Concept of Dynamical Collective Submanifold for Large-Amplitude Collective Motion in the TDHF Theory

by

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Abstract

Concept of "optimum" collective submanifold for large-amplitude collective motion is clarified within the framework of the time-dependent Hartree-Fock (TDHF) theory. It is shown that the optimum collective submanifold is extracted out of the TDHF phase space (symplectic manifold) in such a way that the Hamiltonian on the submanifold is stationary with respect to variations perpendicular to it. The submanifold can be shown to be classified into three regions by means of both stability and separability conditions. It is displayed that these three regions can characterize collective, dissipative and stochastic motions in the TDHF theory.

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§ 1. Introduction

The nuclear dynamics is considered to be governed by interplay between two essentially different types of modes of motion, i.e., the single-particle modes of motion and the collective behavior of the nucleus as a whole. In a finite quantal system such as the nucleus, characteristic difficulties in exploring the dynamics involving such two modes of motion on the basis of the nuclear many-body problem come from the following facts;

(i) The nuclear collective motion is of large amplitude and highly non-linear so that there may be large quantum fluctuations about the Hartree-Fock (-Bogoliubov) "stable" mean field.

(ii) The single-particle states have to alter their features *self-consistently* in accordance with the evolution of the collective coordinates associated with the time-variations of the mean field.

The first step to explore such highly non-linear dynamics, requiring the self-consistency strongly, was to employ the time-dependent Hartree-Fock (TDHF) theory.[1),2)] It is well known that, from the TDHF equation, we can derive the RPA-eigenvalue equation under the small-amplitude harmonic approximation,[3),4)] and the eigenvalue equation gives us the normal collective mode of motion. By definition, the normal collective mode of motion is specified by the dynamical condition that the Hamiltonian provides no coupling between the collective and non-collective modes of motion within the small-amplitude harmonic approximation. It is also well known that the TDHF equation is simply expressed as a set of canonical equations of motion in classical mechanics in the TDHF phase space (symplectic manifold).[5)~8)] Namely, the TDHF equation determines a TDHF trajectory in the TDHF phase space under a given initial condition.

We may thus suppose that the TDHF trajectory representing well-organized large-amplitude collective motion may be realized on a small-dimensional integral surface embedded in the TDHF phase space (manifold). The approximate integral surface, called the collective surface (submanifold), has to satisfy a dynamical condition that the Hamiltonian in the collective surface has no serious coupling with the other irrelevant motion.

With the aim to specify such a global collective submanifold, the self-consistent collective coordinate (SCC) method was proposed by Marumori, et al., in 1980.[9),10)] The method provides us the collective surface (submanifold) in such a way that the total energy of the system is stationary at each point on the surface with respect to the variations perpendicular to the surface. In order to clarify that the surface obtained by the SCC method is really an approximate integral surface, as the next task, it is decisive to investigate the dynamical conditions, which are obtained by calculating the second-order derivatives of the total energy at each point on the surface with respect to the variations perpendicular to the surface.

The first objective of this thesis is to clarify the dynamical conditions. The second objective is to display some specific features of the TDHF trajectories derived by the dynamical conditions and is to elucidate new concepts associated with the specific features.

Generally speaking, the TDHF trajectory is known to be rather sensitive to small changes of the initial conditions at an instant may develop very differently, having lost their initial focussing for a large enough time.[6)] Then, one has to ask oneself following important question. How can one define the very concept of "collectivity" for large-amplitude collective motion?

According to our theory, the collectivity of large-amplitude motion is not subject to properties of a single trajectory but is inevitably related to a characteristic feature displayed by a group of trajectories. If a group of many trajectories gathering together around the surface (submanifold) at an instant can always be confined in a small region around the surface for a large enough time, we may naturally say that the system under consideration displays "large-amplitude collective motion" along a trajectory on the surface, which is a representative of the group of trajectories. Therefore, the large-amplitude collective motion has to be discussed in the connection with not only a characteristic property of the surface but also with an amount of trajectories accumulating in the group.

