	OC	164.5	143.7	123.9	108.2	100.3	84.9	81.7	76.5	73.0	71.2
			IC	33.3	64.9	97.9	133.6	165.2	198.5	231.8	264.5	296.4	330.0
			TC	197.8*	208.6	221.8	241.8	265.5	283.4	313.5	341.0	369.4	401.2
		30	OC	174.6	153.9	134.5	127.0	121.1	112.7	108.8	102.9	104.8	102.0
			IC	40.6	76.5	109.0	141.4	171.9	204.0	233.0	262.3	291.7	324.2
			TC	215.2*	230.4	243.5	268.4	293.0	316.7	341.8	365.2	396.5	426.2
		50#	OC	193.3	194.3	197.3	189.6	195.5	191.9	196.3	198.4	193.3	191.5
			IC	0	0	0	0	0	0	0	0	0	0
			TC	193.3	194.3	197.3	189.6	195.5	191.9	196.3	198.4	193.3	191.5

where #= The case that the Cost Process can never be investigated because B'<0. (i.e., this case generates only operating costs)

Table 14: Average Total Cost over 1000 12-Month Periods with respect to C'=Ch Where M=20

Note that, when h=1, the Exploratory Investigation case results in the Full Investigation case.

As shown in the above tables, when C'=Ch, the smallest values of the average total costs simulated in this numerical analysis are almost found in the Exploratory Investigation cases rather than in the Full Investigation cases. Furthermore, when C increases, h, which gives minimum total costs, decreases. Therefore, if the Full Investigation cost C is large, there is no need to spend as much money as the Full Investigation cost C for the Exploratory Investigation to attain minimum total costs, although the total costs that will be incurred by the Exploratory Investigation may be larger than total costs by the Full Investigation. However, when B'<0, the average total costs have almost the same values for all h, so that it is not possible to determine the best investigation policy. This can be seen for the cases of C=50 when M=20 in the above tables.

For this reason, a decision maker must determine the optimal value of h by simulating how parameters influence the average total costs.

constant: $\mu_1=0, \mu_2=20, g=0.7$ and M=0

ι	σ_2	C		h									
				0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5	5	10	OC	166.8	144.8	130.0	118.1	105.0	96.6	91.2	83.7	76.4	73.1
			IC	0.9	3.1	6.2	10.0	13.8	18.3	23.6	28.3	32.8	38.6
			TC	167.7	147.9	136.2	128.1	118.8	114.9	114.8	112.0	109.2*	111.7
		30	OC	166.1	146.1	127.8	116.6	106.3	96.3	88.2	82.9	78.7	72.9
			IC	2.6	9.0	17.8	28.6	40.3	52.8	60.1	73.5	86.8	98.9
			TC	168.7	155.1	145.6	145.2*	146.6	149.1	148.3	156.4	165.5	171.8
		50	OC	170.6	146.0	127.4	114.8	111.1	96.8	94.5	87.8	89.8	88.3
			IC	4.4	14.9	29.1	42.5	63.0	78.4	89.9	107.2	115.2	126.9
			TC	175.0	160.9	156.5*	157.3	174.1	175.2	184.4	195.0	205.0	215.2
5	20	10	OC	163.9	140.1	125.4	112.5	107.0	97.3	89.4	88.3	83.9	78.0
			IC	1.2	4.7	10.2	15.8	19.9	24.1	28.8	33.7	38.6	43.9
			TC	165.1	144.8	135.6	128.3	126.9	121.4	118.2*	122.0	122.5	121.9
		30	OC	165.9	145.7	131.0	122.0	110.8	105.7	98.5	94.7	92.1	86.0
			IC	3.5	10.6	19.5	29.2	40.4	51.6	58.8	69.6	82.2	93.2
			TC	169.4	156.3	150.5*	151.2	151.2	157.3	157.3	164.3	174.3	179.2
		50	OC	166.1	146.8	133.5	122.6	113.3	105.5	104.2	103.2	103.4	103.2
			IC	5.2	15.4	29.4	41.3	56.9	73.0	83.1	94.3	98.7	114.2
			TC	171.3	162.2*	162.9	163.9	170.2	178.5	187.3	197.5	202.1	217.4
)	5	10	OC	165.8	141.3	121.2	111.0	95.3	87.5	82.0	79.4	71.7	77.7
			IC	1.2	4.8	10.8	19.2	30.0	43.2	58.8	76.8	97.2	120.0
			TC	167.0	146.1	132.0	130.2	125.3*	130.7	140.8	156.2	168.9	197.7
		30	OC	167.5	140.2	124.3	112.7	96.7	82.9	79.6	75.9	78.1	72.5
			IC	3.6	14.4	32.4	57.6	84.8	120.7	161.7	211.2	243.9	300.0
			TC	171.1	154.6*	156.7	170.3	181.5	203.6	241.3	287.1	322.0	372.5
		50	OC	167.8	139.4	126.6	108.7	99.3	94.2	88.0	88.1	95.6	119.4
			IC	6.0	24.0	51.9	88.0	137.5	183.1	223.7	271.1	274.9	122.5
			TC	173.8	163.4*	178.5	196.7	236.8	277.3	311.8	359.2	370.5	241.9

Table 15: Average Total Cost over 1000 12-Month Periods with respect to $C'=Ch^2$ Where M=0

The results tabulated above are shown as in the following figure:

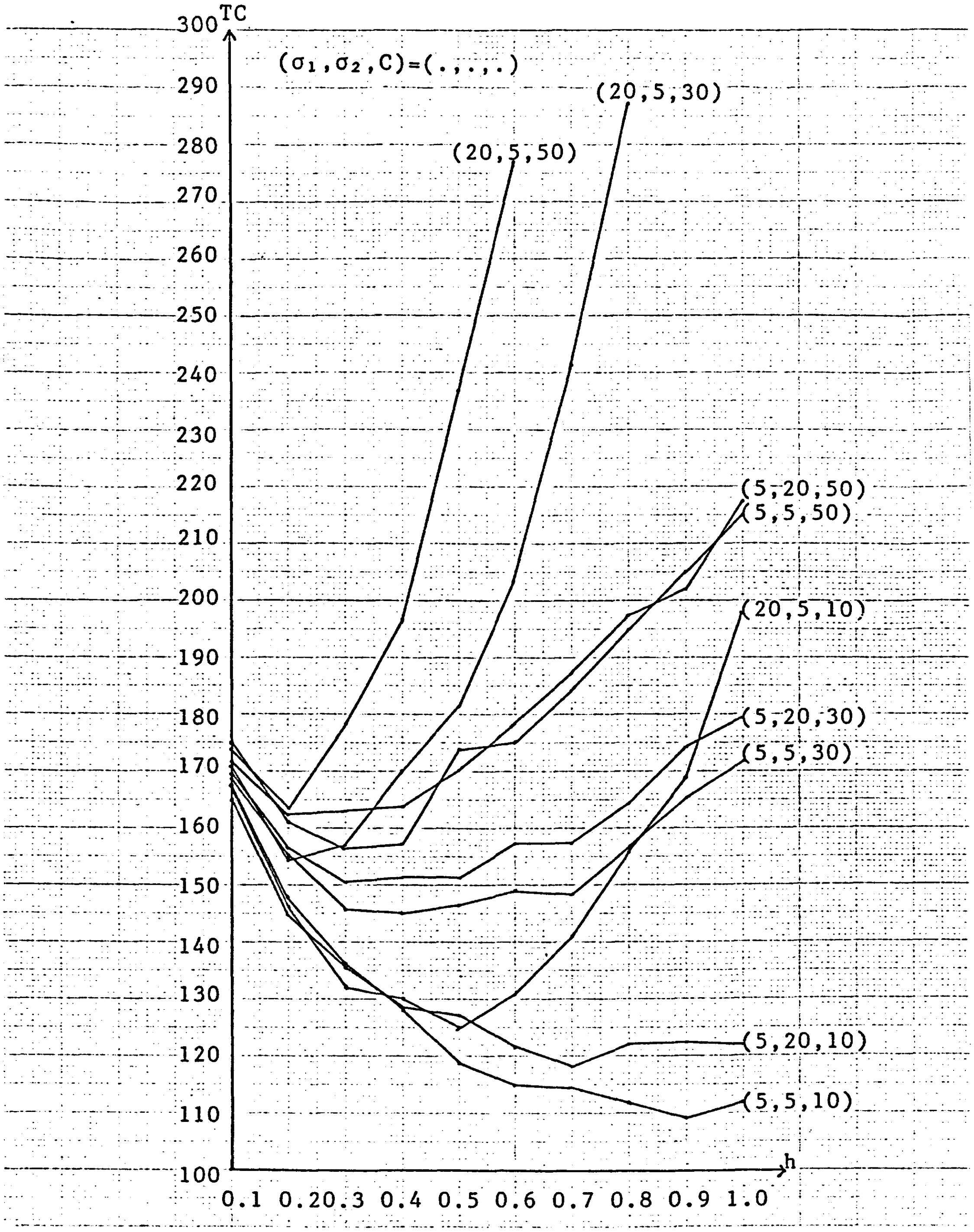


Figure 33: Average Total Cost over 1000 12-Month Periods with respect to $C'=Ch^2$ Where M=0