Thus, our task is to investigate an amount of trajectories accumulating around the surface (submanifold) obtained by SCC method. In order to know the amount of trajectories, we have to calculate a lot of trajectories travelling around the optimum surface, i.e., we have to study various properties of the TDHF phase space (manifold) in the neighborhood of it. Without directly calculating the huge-dimensional coupled differential equations of motion for getting the various trajectories, our theory enables us to investigate the above problem by using both the stability condition of the surface and the separability condition between collective and non-collective degrees of freedom.[11]

In § 2, we formulate the theory of dynamical collective submanifold within the framework of the TDHF theory. In the theory the collective mode of motion is described by the SCC method and the non-collective modes of motion are treated within the RPA-type procedure.[12] In § 3, we investigate the geometrical structure of the theory of dynamical collective submanifold by formulating the TDHF equation in the form of the canonical equations of motion in classical mechanics in the TDHF phase space.[13] In § 4, we discuss the physical meaning of both the stability condition of the collective submanifold and the separability condition between the collective and non-collective modes of motion introduced in our theory. In order to justify the statement in the previous section, in § 5, we apply our theory to a modification of the SU(3) model used by Li, Klein and Dreizler.[11),14] Section 6 will be devoted to conclusion.

§ 2. Theory of Dynamical Collective Submanifold

In this section we formulate the theory of dynamical collective submanifold within the framework of the TDHF theory.[12] In the theory the collective and non-collective modes of motion are treated independently. The former is described within the SCC method and the latter is treated within the RPA-type procedure in the neighborhood of the collective surface.

2.1. *Taylor Expansion of TDHF Equation and Canonical-Variable Representation*

As is well known, the TDHF theory with the basic equation

$$\delta\langle\phi(t)|(i\frac{\partial}{\partial t}-\hat{H})|\phi(t)\rangle = 0 \;,^{*)} \tag{2.1}$$

is an approximation theory to a many-body system obeying the Schrödinger equation

$$(i\frac{\partial}{\partial t}-\hat{H})|\Psi(t)\rangle = 0 \;, \tag{2.2}$$

under the Hartree-Fock approximation

$$|\Psi(t)\rangle \rightarrow |\phi(t)\rangle \;, \tag{2.3}$$

where $|\phi(t)\rangle$ is a single Slater determinant,

$$|\phi(t)\rangle = e^{i\hat{F}}|\phi_0\rangle \;;\; \hat{F} = \sum_{\mu i}\{f_{\mu i}(t)a_\mu^\dagger b_i^\dagger + f_{\mu i}^*(t)b_i a_\mu\} \;. \tag{2.4}$$

Here, $|\phi_0\rangle$ denotes a Hartree-Fock stationary state, and $a_\mu^\dagger$ and $b_i^\dagger$ mean the particle- and hole-creation operators with respect to $|\phi_0\rangle$,

$$a_\mu|\phi_0\rangle = 0 \;;\; \mu = 1,2,\cdots,M,$$

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*) Throughout this thesis, we adopt the convention of using $\hbar=1$.

$$b_i \mid \phi_0 \rangle = 0 \; ; \; i = 1,2,\cdots,N,^{*)} \tag{2.5}$$

M and N being numbers of the single-particle and single-hole states, respectively. Instead of $2MN$ variables $(f_{\mu i}(t), f_{\mu i}^*(t))$ in Eq. (2.4), we introduce a set of new variables

$$\eta_r, \eta_r^* \; ; \; r = 1,2,\cdots,K \ll MN,$$
$$\xi_\alpha, \xi_\alpha^* \; ; \; \alpha = 1,2,\cdots,MN-K, \tag{2.6}$$

which is related to the original variables through a general variable transformation,

$$f_{\mu i} = f_{\mu i}(\eta_r, \eta_r^*; \xi_\alpha, \xi_\alpha^*) \; , \qquad f_{\mu i}^* = f_{\mu i}^*(\eta_r, \eta_r^*; \xi_\alpha, \xi_\alpha^*) \; . \tag{2.7}$$

Here, $(\eta_r, \eta_r^*; r=1,2,\cdots,K)$ are supposed to describe the collective motion under consideration and are called collective variables, and $(\xi_\alpha, \xi_\alpha^*; \alpha=1,2,\cdots,MN-K)$ represent the rest degrees of freedom and are called non-collective variables. Such a supposition may be possible if there exists an approximate invariant subspace of the Hamiltonian, which is characterized by the variables (η, η^*) within the TDHF theory. For simplicity of discussion, hereafter we restrict ourselves to a single pair of collective variables (η, η^*), which corresponds to the simplest case with $K=1$. An extension of the theory to any finite number of pairs of parameters is, of course, straightforward. The local infinitesimal generators with respect to the new variables are defined by one-body operators

$$\hat{O}_{\rm coll}^\dagger \equiv e^{-i\hat{F}} \frac{\partial}{\partial \eta} e^{i\hat{F}} \; , \qquad \hat{O}_{\rm coll} \equiv -e^{-i\hat{F}} \frac{\partial}{\partial \eta^*} e^{i\hat{F}} \; ,$$

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*) We use the convention of denoting occupied single-particle states by indices $i, j, \cdots$, and unoccupied states by $\mu, \nu, \cdots$.

$$\hat{O}^\dagger_\alpha \equiv e^{-i\hat{F}}\frac{\partial}{\partial \xi_\alpha}e^{i\hat{F}} \ , \qquad \hat{O}_\alpha \equiv -e^{-i\hat{F}}\frac{\partial}{\partial \xi^*_\alpha}e^{i\hat{F}} \ . \tag{2.8}$$

If the non-collective modes of motion described by $(\xi_\alpha, \xi^*_\alpha)$ are of small amplitude, we may introduce a Taylor expansion of $2MN$ variables $(f_{\mu i}, f^*_{\mu i})$ with respect to the non-collective variables $(\xi_\alpha, \xi^*_\alpha)$ around a collective surface $(\eta, \eta^*; \xi_\alpha = \xi^*_\alpha = 0)$,

$$f_{\mu i} = [f_{\mu i}] + \sum_\alpha \{\xi_\alpha [\frac{\partial f_{\mu i}}{\partial \xi_\alpha}] + \xi^*_\alpha [\frac{\partial f_{\mu i}}{\partial \xi^*_\alpha}]\} + \cdots \ , \tag{2.9}$$

where the symbol $[f]$ for any function $f(\eta, \eta^*; \xi_\alpha, \xi^*_\alpha)$ denotes the value at the collective surface $(\eta, \eta^*; \xi_\alpha = \xi^*_\alpha = 0)$,

$$[f] = f(\eta, \eta^*; \xi_\alpha = \xi^*_\alpha = 0), \tag{2.10}$$

and is a function of the collective variables (η, η^*) alone. In the same way, the Taylor expansions of the $2MN$ one-body operators (2.8) with respect to the non-collective variables are written as

$$\hat{O}_{\rm coll}^\dagger = [\hat{O}_{\rm coll}^\dagger] + \sum_\alpha \{\xi_\alpha [\frac{\partial \hat{O}_{\rm coll}^\dagger}{\partial \xi_\alpha}] + \xi^*_\alpha [\frac{\partial \hat{O}_{\rm coll}^\dagger}{\partial \xi^*_\alpha}]\} + \cdots \ ,$$

$$\hat{O}^\dagger_\alpha = [\hat{O}^\dagger_\alpha] + \sum_\beta \{\xi_\beta [\frac{\partial \hat{O}^\dagger_\alpha}{\partial \xi_\beta}] + \xi^*_\beta [\frac{\partial \hat{O}^\dagger_\alpha}{\partial \xi^*_\beta}]\} + \cdots \ , \tag{2.11}$$