constant: $\mu_1=0, \mu_2=20, g=0.7$ and M=20

ı	σ_2	C		h									
				0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5	5	10	OC	163.2	143.7	128.6	114.9	105.5	93.5	91.3	82.5	79.1	73.3
			IC	17.0	31.1	42.4	52.0	62.2	70.1	80.4	87.6	97.8	103.6
			TC	180.2	174.8	171.0	166.9	167.7	163.6*	171.7	170.1	177.0	176.9
		30	OC	167.4	147.6	127.2	117.8	111.2	101.9	92.7	92.8	88.3	88.0
			IC	17.8	35.1	51.6	69.4	77.3	92.2	105.6	106.9	118 3	130.3
			TC	185.2	182.7	178.8*	187.2	188.5	194.1	198.3	199.7	206.6	218.3
		50	OC	167.3	144.9	129.4	119.0	118.3	108.7	110.9	126.1	173.3	195.8
			IC	19.5	40.3	55.1	75.9	87.9	107.7	109.3	92.9	29.0	0
			TC	186.8	185.2	184.5*	194.9	206.2	216.4	220.2	219.0	202.3	195.8
5	20	10	OC	164.2	143.0	127.7	116.7	104.8	103.1	93.0	90.2	85.9	82.1
			IC	24.0	46.9	60.8	66.8	74.1	79.7	87.3	94.1	98.8	105.6
			TC	188.2	189.9	188.5	183.5	178.9*	182.8	180.3	184.3	184.7	187.7
		30	OC	171.0	148.4	129.8	120.5	117.8	111.1	102.4	106.3	101.8	101.4
			IC	23.1	38.6	52.9	66.0	73.1	85.6	95.2	97.2	108.2	117.0
			TC	194.1	187.0	182.7*	186.5	190.9	196.7	197.6	203.5	210.0	218.4
		50	OC	162.0	148.1	137.6	128.0	122.2	117.9	123.0	138.0	176.9	193.9
			IC	21.3	39.0	54.3	71.6	79.4	93.4	95.1	80.2	23.4	0
			TC	183.3*	187.1	191.9	199.6	201.6	211.3	218.1	218.2	200.3	193.9
)	5	10	OC	167.3	144.4	121.4	113.3	93.8	86.4	85.4	75.8	75.1	76.0
			IC	22.1	48.4	74.5	105.8	138.6	171.4	208.1	246.4	288.2	330.0
			TC	189.4*	192.8	195.9	219.1	232.4	257.8	293.5	322.2	363.3	406.0
		30	OC	167.9	139.6	124.6	112.7	103.5	96.4	85.8	92.9	93.8	100.4
			IC	25.2	57.5	95.4	131.6	175.2	210.4	260.8	285.5	316.5	324.3
			TC	193.1*	197.1	220.0	244.3	278.7	306.8	346.6	378.4	410.3	424.7
		50	OC	164.8	145.7	121.2	116.7	110.1	119.1	138.8	149.1	193.3	195.2
			IC	26.9	64.0	105.8	149.4	186.9	203.5	97.6	70.9	0	0
			TC	191.7*	209.7	227.0	266.1	297.0	322.6	236.4	220.0	193.3	195.2

Table 16: Average Total Cost over 1000 12-Month Periods with respect to $C'=Ch^2$ Where M=20

Different from $C'=Ch$, as shown in the table above, when $C'=Ch^2$, the smallest values of the average total costs simulated in these numerical examples are always found in the Exploratory Investigation case rather than in the Full Investigation case. Furthermore, when C increases, h, which gives minimum total costs, decreases. For this reason, a decision maker must determine the optimal value of h by simulating how parameters influence the average total costs.

Note that the author obtained similar results to the above results in the cases, $g=0.5$ and $g=0.9$.

The results obtained in this simulation analysis are summarized as in the following proposition:

[PROPOSITION 3]

> The value of h that gives minimum total costs will almost exist when $C'=Ch$ and will always exist when $C'=Ch^2$, in the range of the Exploratory Investigation. Furthermore, when the value of C increases, the value of h that gives minimum total costs will decrease. Therefore, if the Full Investigation cost C is large, there is no need to spend as much money as the Full Investigation cost C for the Exploratory Investigation to attain minimum total costs, although the total costs that will be incurred by the Exploratory Investigation may be larger than total costs by the Full Investigation.

CHAPTER EIGHT

CONCLUSION

1) Overview

After the Second World War, there was an increasing awareness of the view that accounting information should be appropriate to the needs of users, especially managers. Thus management accounting became recognized such that accounting information could be widely used in both managerial planning and managerial control.

With this recognition, management accounting placed the notion of responsibility accounting at the very center of the management control system. Responsibility accounting was confirmed to the principles of management which emphasized lines of authority and responsibility. These principles had a substantial impact on organizational design, and it was argued that the responsibility accounting system should be founded upon the company's organization structure.

Although managerial control system, such as standard costing and budget control, had been developed in the early decades of the twentieth century, the responsibility accounting developed rapidly, in the vanguard of advances in management accounting. Its development and popularity was a major step in the movement from cost control to managerial control which typified the emergence of management. The use of standard costs and/or budgets to quantify plans for responsibility centers and the measurement of performance in terms of variances therefrom became the main method of managerial control.

Due to the recognition for managerial control, new methods of evaluating variances were derived from the economic framework. This implies economic decisions on whether or not the process should be investigated.

The previous studies were classified into three kinds according to how control variables were established. The first was the Decision Theoretic Approach in which the control variable was denoted as a Bayesian posterior probability with respect to opportunity costs. The second was the Bayesian Dynamic Programming Approach, in which the control variable was denoted as a Bayesian posterior probability relating whether to investigate a Cost Process with respect to operating costs and investigation costs. The third was the Markovian Approach in which the control variable was denoted as a cost variance itself in order to decide whether to investigate a Cost Process with respect to operating costs and investigation costs. However, these approaches may entail trade-offs with respect to the "best policy" as pointed out by Magee(1976), Dittman and Prakash(1979) and other comparison studies.

Accordingly, the present study does not discuss the comparison between the proposed model and the above three approaches, because the simulation results of Magee(1976) show that the differences between Dyckman's(1969) and Kaplan's(1969) models may have little effect on the incremental cost savings; and the Normal Form of this study and Extensive Form of Dyckman's model are mathematically equivalent and lead to identical results whether the pre-experimental viewpoint or post-experimental viewpoint is taken.

However, as pointed out in the Literature Review chapter, the Markovian Approach has the problem that the existence of x^* is restricted within the condition $0<(1-g)K+I-g\Delta\mu<gI$ and $F_1(x^*)>((1-g)K+I-g\Delta\mu)/gI$, and that x^* is sought by minimizing the total cost function with respect to the trial and error method because the total cost function is not easily differentiated. Also the Bayes Dynamic Programming Approach has the problem that the models are complex and have a higher solution cost.

Thus, this study concentrated on developing another form of the Decision Theoretic Approach that can capture much of the benefits obtainable from the other models. The control variable is denoted as a cost variance itself in place of the Bayes posterior probability used in the previous Decision Theoretic Approach models.

Section 2 discusses, the relation between the proposed and previous models, and the contribution from the proposed model against the previous models.