For later convenience, we use the following notations for the zeroth-order quantities with respect to $(\xi_\alpha, \xi^*_\alpha)$,

$$\hat{G} \equiv [\hat{F}] = \sum_{\mu i} \{ [f_{\mu i}] a^\dagger_\mu b^\dagger_i + [f^*_{\mu i}] b_i a_\mu \} \ ,$$

$$\hat{X}_{\rm coll}^\dagger \equiv [\hat{O}_{\rm coll}^\dagger] = e^{-i\hat{G}}\frac{\partial}{\partial \eta}e^{i\hat{G}} \ , \qquad \hat{X}^\dagger_\alpha \equiv [\hat{O}^\dagger_\alpha] \ ,$$

$$\hat{X}_{\rm coll} \equiv [\hat{O}_{\rm coll}] = -e^{-i\hat{G}}\frac{\partial}{\partial \eta^*}e^{i\hat{G}} \ , \qquad \hat{X}_\alpha \equiv [\hat{O}_\alpha] \ . \tag{2.12}$$

Using the above notations, we obtain an expansion form of the time-dependent Slater determinant,

$$e^{i\hat{F}}|\phi_0> = e^{i\hat{G}}\{1+\sum_\alpha(\xi_\alpha\hat{X}^\dagger_\alpha-\xi^*_\alpha\hat{X}_\alpha)+\cdots\}|\phi_0> . \tag{2.13}$$

By substituting Eq.(2.13) for Eq.(2.1) and by using the relation

$$<\phi_0|e^{-i\hat{F}}i\frac{\partial}{\partial t}e^{i\hat{F}}|\phi_0>$$

$$= <\phi_0|\{i\dot{\eta}\hat{O}_{\rm coll}^\dagger-i\dot{\eta}^*\hat{O}_{\rm coll}+i\sum_\alpha(\dot{\xi}_\alpha\hat{O}^\dagger_\alpha-\dot{\xi}^*_\alpha\hat{O}_\alpha)\}|\phi_0>$$

$$= <\phi_0|\{i\dot{\eta}\hat{X}_{\rm coll}^\dagger-i\dot{\eta}^*\hat{X}_{\rm coll}+i\dot{\eta}\sum_\alpha(\xi_\alpha[\frac{\partial\hat{O}_{\rm coll}^\dagger}{\partial\xi_\alpha}]+\xi^*_\alpha[\frac{\partial\hat{O}_{\rm coll}^\dagger}{\partial\xi^*_\alpha}])$$

$$-i\dot{\eta}^*\sum_\alpha(\xi_\alpha[\frac{\partial\hat{O}_{\rm coll}}{\partial\xi_\alpha}]+\xi^*_\alpha[\frac{\partial\hat{O}_{\rm coll}}{\partial\xi^*_\alpha}])+i\sum_\alpha(\dot{\xi}_\alpha\hat{X}^\dagger_\alpha-\dot{\xi}^*_\alpha\hat{X}_\alpha)+\cdots\}|\phi_0> , \tag{2.14}$$

we obtain an expansion form of the TDHF equation with respect to $(\xi_\alpha,\xi^*_\alpha)$. The zeroth-order equation, which is called the *invariance principle of the time-dependent Schrödinger equation*,[4),9),10)] is written as

$$\delta<\phi_0|\{i\dot{\eta}\hat{X}_{\rm coll}^\dagger-i\dot{\eta}^*\hat{X}_{\rm coll}-e^{-i\hat{G}}\hat{H}e^{i\hat{G}}\}|\phi_0> = 0 , \tag{2.15}$$

and the first-order equation is given by

$$\delta<\phi_0|\{i\dot{\eta}\sum_\alpha(\xi_\alpha[\frac{\partial\hat{O}_{\rm coll}^\dagger}{\partial\xi_\alpha}]+\xi^*_\alpha[\frac{\partial\hat{O}_{\rm coll}^\dagger}{\partial\xi^*_\alpha}])-i\dot{\eta}^*\sum_\alpha(\xi_\alpha[\frac{\partial\hat{O}_{\rm coll}}{\partial\xi_\alpha}]+\xi^*_\alpha[\frac{\partial\hat{O}_{\rm coll}}{\partial\xi^*_\alpha}])$$

$$+i\sum_\alpha(\dot{\xi}_\alpha\hat{X}^\dagger_\alpha-\dot{\xi}^*_\alpha\hat{X}_\alpha)-[e^{-i\hat{G}}\hat{H}e^{i\hat{G}},\sum_\alpha(\xi_\alpha\hat{X}^\dagger_\alpha-\xi^*_\alpha\hat{X}_\alpha)]\}|\phi_0> = 0 . \tag{2.16}$$