The limitations of the proposed model and further direction are discussed in section 3.

2) The Results of the Proposed Model

This study concentrated on developing a new method, using a Normal Form of Decision Theoretic Approach, and investigating the relation between the Full and Exploratory Investigation cases with respect to investigation costs.

Thus, we can summarize the results and contributions from the proposed model against the previous models as follows:

a) The proposed model provides a new method with respect to a Normal Form of analysis according to the Decision Theoretic Approach, and it shows how to determine the investigating regions S_2 depending on the variance ratio σ_2^2/σ_1^2 with respect to the cost variance x itself. Therefore, this proposed model simply requires the matching of a reported cost variance against a given investigating region depending on the prior probability, different from the Markovian Approach model having the same critical control limit for every period. As a result, this model has analytical merits because control actions are sought within the sample space, while the Extensive Form has a condensed meaning because it is discussed within a [0,1] probability space.

b) This study shows how the optimal set of processes to be investigated in any period can be selected in N-Cost Processes with each cost process being treated independently of the others. This differs from the model of N-Cost Processes by Dyckman(1969).

c) This study developed a Exploratory Investigation model as an extension of the Full Investigation model with respect to the Normal Form of Bayesian analysis.

d) The numerical analysis can seek the lower or upper bound of the prior probability, the critical value for determining when a Cost Process should be never or always investigated. Therefore one can partially determine whether or not to investigate according to the prior probability as shown in the numerical analysis.

e) The results simulated in the numerical analysis chapter are summarized as follows:
The value of h that gives minimum total costs will almost exist when $C'=Ch$ and will always exist when $C'=Ch^2$, in the range of the Exploratory Investigation. Furthermore, when C increases, the value of h, which gives minimum total costs, will decrease. Therefore, if the Full Investigation cost C is large, there is no need to spend as much money

as the Full Investigation cost C for the Exploratory Investigation to attain minimum total costs, although the total costs that will be incurred by the Exploratory Investigation may be larger than total costs by the Full Investigation. For this reason, a decision maker must determine the optimal value of h by simulating how parameters influence the average total costs.

3) <u>Limitations and Further Direction</u>

This study was set up based on the assumption that an accounting report did not change the behavior of a second party. Thus, as discussed in the Motivation section of chapter one, further investigation into the Decision-Influencing case when the accounting report changes the behavior of a second party, should be undertaken.

This study also showed that the sample space with respect to the investigating region was split into three parts under the case that $\sigma_1 \neq \sigma_2$. Thus Bayesian investigating region differs from the two-parts-investigating region of Magee's(1976) study and Dittman and Prakash's(1978) model. However, Dittman and Prakash(1979) showed that Markovian control allegedly performed almost as well as Bayesian optimal contol unless the in-control cost had at least a moderately large coefficient of intrusion and a substantially greater dispersion than the out-of-control cost. This is due to the fact that Markovian two-parts control differs from Bayesian three-parts control. Additionally, Magee(1976) found that the differences between Bayesian-optimal control and two-standard-deviations control were not terribly large. However, these were based only on the case when $\sigma_1=\sigma_2$, therefore the case when $\sigma_1\neq\sigma_2$ should be analyzed.

APPENDIX A: Properties of Likelihood Ratio, $f(x|\theta_1)/f(x|\theta_2)$

$$\lambda(x) = f(x|\theta_1)/f(x|\theta_2) = (\sigma_2/\sigma_1)\exp\{(-1/2)[((x-\mu_1)^2/\sigma_1^2) - ((x-\mu_2)^2/\sigma_2^2)]\}.$$

If we consider $\log\lambda(x)$ instead of $\lambda(x)$, then
$\log\lambda(x) = \log(\sigma_2/\sigma_1) - (1/2)[((x-\mu_1)^2/\sigma_1^2) - ((x-\mu_2)^2/\sigma_2^2)]$.
If we calculate the first derivative of $\lambda(x)$ with respect to x, then $d(\log\lambda(x))/dx = [(\sigma_2^2-\sigma_1^2)x/\sigma_1^2\sigma_2^2] - [(\mu_2\sigma_1^2-\mu_1\sigma_2^2)/\sigma_1^2\sigma_2^2]$.
When $d(\log\lambda(x))/dx=0$, then $x=\mu_1+(\mu_1-\mu_2)\sigma_1^2/(\sigma_2^2-\sigma_1^2)=\mu_2+(\mu_1-\mu_2)\sigma_2^2/(\sigma_2^2-\sigma_1^2)$, and the maximum or minimum value of $\lambda(x)$ with respect to x is $(\sigma_2/\sigma_1)\exp[(\mu_1-\mu_2)^2/2(\sigma_2^2-\sigma_1^2)]$. However, if $\sigma_i^2=\sigma^2 (i=1,2)$, then $d\lambda(x)/dx = (-(\mu_2-\mu_1)/\sigma^2)\exp[(-(\mu_2-\mu_1)(2x-(\mu_1+\mu_2))/2\sigma^2]<0$. Thus $\lambda(x)$ with respect to x is a monotone decreasing function.

If we calculate the second derivative of $\lambda(x)$ with respect to x, the operating of taking the derivative of $\lambda(x)$ twice can be shown as follows:
Let $\lambda(x)$ denote $f(x)$. Then
$\log f(x) = \log\lambda(x)$
$\Rightarrow d(\log f(x))/dx = d(\log\lambda(x))/dx$
$\Rightarrow f'(x)/f(x) = d(\log\lambda(x))/dx$
$\Rightarrow f'(x) = [d(\log\lambda(x))/dx]f(x)$
$\Rightarrow df'(x)/dx = f''(x) = [d(d(\log\lambda(x))/dx)/dx]f(x) + [d(\log\lambda(x))/dx]f'(x)$
$= [d(d(\log\lambda(x))/dx)/dx]\lambda(x) + [d(\log\lambda(x))/dx]^2\lambda(x)$
$= \lambda(x)\{[-(\sigma_2^2-\sigma_1^2)/\sigma_1^2\sigma_2^2] + [(-(\sigma_2^2-\sigma_1^2)x/\sigma_1^2\sigma_2^2) - ((\mu_2\sigma_1^2-\mu_1\sigma_2^2)/\sigma_1^2\sigma_2^2)]^2\}$.

If $\sigma_2^2>\sigma_1^2$, then $f''(\mu_1+\sigma_1^2(\mu_1-\mu_2)/(\sigma_2^2-\sigma_1^2))$
$= \lambda(\mu_1+(\sigma_1^2(\mu_1-\mu_2)/(\sigma_2^2-\sigma_1^2)))[-(\sigma_2^2-\sigma_1^2)/\sigma_1^2\sigma_2^2]<0$.

If $\sigma_2^2<\sigma_1^2$, then $f''(\mu_2+\sigma_2^2(\mu_1-\mu_2)/(\sigma_2^2-\sigma_1^2))$
$= \lambda(\mu_2+(\sigma_2^2(\mu_1-\mu_2)/(\sigma_2^2-\sigma_1^2)))[-(\sigma_2^2-\sigma_1^2)/\sigma_1^2\sigma_2^2]>0$.

Thus, if $\sigma_1^2 > \sigma_2^2$, then $\lambda(x)$ with respect to x is a convex function that has a minimum value $(\sigma_2/\sigma_1)\exp[-(\mu_1-\mu_2)^2/2(\sigma_1^2-\sigma_2^2)]$ when $x = \mu_2 + |(\mu_1-\mu_2)\sigma_2^2/(\sigma_2^2-\sigma_1^2)|$.

If $\sigma_1^2 < \sigma_2^2$, then $\lambda(x)$ with respect to x is a concave function that has a maximum value $(\sigma_2/\sigma_1)\exp[(\mu_1-\mu_2)^2/2(\sigma_2^2-\sigma_1^2)]$ when $x = \mu_1 - |(\mu_1-\mu_2)\sigma_2^2/(\sigma_2^2-\sigma_1^2)|$.

APPENDIX B: Developing the Objective Function in N Cost Processes of the Full Investigation Case

$$\text{Min } \sum E[\ell(\delta(x_j,y_j);\theta_j)(1-y_j)]$$

$$= \text{Min } \sum\{E[\ell(\delta(x_j,y_j);\theta_j)I_{s_{j2}}(x_j)(1-y_j)] + E[\ell(\delta(x_j,y_j);\theta_j)I_{s_{j1}}(x_j)(1-y_j)]\}$$

$$= \text{Min}\{ \sum E[\ell(\delta(x_j,y_j);\theta_j)I_{s_{j2}}(x_j)(1-y_j)] + \sum E[\ell(\delta(x_j,y_j);\theta_j)I_{s_{j1}}(x_j)(1-y_j)]\}$$

$$= \text{Min } \{\sum E[\ell(\delta(x_j,y_j);\theta_j)I_{s_{j2}}(x_j)(1-y_j)] + \sum E[\ell(\delta(x_j);\theta_j)I_{s_{j1}}(x_j)]\}$$

the j that satisfies the above equation is equivalent to the following equation:

$$\text{Min } \sum_{j\varepsilon\rho} E[\ell(\delta(x_j,y_j);\theta_j)I_{s_{j2}}(x_j)(1-y_j)]$$

$$\begin{aligned}= \text{Min} \sum_{j\varepsilon\rho}\{&\ell(\delta(x_j,y_j);\theta_{j1})I_{\{0\}}(y_j)(1-y_j)P[x_j\varepsilon S_{j2}|\theta_{j1}]P[\theta_{j1}]+\\ &\ell(\delta(x_j,y_j);\theta_{j1})I_{\{1\}}(y_j)(1-y_j)P[x_j\varepsilon S_{j2}|\theta_{j1}]P[\theta_{j1}]+\\ &\ell(\delta(x_j,y_j);\theta_{j2})I_{\{0\}}(y_j)(1-y_j)P[x_j\varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]+\\ &\ell(\delta(x_j,y_j);\theta_{j2})I_{\{1\}}(y_j)(1-y_j)P[x_j\varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]\}\end{aligned}$$

$$\begin{aligned}= \text{Min} \sum_{j\varepsilon\rho}\{&\ell(a_1;\theta_{j1})I_{\{0\}}(y_j)(1-y_j)P[x_j\varepsilon S_{j2}|\theta_{j1}]P[\theta_{j1}]+\\ &\ell(a_2;\theta_{j1})I_{\{1\}}(y_j)(1-y_j)P[x_j\varepsilon S_{j2}|\theta_{j1}]P[\theta_{j1}]+\\ &\ell(a_1;\theta_{j2})I_{\{0\}}(y_j)(1-y_j)P[x_j\varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]+\\ &\ell(a_2;\theta_{j2})I_{\{1\}}(y_j)(1-y_j)P[x_j\varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]\}\end{aligned}$$

$$= \text{Min} \sum_{j\varepsilon\rho}\ell(a_1;\theta_{j2})I_{\{0\}}(y_j)(1-y_j)P[x_j\varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]$$

$$= \text{Min} \sum_{j\varepsilon\rho}\ell(a_1;\theta_{j2})(1-y_j)P[x_j\varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]$$

$$= \text{Max} \sum_{j\varepsilon\rho}\ell(a_1;\theta_{j2})P[x_j\varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]y_j$$

APPENDIX C: Developing the Generalized Posterior Probability $\pi(\theta_1 | x_1, \ldots, x_n)$

For every natural number n,
posterior probability, $\pi(\theta_1 | x_1, \ldots, x_n)$

$$= [1 + [f(x_n|\theta_2)(1-g)/f(x_n|\theta_1)g] + [f(x_n|\theta_2)f(x_{n-1}|\theta_2)(1-g) / f(x_n|\theta_1)f(x_{n-1}|\theta_1)g^2] + \ldots + [f(x_n|\theta_2)\ldots f(x_2|\theta_2)(1-g) / f(x_n|\theta_1)\ldots f(x_2|\theta_1)g^{n-1}] + [f(x_n|\theta_2)\ldots f(x_1|\theta_2)(1-\pi g) / f(x_n|\theta_1)\ldots f(x_1|\theta_1)\pi g^n]]^{-1}.$$

[Proof]

For n=1,

$$\pi(\theta_1|x_1) = f(x_1|\theta_1)\pi g / [f(x_1|\theta_1)\pi g + f(x_1|\theta_2)(1-\pi g)] \text{ (by Bayes formula)}$$
$$= [1 + f(x_1|\theta_2)(1-\pi g)/f(x_1|\theta_1)\pi g\ ]^{-1}$$

For n=2,

$$\pi(\theta_1|x_1,x_2) = f(x_2|\theta_1)\pi(\theta_1|x_1)g / [f(x_2|\theta_1)\pi(\theta_1|x_1)g + f(x_2|\theta_2)(1-\pi(\theta_1|x_1)g)] \text{ (by the Bayes formula)}$$
$$= [1 + f(x_2|\theta_2)(1-\pi(\theta_1|x_1)g)/f(x_2|\theta_1)\pi(\theta_1|x_1)g]^{-1}$$
$$= [1 + f(x_2|\theta_2)(1-g)/f(x_2|\theta_1)g + f(x_2|\theta_2)f(x_1|\theta_2)(1-\pi g)/f(x_2|\theta_1)f(x_1|\theta_1)\pi g^2]^{-1}$$

For n=k, we assume that $\pi(\theta_1|x_1, \ldots, x_k)$

$$= f(x_k|\theta_1)\pi(\theta_1|x_1, \ldots, x_{k-1})g / [f(x_k|\theta_1)\pi(\theta_1|x_1, \ldots, x_{k-1})g + f(x_k|\theta_2)(1-\pi(\theta_1|x_1, \ldots, x_{k-1})g)] \text{ (by Bayes formula)}$$

$$= [1 + [f(x_k|\theta_2)(1-g)/f(x_k|\theta_1)g] + [f(x_k|\theta_2)f(x_{k-1}|\theta_2)(1-g) / f(x_k|\theta_1)f(x_{k-1}|\theta_1)g^2] + \ldots + [f(x_k|\theta_2)\ldots f(x_2|\theta_2)(1-g) / f(x_k|\theta_1)\ldots f(x_2|\theta_1)g^{k-1}] + [f(x_k|\theta_2)\ldots f(x_1|\theta_2)(1-\pi g) / f(x_k|\theta_1)\ldots f(x_1|\theta_1)\pi g^k]]^{-1}.$$

Then for n=k+1, $\pi(\theta_1|x_1,...,x_{k+1})$

$$= f(x_{k+1}|\theta_1)\pi(\theta_1|x_1,...,x_k)g/[f(x_{k+1}|\theta_1)\pi(\theta_1|x_1,...,x_k)g+ f(x_{k+1}|\theta_2)(1-\pi(\theta_1|x_1,...,x_k)g)]$$ (by Bayes formula)

$$=[1+f(x_{k+1}|\theta_2)(1-\pi(\theta_1|x_1,...,x_k)g)/f(x_{k+1}|\theta_1)\pi(\theta_1|x_1,...,x_k)g]^{-1}$$

$$=[1-f(x_{k+1}|\theta_2)/f(x_{k+1}|\theta_1)+f(x_{k+1}|\theta_2)/f(x_{k+1}|\theta_1)\pi(\theta_1|x_1,...,x_k)g]^{-1}$$

$$=[1-f(x_{k+1}|\theta_2)/f(x_{k+1}|\theta_1)+f(x_{k+1}|\theta_2)/f(x_{k+1}|\theta_1)g* \{1+[f(x_k|\theta_2)(1-g)/f(x_k|\theta_1)g]+[f(x_k|\theta_2)f(x_{k-1}|\theta_2)(1-g) /f(x_k|\theta_1)f(x_{k-1}|\theta_1)g^2]+...+[f(x_k|\theta_2)...f(x_2|\theta_2)(1-g) /f(x_k|\theta_1)...f(x_2|\theta_1)g^{k-1}]+[f(x_k|\theta_2)...f(x_1|\theta_2)(1-\pi g) /f(x_k|\theta_1)...f(x_1|\theta_1)\pi g^k\}]^{-1}.$$

$$= [1+[f(x_{k+1}|\theta_2)(1-g)/f(x_{k+1}|\theta_1)g]+[f(x_{k+1}|\theta_2)f(x_k|\theta_2)(1-g) /f(x_{k+1}|\theta_1)f(x_k|\theta_1)g^2]+...+[f(x_{k+1}|\theta_2)...f(x_2|\theta_2)(1-g) /f(x_{k+1}|\theta_1)...f(x_2|\theta_1)g^k]+[f(x_{k+1}|\theta_2)...f(x_1|\theta_2)(1-\pi g) /f(x_{k+1}|\theta_1)...f(x_1|\theta_1)\pi g^{k+1}]]^{-1}.$$

We show that $\pi(\theta_1|x_1,...,x_{k+1})$ is valid for n=k+1 if it is valid for n=k. Thus by mathematical induction, the generalized posterior probability, $\pi(\theta_1|x_1,...,x_n)$ is true for all natural numbers n.