In getting Eq.(2.15), we have assumed that the order of $(\dot{\xi}_\alpha,\dot{\xi}^*_\alpha)$ is the same as that of $(\xi_\alpha,\xi^*_\alpha)$ in accordance with the small-amplitude assumption on the non-collective modes of motion.

Except for the small-amplitude assumption on the non-collective modes of motion, we have not yet imposed any condition on the variable transformation in Eq.(2.7). Here we require that the new variables $(\eta,\eta^*;\xi_\alpha,\xi_\alpha^*)$ should satisfy the canonical equations of motion within the first-order of $(\xi_\alpha,\xi_\alpha^*)$-expansion. Then, we obtain the canonical equations of motion for the collective mode of motion

$$i\dot{\eta} = \frac{\partial [H]}{\partial \eta^*}+\sum_\alpha \{\xi_\alpha [\frac{\partial^2 H}{\partial \eta^* \partial \xi_\alpha}] + \xi_\alpha^* [\frac{\partial^2 H}{\partial \eta^* \partial \xi_\alpha^*}]\} ,$$

$$i\dot{\eta}^* = -\frac{\partial [H]}{\partial \eta}-\sum_\alpha \{\xi_\alpha [\frac{\partial^2 H}{\partial \eta \partial \xi_\alpha}] + \xi_\alpha^* [\frac{\partial^2 H}{\partial \eta \partial \xi_\alpha^*}]\} , \tag{2.17}$$

and that for the non-collective modes of motion

$$i\dot{\xi}_\beta = [\frac{\partial H}{\partial \xi_\beta^*}] +\sum_\alpha \{\xi_\alpha [\frac{\partial^2 H}{\partial \xi_\alpha \partial \xi_\beta^*}] + \xi_\alpha^* [\frac{\partial^2 H}{\partial \xi_\alpha^* \partial \xi_\beta^*}]\} ,$$

$$i\dot{\xi}_\beta^* = -[\frac{\partial H}{\partial \xi_\beta}] -\sum_\alpha \{\xi_\alpha [\frac{\partial^2 H}{\partial \xi_\alpha \partial \xi_\beta}] + \xi_\alpha^* [\frac{\partial^2 H}{\partial \xi_\alpha^* \partial \xi_\beta}]\} . \tag{2.18}$$

where the classical Hamiltonian H is defined by

$$H \equiv <\phi_0| e^{-i\hat{F}}\hat{H}e^{i\hat{F}} |\phi_0>-<\phi_0|\hat{H}|\phi_0> . \tag{2.19}$$

We further demand that the classical Hamiltonian satisfies the maximal-decoupling condition,[4),5),9),10)]

$$[\frac{\partial H}{\partial \xi_\alpha}] = -<\phi_0| [\hat{X}_\alpha^\dagger, e^{-i\hat{C}}\hat{H}e^{i\hat{C}}] |\phi_0> = 0 ,$$

$$[\frac{\partial H}{\partial \xi_\alpha^*}] = <\phi_0| [\hat{X}_\alpha, e^{-i\hat{C}}\hat{H}e^{i\hat{C}}] |\phi_0> = 0 , \tag{2.20}$$

where we have used the definition of the operators $(\hat{X}_\alpha,\hat{X}_\alpha^\dagger)$ in Eq.(2.12). Then, Eqs.(2.17) and (2.18) are reduced to

$$i\dot{\eta} = \frac{\partial [H]}{\partial \eta^*} ,$$

$$i\dot{\eta}^* = -\frac{\partial [H]}{\partial \eta} , \qquad (2.21)$$

and

$$i\dot{\xi}_\beta = \sum_\alpha \{ \xi_\alpha [\frac{\partial^2 H}{\partial \xi_\alpha \partial \xi^*_\beta}] + \xi^*_\alpha [\frac{\partial^2 H}{\partial \xi^*_\alpha \partial \xi^*_\beta}] \} ,$$

$$i\dot{\xi}^*_\beta = -\sum_\alpha \{ \xi_\alpha [\frac{\partial^2 H}{\partial \xi_\alpha \partial \xi_\beta}] + \xi^*_\alpha [\frac{\partial^2 H}{\partial \xi^*_\alpha \partial \xi_\beta}] \} . \qquad (2.22)$$