APPENDIX D: Developing the Objective Function in N Cost Processes of the Exploratory Investigation Case

$$\mathrm{Min}\ \sum E[\ell(\delta(x_j,y_j);\theta_j)(1-y_j)]$$

$$= \mathrm{Min}\ \sum\{E[\ell(\delta(x_j,y_j);\theta_j)I_{s_{j2}}(x_j)(1-y_j)] + E[\ell(\delta(x_j,y_j);\theta_j)I_{s_{j1}}(x_j)(1-y_j)]\}$$

$$= \mathrm{Min}\{\ \sum E[\ell(\delta(x_j,y_j);\theta_j)I_{s_{j2}}(x_j)(1-y_j)] + \sum E[\ell(\delta(x_j,y_j);\theta_j)I_{s_{j1}}(x_j)(1-y_j)]\}$$

$$= \mathrm{Min}\ \{\sum E[\ell(\delta(x_j,y_j);\theta_j)I_{s_{j2}}(x_j)(1-y_j)] + \sum E[\ell(\delta(x_j);\theta_j)I_{s_{j1}}(x_j)]\}$$

the j that satisfies the above equation is equivalent to the following equation:

$$\mathrm{Min}\ \sum_{j\varepsilon\rho} E[\ell(\delta(x_j,y_j);\theta_j)I_{s_{j2}}(x_j)(1-y_j)]$$

$$\begin{aligned}= \mathrm{Min}\sum_{j\varepsilon\rho}\{&\ell(\delta(x_j,y_j);\theta_{j1})I_{\{0\}}(y_j)(1-y_j)P[x_j\varepsilon S_{j2}|\theta_{j1}]P[\theta_{j1}]+\\ &\ell(\delta(x_j,y_j);\theta_{j1})I_{\{1\}}(y_j)(1-y_j)P[x_j\varepsilon S_{j2}|\theta_{j1}]P[\theta_{j1}]+\\ &\ell(\delta(x_j,y_j);w_1|\theta_{j2})I_{\{0\}}(y_j)(1-y_j)*\\ &\qquad P[w_1|\theta_{j2}]P[x_j\varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]+\\ &\ell(\delta(x_j,y_j);w_1|\theta_{j2})I_{\{1\}}(y_j)(1-y_j)*\\ &\qquad P[w_1|\theta_{j2}]P[x_j\varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]+\\ &\ell(\delta(x_j,y_j);w_2|\theta_{j2})I_{\{0\}}(y_j)(1-y_j)*\\ &\qquad P[w_2|\theta_{j2}]P[x_j\varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]+\\ &\ell(\delta(x_j,y_j);w_2|\theta_{j2})I_{\{1\}}(y_j)(1-y_j)*\\ &\qquad P[w_2|\theta_{j2}]P[x_j\varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]\}\end{aligned}$$

$$= \quad Min_{j}\sum_{\varepsilon\rho}\{\ell(a_1;\theta_{j1})I_{\{0\}}(y_j)(1-y_j)P[x_j \varepsilon S_{j2}|\theta_{j1}]P[\theta_{j1}]+$$
$$\ell(a_2;\theta_{j1})I_{\{1\}}(y_j)(1-y_j)P[x_j \varepsilon S_{j2}|\theta_{j1}]P[\theta_{j1}]+$$
$$\ell(a_1;w_1|\theta_{j2})I_{\{0\}}(y_j)(1-y_j)P[w_1|\theta_{j2}]P[x_j \varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]+$$
$$\ell(a_2;w_1|\theta_{j2})I_{\{1\}}(y_j)(1-y_j)P[w_1|\theta_{j2}]P[x_j \varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]+$$
$$\ell(a_1;w_2|\theta_{j2})I_{\{0\}}(y_j)(1-y_j)P[w_2|\theta_{j2}]P[x_j \varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]+$$
$$\ell(a_2;w_2|\theta_{j2})I_{\{1\}}(y_j)(1-y_j)P[w_2|\theta_{j2}]P[x_j \varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]\}$$

$$= \quad Min_{j}\sum_{\varepsilon\rho}\{\ell(a_1;w_1|\theta_{j2})I_{\{0\}}(y_j)(1-y_j)P[w_1|\theta_{j2}]P[x_j \varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]+$$
$$\ell(a_1;w_2|\theta_{j2})I_{\{0\}}(y_j)(1-y_j)P[w_2|\theta_{j2}]P[x_j \varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]\}$$

$$= \quad Min_{j}\sum_{\varepsilon\rho}\{\ell(a_1;w_1|\theta_{j2})(1-y_j)P[w_1|\theta_{j2}]P[x_j \varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]+$$
$$\ell(a_1;w_2|\theta_{j2})(1-y_j)P[w_2|\theta_{j2}]P[x_j \varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]\}$$

$$= \quad Max_{j}\sum_{\varepsilon\rho}\{\ell(a_1;w_1|\theta_{j2})P[w_1|\theta_{j2}]P[x_j \varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]+$$
$$\ell(a_1;w_2|\theta_{j2})P[w_2|\theta_{j2}]P[x_j \varepsilon S_{j2}|\theta_{j2}]P[\theta_{j2}]\}y_j$$

APPENDIX E: Developing the Generalized Prior Probability in the Case of Exploratory Investigation

For every natural number n,

prior probability in the beginning of period n+1, $\pi'(\theta_1|x_1,...,x_n)$

$$\pi'(\theta_1|x_1,...,x_n)= \pi(\theta_1|x_1,...,x_n)/[\pi(\theta_1|x_1,...,x_n)+\pi(\theta_2|x_1,...,x_n)(1-h)]$$

$$= [1+[f(x_n|\theta_2)(1-g)(1-h)/f(x_n|\theta_1)g]+[f(x_n|\theta_2)f(x_{n-1}|\theta_2)(1-g)(1-h)^2 /f(x_n|\theta_1)f(x_{n-1}|\theta_1)g^2]+...+[f(x_n|\theta_2)...f(x_2|\theta_2)(1-g)(1-h)^{n-1} /f(x_n|\theta_1)...f(x_2|\theta_1)g^{n-1}]+[f(x_n|\theta_2)...f(x_1|\theta_2)(1-\pi g)(1-h)^n /f(x_n|\theta_1)...f(x_1|\theta_1)\pi g^n]]^{-1}.$$

[Proof]

For n=1, $\pi'(\theta_1|x_1)=\pi(\theta_1|x_1)/[\pi(\theta_1|x_1)+\pi(\theta_2|x_1)(1-h)]$

$$=f(x_1|\theta_1)\pi g/[f(x_1|\theta_1)\pi g +f(x_1|\theta_2)(1-\pi g)(1-h)]$$

(by Bayes formula)

$$=[1+f(x_1|\theta_2)(1-\pi g)(1-h)/f(x_1|\theta_1)\pi g]^{-1}.$$

For n=2, $\pi'(\theta_1|x_1,x_2)=\pi(\theta_1|x_1,x_2)/[\pi(\theta_1|x_1,x_2)+\pi(\theta_2|x_1,x_2)(1-h)]$

$$=f(x_2|\theta_1)\pi'(\theta_1|x_1)g/[f(x_2|\theta_1)\pi'(\theta_1|x_1)g+ f(x_2|\theta_2)(1-\pi'(\theta_1|x_1)g)(1-h)]$$ (by the Bayes formula)

$$=[1+f(x_2|\theta_2)(1-\pi'(\theta_1|x_1)g)(1-h)/f(x_2|\theta_1)\pi'(\theta_1|x_1)g]^{-1}$$

$$=[1+f(x_2|\theta_2)(1-g)(1-h)/f(x_2|\theta_1)g+ f(x_2|\theta_2)f(x_1|\theta_2)(1-\pi g)(1-h)^2/f(x_2|\theta_1)f(x_1|\theta_1)\pi g^2]^{-1}$$