The expanded TDHF equations (2.15), (2.16) and the canonical equations of motion (2.21), (2.22) with the maximal-decoupling condition (2.20) play essential roles in our theory. Comparing the expanded TDHF equations (2.15) and (2.16) with the canonical equations of motion (2.21) and (2.22) with (2.20), we can find some consistency conditions under which the expanded TDHF equations are equivalent to the canonical equations of motion with the maximal-decoupling condition. We may thus expect that these consistency conditions and the expanded TDHF equations properly specify the large-amplitude collective motion as well as the non-collective modes of motion under consideration.

2.2. *Zeroth-order TDHF Equation and the Self-consistent Collective Coordinate Method*

First, we discuss Eqs. (2.15), (2.21) and the maximal-decoupling condition (2.20), which are of the zeroth-order with respect to $(\xi_\alpha, \xi^*_\alpha)$. Equation (2.15) can be reduced to Eq. (2.21) under the following condition,

$$<\phi_0 | [\hat{X}_{coll}, \hat{X}_{coll}^\dagger] | \phi_0> = 1 . \qquad (2.23)$$

By taking $\delta|\phi_0> \propto :\hat{X}_{coll}^\dagger: |\phi_0>$ and $:\hat{X}_{coll}: |\phi_0>$, and then by using Eq. (2.23), we can easily see that the zeroth-order TDHF equation directly results in the canonical equations of motion (2.21) for the large-amplitude collective motion.

Equation (2.15) can be reduced to the maximal-decoupling condition (2.20) under the following condition,

$$<\phi_0|\,[\hat{X}_\alpha,\hat{X}_{\mathrm{coll}}^\dagger]\,|\phi_0> = 0\ ,\qquad <\phi_0|\,[\hat{X}_\alpha^\dagger,\hat{X}_{\mathrm{coll}}^\dagger]\,|\phi_0> = 0\ . \tag{2.24}$$

By taking $\delta|\phi_0>\propto :\hat{X}_\alpha^\dagger:|\phi_0>$ and $:\hat{X}_\alpha:|\phi_0>$, and then by using Eq. (2.24), the zeroth-order TDHF equation (2.15) leads to the maximal-decoupling condition (2.20).

The SCC method[4),9),10)] is formulated just within the zeroth-order approximation in our expansion method. With the aid of the theorem of Frobenius-Darboux,[15)] in the SCC method, the condition (2.23) can be specified in a more convenient form

$$<\phi_0|\hat{X}_{\mathrm{coll}}^\dagger|\phi_0> = \frac{1}{2}\eta^*\ ,\qquad <\phi_0|\hat{X}_{\mathrm{coll}}|\phi_0> = \frac{1}{2}\eta\ , \tag{2.25}$$

because the necessary condition (2.23) still allows us to have the freedom of choice of (η,η^*) within the canonical transformation satisfying Eq. (2.21).[4),10),16)] From Eq. (2.25), we can easily obtain the condition (2.23) through the relation

$$\frac{\partial}{\partial\eta}\Big(\frac{1}{2}\eta\Big)+\frac{\partial}{\partial\eta^*}\Big(\frac{1}{2}\eta^*\Big) = \frac{\partial}{\partial\eta}<\phi_0|\hat{X}_{\mathrm{coll}}|\phi_0>+\frac{\partial}{\partial\eta^*}<\phi_0|\hat{X}_{\mathrm{coll}}^\dagger|\phi_0>$$

$$= <\phi_0|\,[\hat{X}_{\mathrm{coll}},\hat{X}_{\mathrm{coll}}^\dagger]\,|\phi_0> = 1\ . \tag{2.26}$$