For n=k, we assume that $\pi'(\theta_1|x_1,...,x_k)$

$$=\pi(\theta_1|x_1,...,x_k)/[\pi(\theta_1|x_1,...,x_k)+\pi(\theta_2|x_1,...,x_k)(1-h)]$$

$$= [1+[f(x_k|\theta_2)(1-g)(1-h)/f(x_k|\theta_1)g]+[f(x_k|\theta_2)f(x_{k-1}|\theta_2)(1-g)(1-h)^2 /f(x_k|\theta_1)f(x_{k-1}|\theta_1)g^2]+...+[f(x_k|\theta_2)...f(x_2|\theta_2)(1-g)(1-h)^{k-1} /f(x_k|\theta_1)...f(x_2|\theta_1)g^{k-1}]+[f(x_k|\theta_2)...f(x_1|\theta_2)(1-\pi g)(1-h)^k /f(x_k|\theta_1)...f(x_1|\theta_1)\pi g^k]]^{-1}.$$

Then for n=k+1, $\pi'(\theta_1|x_1,...,x_{k+1})$

$= \pi(\theta_1|x_1,...,x_{k+1})/[\pi(\theta_1|x_1,...,x_{k+1})+\pi(\theta_2|x_1,...,x_{k+1})(1-h)]$

$= f(x_{k+1}|\theta_1)\pi'(\theta_1|x_1,...,x_k)g/[f(x_{k+1}|\theta_1)\pi'(\theta_1|x_1,...,x_k)g+ f(x_{k+1}|\theta_2)(1-\pi'(\theta_1|x_1,...,x_k)g)(1-h)]$ (by Bayes formula)

$=[1+f(x_{k+1}|\theta_2)(1-\pi'(\theta_1|x_1,...,x_k)g)(1-h) /f(x_{k+1}|\theta_1)\pi'(\theta_1|x_1,...,x_k)g]^{-1}$

$=[1-f(x_{k+1}|\theta_2)(1-h)/f(x_{k+1}|\theta_1)+f(x_{k+1}|\theta_2)(1-h)/f(x_{k+1}|\theta_1)g* \{1+[f(x_k|\theta_2)(1-g)(1-h)/f(x_k|\theta_1)g]+[f(x_k|\theta_2)f(x_{k-1}|\theta_2)(1-g)(1-h)^2 /f(x_k|\theta_1)f(x_{k-1}|\theta_1)g^2]+...+[f(x_k|\theta_2)...f(x_2|\theta_2)(1-g)(1-h)^{k-1} /f(x_k|\theta_1)...f(x_2|\theta_1)g^{k-1}]+[f(x_k|\theta_2)...f(x_1|\theta_2)(1-\pi g)(1-h)^k /f(x_k|\theta_1)...f(x_1|\theta_1)\pi g^k\}]^{-1}.$

$= [1+[f(x_{k+1}|\theta_2)(1-g)(1-h)/f(x_{k+1}|\theta_1)g]+[f(x_{k+1}|\theta_2)f(x_k|\theta_2)(1-g) (1-h)^2/f(x_{k+1}|\theta_1)f(x_k|\theta_1)g^2]+...+[f(x_{k+1}|\theta_2)...f(x_2|\theta_2)(1-g) (1-h)^k/f(x_{k+1}|\theta_1)...f(x_2|\theta_1)g^k]+[f(x_{k+1}|\theta_2)...f(x_1|\theta_2)(1-\pi g) (1-h)^{k+1}/f(x_{k+1}|\theta_1)...f(x_1|\theta_1)\pi g^{k+1}]]^{-1}.$

We show that $\pi'(\theta_1|x_1,...,x_{k+1})$ is valid for n=k+1 if it is valid for n=k. Thus by mathematical induction, the generalized posterior probability, $\pi'(\theta_1|x_1,...,x_n)$ is true for all natural numbers n.

APPENDIX F: Computer Program for the Property of Likelihood-Ratio λ(x)

```
Program NumericalAnalysisI;
   {$u+}
   {This is a program to explain a property of
    likelihood function, f(x|in control)/f(x|out of control)}
const
  u2=20;
  s1max=30;
  xmax=50;
  s2max=30;
var
  s1,s2,x:integer;
  y:real;
begin
  writeln(lst,'s1':15,'s2':15,'x':15,'y':30);
  writeln;
  s1:=5;
  while s1<=s1max do
  begin
    s2:=5;
    while s2<=s2max do
      begin
        x:=-30;
        while x<=xmax do
        begin
          y:=(s2/s1)*exp((-1/2)*(sqr(x/s1)-sqr((x-u2)/s2)));
          writeln(lst,s1:15,s2:15,x:15,y:30:3);
          x:=x+5;
        end;
        s2:=s2+5;
      end;
      s1:=s1+5;
  end
end.
```

APPENDIX G: Computer Program for the Lower or Upper Bounds of the Prior Probability in the Full Investigation Case

```
Program NumericalAnalysisII;
    {$u+}
    {This is a program to explain an upper or lower bound
     of the prior probability in the Full Investigation case}
const
  u2=20;
  a=0.98;
  gmax=9;
  cmax=60;
  s1max=30;
  s2max=30;
var
  g,c,s1,s2:integer;
  k,t,p,g1,g2,y:real;
  function cost(gx:real;cx:integer):real;
    begin
      cost:=(u2/(cx*(1-a*gx)))-1
    end;
begin
  writeln;
  writeln;
  writeln(lst,'c':15,'g':15,'s1':15,'s2':15,'p':15);
  writeln;
  c:=10;
  while c<=cmax do
  begin
    g:=5;
    while g<=gmax do
    begin
      s1:=5;
      while s1<=s1max do
      begin
        s2:=5;
        while s2<=s2max do
        begin
          g1:=g;
          g2:=g1/10;
          k:=cost(g2,c);
          if k>0 then
          begin
            if s1<>s2 then
            if S1<s2 then
            begin
              p:=k/(g2*(k+(s2/s1)*exp(sqr(u2)/(2*(sqr(s2)-sqr(s1))))));
              writeln(lst,c:15,g2:15:2,s1:15,s2:15,p:15:4);
            end
            else
            begin
              p:=k/(g2*(k+(s2/s1)*exp(-sqr(u2)/(2*(sqr(s1)-sqr(s2))))));
              writeln(lst,c:15,g2:15:2,s1:15,s2:15,p:15:4);
            end;
          end;
          if k<=0 then
          begin
            writeln(lst,c:15,g2:15:2,s1:15,s2:15,'B<0(never investigate)'
          end;
```

```
          s2:=s2+5;
        end;
        s1:=s1+5;
      end;
      g:=g+2;
    end;
    c:=c+10;
  end
end.
```

'ENDIX H: Computer Program for the Lower or Upper Bounds of the Prior Probability in the Exploratory Investigation Case When C'=Ch²

```
ogram NumericalAnalysisIII;
 {$u+}
 {This is a program to explain an upper or lower bounds
  of the prior probability in the Exploratory Investigation
  case when C'=Csqr(h)}
nst
u2=20;
a=0.98;
gmax=9;
cmax=60;
hmax=9;
s1max=30;
s2max=30;
r
h,g,c,s1,s2:integer;
k,t,p,g1,g2,h1,h2,c1,c2:real;
function cost(gx,hx,cx:real):real;
  begin
    cost:=((u2/(1-a*gx))-cx*hx)/(cx*hx)
  end;
gin
writeln;
writeln(lst,'c':15,'c!':15,'g':15,'h':15,'s1':15,'s2':15,'p':15);
writeln;
c:=10;
while c<=cmax do
begin
  g:=5;
  while g<=gmax do
  begin
    h:=5;
    while h<=hmax do
    begin
      s1:=5;
      while s1<=s1max do
      begin
        s2:=5;
        while s2<=s2max do
        begin
          g1:=g;
          g2:=g1/10;
          h1:=h;
          h2:=h1/10;
          c1:=c;
          c2:=c1*sqr(h2);
          k:=cost(g2,h2,c1);
          if k>0 then
          begin
            if s1<>s2 then
            if S1<s2 then
            begin
              p:=k/(g2*(k+(s2/s1)*exp(sqr(u2)/(2*(sqr(s2)-sqr(s1))))));
              writeln(lst,c:15,c2:15:2,g2:15:2,h2:15:2,s1:15,s2:15,p:15:4)
            end
            else
            begin
```

```
            p:=k/(g2*(k+(s2/s1)*exp(-sqr(u2)/(2*(sqr(s1)-sqr(s2))))));
            writeln(lst,c:15,c2:15:2,g2:15:2,h2:15:2,s1:15,s2:15,p:15:4);
          end;
        end;
        if k<=0 then
        begin
          writeln(lst,c:15,c2:15:2,g2:15:2,h2:15:2,s1:15,s2:15,
                  'B!<0(never investigate)':30);
        end;
        s2:=s2+5;
      end;
      s1:=s1+5;
    end;
    h:=h+2;
  end;
  g:=g+2;
 end;
 c:=c+10;
nd
.
```

APPENDIX I: Computer Program for Seeking an Investigating Region S_2 in the Full Investigation Case

```
Program NumericalAnalysisIV;
   {$u+}
   {This is a program to explain how to seek an investigating
   region in the Full Investigation case}
const
  u2=20;
  a=0.98;
  gmax=9;
  cmax=60;
  s1max=30;
  s2max=30;
  pmax=10;
var
  g,c,s1,s2,p:integer;
  k,t,p1,p2,g1,g2,x,x1,x2,dis:real;
  function cost(gx:real;cx:integer):real;
    begin
      cost:=(u2/(cx*(1-a*gx)))-1
    end;
begin
  writeln;
  writeln(lst,'c':15,'g':15,'s1':15,'s2':15,
          'p':15,'x1':15,'x2':15);
  writeln;
  c:=10;
  while c<=cmax do
  begin
    g:=5;
    while g<=gmax do
    begin
      s1:=5;
      while s1<=s1max do
      begin
        s2:=5;
        while s2<=s2max do
        begin
          p:=2;
          while p<=pmax do
          begin
            g1:=g;
            g2:=g1/10;
            p1:=p;
            p2:=p1/10;
            k:=cost(g2,c);
            if k>0 then
            begin
              if s1=s2 then
              begin
                x:=(u2/2)-(sqr(s1)/u2)*ln((1-p2*g2)*k/(p2*g2));
                writeln(lst,c:15,g2:15:2,s1:15,s2:15,p2:15:2,x:15:4);
              end
              else
              begin
                if s1<s2 then
                begin
                  dis:=sqr(u2*sqr(s1))-(sqr(s2)-sqr(s1))*(2*sqr(s1*s2)*
```

```
                    ln((s1*(1-p2*g2)*k)/(s2*p2*g2))-sqr(s1)*sqr(u2));
               if dis>0.0 then
               begin
                 x1:=(-u2*sqr(s1)-sqrt(dis))/(sqr(s2)-sqr(s1));
                 x2:=(-u2*sqr(s1)+sqrt(dis))/(sqr(s2)-sqr(s1));
                 writeln(lst,c:15,g2:15:2,s1:15,s2:15,p2:15:2,
                         x1:15:4,x2:15:4);
               end;
               if dis=0.0 then
               begin
                 x:=(-u2*sqr(s1))/(sqr(s2)-sqr(s1));
                 writeln(lst,c:15,g2:15:2,s1:15,s2:15,p2:15:2,x:15:3);
               end;
               if dis<0.0 then
               begin
                 writeln(lst,c:15,g2:15:2,s1:15,s2:15,p2:15:2,
                         'always investigate':20);
               end
             end;
             if s1>s2 then
             begin
               dis:=sqr(u2*sqr(s1))-(sqr(s1)-sqr(s2))*((sqr(s1)*sqr(u2))-
                    2*sqr(s1*s2)*ln((s1*(1-p2*g2)*k)/(s2*p2*g2)));
               if dis>0.0 then
               begin
                 x1:=(u2*sqr(s1)-sqrt(dis))/(sqr(s1)-sqr(s2));
                 x2:=(u2*sqr(s1)+sqrt(dis))/(sqr(s1)-sqr(s2));
                 writeln(lst,c:15,g2:15:2,s1:15,s2:15,p2:15:2,
                         x1:15:4,x2:15:4);
               end;
               if dis=0.0 then
               begin
                 x:=(u2*sqr(s1))/(sqr(s1)-sqr(s2));
                 writeln(lst,c:15,g2:15:2,s1:15,s2:15,p2:15:2,x:15:3);
               end;
               if dis<0.0 then
               begin
                 writeln(lst,c:15,g2:15:2,s1:15,s2:15,p2:15:2,
                         'never investigate':20);
               end
             end
           end;
         end;
         if k<=0 then
         begin
           writeln(lst,c:15,g2:15:2,s1:15,s2:15,p2:15:2,'no solution':15);
         end;
         p:=p+2;
       end;
       s2:=s2+5;
     end;
     s1:=s1+5;
   end;
   g:=g+2;
 end;
 c:=c+10;
```

```
  end
end.
```

APPENDIX J: Computer Program for Seeking an Investigating Region S_2 in the Exploratory Investigation Case When $C'=Ch^2$

```
Program NumericalAnalysisV;
   {$u+}
   {This is a program to explain how to an investigation
    region when C'=Csqr(h) in the Exploratory Investigation case}
const
  u2=20;
  a=0.98;
  gmax=9;
  cmax=60;
  hmax=9;
  s1max=30;
  s2max=30;
  pmax=8;
var
  h,g,c,s1,s2,p:integer;
  k,t,p1,p2,g1,g2,h1,h2,c1,c2,x,x1,x2,dis:real;
  function cost(gx,hx,cx:real):real;
    begin
      cost:=((u2/(1-a*gx))*hx-cx)/cx
    end;
begin
  writeln;
  writeln(lst,'c':15,'c!':15,'g':15,'h':15,'s1':15,
          's2':15,'p':15,'x1':15,'x2':15);
  writeln;
  c:=10;
  while c<=cmax do
  begin
    g:=5;
    while g<=gmax do
    begin
      h:=5;
      while h<=hmax do
      begin
        s1:=5;
        while s1<=s1max do
        begin
          s2:=5;
          while s2<=s2max do
          begin
            p:=2;
            while p<=pmax do
            begin
              g1:=g;
              g2:=g1/10;
              p1:=p;
              p2:=p1/10;
              h1:=h;
              h2:=h1/10;
              c1:=c;
              c2:=c1*sqr(h2);
              k:=cost(g2,h2,c2);
              if s1=s2 then
              begin
                x:=(u2/2)-(sqr(s1)/u2)*ln((1-p2*g2)*k/(p2*g2));
                writeln(lst,c:15,c2:15:2,g2:15:2,h2:15:2,s1:15,
```

```
              s2:15,p2:15:2,x:15:4);
      end
      else
      begin
        if s1<s2 then
        begin
          dis:=sqr(u2*sqr(s1))-(sqr(s2)-sqr(s1))*(2*sqr(s1*s2)*
               ln((s1*(1-p2*g2)*k)/(s2*p2*g2))-sqr(s1)*sqr(u2));
          if dis>0.0 then
          begin
            x1:=(-u2*sqr(s1)-sqrt(dis))/(sqr(s2)-sqr(s1));
            x2:=(-u2*sqr(s1)+sqrt(dis))/(sqr(s2)-sqr(s1));
            writeln(lst,c:15,c2:15:2,g2:15:2,h2:15:2,s1:15,
                    s2:15,p2:15:2,x1:15:4,x2:15:4);
          end;
          if dis=0.0 then
          begin
            x:=(-u2*sqr(s1))/(sqr(s2)-sqr(s1));
            writeln(lst,c:15,c2:15:2,g2:15:2,h2:15:2,s1:15,
                    s2:15,p2:15:2,x:15:3);
          end;
          if dis<0.0 then
          begin
          writeln(lst,c:15,c2:15:2,g2:15:2,h2:15:2,s1:15,s2:15,
          p2:15:2,'always investigate':20);
          end
        end;
        if s1>s2 then
        begin
          dis:=sqr(u2*sqr(s1))-(sqr(s1)-sqr(s2))*((sqr(s1)*sqr(u2))-
               2*sqr(s1*s2)*ln((s1*(1-p2*g2)*k)/(s2*p2*g2)));
          if dis>0.0 then
          begin
            x1:=(u2*sqr(s1)-sqrt(dis))/(sqr(s1)-sqr(s2));
            x2:=(u2*sqr(s1)+sqrt(dis))/(sqr(s1)-sqr(s2));
            writeln(lst,c:15,c2:15:2,g2:15:2,h2:15:2,s1:15,s2:15,
            p2:15:2,x1:15:4,x2:15:4);
          end;
          if dis=0.0 then
          begin
            x:=(u2*sqr(s1))/(sqr(s1)-sqr(s2));
            writeln(lst,c:15,c2:15:2,g2:15:2,h2:1:2,s1:15,s2:15,
            p2:15:2,x:15:3);
          end;
          if dis<0.0 then
          begin
            writeln(lst,c:15,c2:15:2,g2:15:2,h2:15:2,s1:15,s2:15,
            p2:15:2,'never investigate':20);
          end
        end
      end;
      p:=p+3;
    end;
    s2:=s2+5;
  end;
  s1:=s1+5;
```

```
        end;
        h:=h+2;
      end;
      g:=g+2;
    end;
    c:=c+10;
  end
end.
```

APPENDIX K: Computer Program for Simulating the Relation between the Full and Exploratory Investigation Cases

```
Program NumericalAnalysisVI;
{$u+}
{This is a program to simulate the relation between
 the Full and Exploratory Investigation cases}
Const
  U2=20;
  g=0.7;
  hMax=10;
  CMax=50;
  IMin=1;
  S1max=20;
  S2max=20;
  M=0;
Var
  C,h,i,j,i1,l,s1,s2:integer;
  h1,h2,c1,c2,p,g1,k,k1,a,t,r,sum1,r1,ic,x1,x2,TC,
  f,dis,t1,t2,oc,oc1,ic1,ic2,tc1,tc2:real;
begin
  writeln(lst,'s1':10,'s2':10,'C':10,'M':10,'Partial C':10,
          'h':10,'l':10,'OC':10,'IC':10,'TC':10);
  s1:=5;
  while s1<=s1max do
  begin
    s2:=5;
    while s2<=s2max do
    begin
      C:=10;
      while C<=CMax do
      begin
        h:=3;
        while h<=hMax do
        begin
          h1:=h;
          h2:=h1/10;
          C1:=C;
          C2:=C1*h2;
          OC:=0;
          OC1:=0;
          IC1:=0;
          IC2:=0;
          TC1:=0;
          TC2:=0;
          for l:=1 to 1000 do
          begin
            p:=1;
            i:=12;
            IC:=0;
            X2:=0;
            TC:=0;
            while i>=imin do
            begin
              j:=1;
              g1:=1;
              while j<=i do
              begin
                g1:=g1*g;
```

```
  j:=j+1
end;
k1:=(U2*(1-g1))/(1-g);
k:=((k1-M)*h2-C2)/C2;
r:=random;
if r<=p*g then
begin
  sum1:=0;
  for i1:=1 to 12 do
  begin
    r1:=random;
    sum1:=r1+sum1
  end;
  x1:=s1*(sum1-6)
end
else
begin
  sum1:=0;
  for i1:=1 to 12 do
  begin
    r1:=random;
    sum1:=r1+sum1
  end;
  x1:=S2*(sum1-6)+u2
end;
if s1=s2 then
begin
  if K>0 then
  begin
    t:=(u2/2)-(sqr(s1)/u2)*ln((1-p*g)*k/(p*g));
    if x1>=t then
    begin
      a:=random;
      if a<=h2 then
      begin
        IC:=IC+C2+M;
        p:=1
      end
      else
      begin
        IC:=IC+C2;
        P:=1/(1+Exp((2*x1*u2-sqr(u2))/(2*sqr(s1))*
        ((1-p*g)*(1-h2)/(p*g)))
      end;
    end
    else
    p:=1/(1+Exp((2*x1*u2-sqr(u2))/(2*sqr(s1))*((1-p*g)/(p*g)));
  end
  else
  p:=1/(1+Exp((2*x1*u2-sqr(u2))/(2*sqr(s1))*((1-p*g)/(p*g)));
end
else
begin
  f:=(sqr(x1*s2)-(sqr(x1-u2)*sqr(s1)))/(2*sqr(s1*s2));
  if f > 80 then f:=80;
  if f < -80 then f:=-80;
```

```
if s1<s2 then
begin
  if K>0 then
  begin
    dis:=sqr(u2*sqr(s1))-(sqr(s2)-sqr(s1))*(2*sqr(s1*s2)*
         ln((s1*(1-p*g)*k)/(s2*p*g))-sqr(s1*u2));
    if dis>0 then
    begin
      t1:=(-u2*sqr(s1)-sqrt(dis))/(sqr(s2)-sqr(s1));
      t2:=(-u2*sqr(s1)+sqrt(dis))/(sqr(s2)-sqr(s1));
      if (x1<=t1) or (x1>=t2) then
      begin
        a:=random;
        if a<=h2 then
        begin
          IC:=IC+C2+M;
          p:=1
        end
        else
        begin
          IC:=IC+C2;
          p:=1/(1+(s1/s2)*Exp(f)*((1-p*g)*(1-h2)/(p*g)))
        end;
      end
      else
      p:=1/(1+(s1/s2)*Exp(f)*((1-p*g)/(p*g)));
    end
    else
    begin
      a:=random;
      if a<=h2 then
      begin
        IC:=IC+C2+M;
        p:=1
      end
      else
      begin
        IC:=IC+C2;
        p:=1/(1+(s1/s2)*((1-p*g)*(1-h2)/(p*g))*
             Exp(f))
      end;
    end;
  end
  else
  p:=1/(1+(s1/s2)*Exp(f)*((1-p*g)/(p*g)));
end;
if s1>s2 then
begin
  if K>0 then
  begin
    dis:=sqr(u2*sqr(s1))-(sqr(s1)-sqr(s2))*(sqr(u2*s1)-
         2*sqr(s1*s2)*ln((s1*(1-p*g)*k)/(s2*p*g)));
    if dis>0 then
    begin
      t1:=(u2*sqr(s1)-sqrt(dis))/(sqr(s1)-sqr(s2));
      t2:=(u2*sqr(s1)+sqrt(dis))/(sqr(s1)-sqr(s2));
```

```
                    if (x1>=t1) or (x1<=t2) then
                    begin
                      a:=random;
                      if a<=h2 then
                      begin
                        IC:=IC+C2+M;
                        p:=1
                      end
                      else
                      begin
                        IC:=IC+C2;
                        p:=1/(1+(s1/s2)*Exp(f)*((1-p*g)*(1-h2)/(p*g)))
                      end;
                    end
                    else
                    p:=1/(1+(s1/s2)*Exp(f)*((1-p*g)/(p*g)));
                  end
                  else
                  p:=1/(1+(s1/s2)*Exp(f)*((1-p*g)/(p*g)));
                end
                else
                p:=1/(1+(s1/s2)*Exp(f)*((1-p*g)/(p*g)));
              end;
            end;
            if p<=0.001 then p:=0.001;
            x2:=x1+x2;
            TC:=IC+x2;
            i:=i-1
          end;
          OC:=OC+x2;
          IC1:=IC1+IC;
          TC1:=TC1+TC;
        end;
        OC1:=OC/l;
        IC2:=IC1/l;
        TC2:=TC1/l;
        writeln(lst,S1:10,S2:10,C:10,M:10,c2:10:2,h2:10:2,
                l:10,oc1:10:2,ic2:10:2,tc2:10:2);
        h:=h+1;
      end;
      C:=C+20;
    end;
    s2:=s2+15;
  end;
  s1:=s1+15;
 end
end.
```

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