In the SCC method, furthermore, the maximal-decoupling condition (2.20) is expressed in the form,

$$\delta<\phi_0|\,\{e^{-i\hat{G}}\hat{H}e^{i\hat{G}}-\frac{\partial[H]}{\partial\eta^*}\hat{X}_{\mathrm{coll}}^\dagger-\frac{\partial[H]}{\partial\eta}\hat{X}_{\mathrm{coll}}\}\,|\phi_0> = 0\ , \tag{2.27}$$

which is derived from Eq. (2.15) with Eq. (2.21). The advantage of the use of

Eq. (2.27) instead of the maximal-decoupling condition (2.20) is that we can evaluate the condition (2.20) without having to know the explicit forms of the operators $(\hat{X}_\alpha, \hat{X}_\alpha^\dagger)$, which can be obtained only after having determined the collective mode of motion $(\hat{X}_{\rm coll}, \hat{X}_{\rm coll}^\dagger)$.

Equations (2.25) and (2.27), which consist of the basic equations of the SCC method, enable us to uniquely determine the functional form of $([f_{\mu i}], [f_{\mu i}^*])$ in Eq. (2.9) as well as the collective Hamiltonian $[H]$ with respect to (η, η^*), provided that a specific boundary condition appropriate for the collective motion under consideration is set up.[4),10)] Namely, we can obtain a mapping

$$M^2: \{\eta, \eta^*\} \rightarrow \Sigma^2: \{[f_{\mu i}], [f_{\mu i}^*]\} \tag{2.28}$$

which defines a two-dimensional collective submanifold Σ^2, i.e. an approximate integral surface embedded in the $2MN$-dimensional TDHF manifold (phase space) M^{2MN}. (See Fig. 1.) In the general case, we employ a perturbative treatment for determining functional forms of $([f_{\mu i}], [f_{\mu i}^*])$ with respect to (η, η^*). The perturbative treatment with respect to (η, η^*) is given in Appendix. The trajectory determined by Eq. (2.21) is thus mapped on the collective submanifold Σ^2 by the mapping (2.28), and it is called the SCC trajectory hereafter.

The maximal-decoupling condition (2.20) demonstrates that the classical Hamiltonian H is stationary at each point on the collective submanifold Σ^2 with respect to the variations toward non-collective directions characterized by $(\hat{X}_\alpha, \hat{X}_\alpha^\dagger)$, which have to be orthogonal to the variations toward collective directions characterized by $(\hat{X}_{\rm coll}, \hat{X}_{\rm coll}^\dagger)$, i.e., the tangential directions of the collective submanifold Σ^2.

In determining the collective submanifold within the zeroth-order of our expansion, the condition (2.24) has not been explicitly used. As is shown in the next subsection, however, the condition (2.24) plays a crucial role in specifying the functional form of the operators $(\hat{X}_\alpha, \hat{X}_\alpha^\dagger)$, together with the first-order TDHF equation of our expansion method.

2.3. *First-order TDHF Equation and Non-collective Modes of Motion*

As the next step, we discuss Eqs. (2.16) and (2.22), which are of first-order of $(\xi_\alpha, \xi_\alpha^\dagger)$. For the purpose of obtaining the self-consistency conditions under which the first-order TDHF equation (2.16) is equivalent to the canonical equations of non-collective motion (2.22), it is convenient to start with the following relations,

$$\frac{\partial \hat{O}_{\rm coll}^\dagger}{\partial \xi_\alpha} - \frac{\partial \hat{O}_\alpha^\dagger}{\partial \eta} = [\hat{O}_{\rm coll}^\dagger, \hat{O}_\alpha^\dagger] \quad ,$$

$$\frac{\partial \hat{O}_{\rm coll}^\dagger}{\partial \xi_\alpha^*} + \frac{\partial \hat{O}_\alpha}{\partial \eta} = [\hat{O}_\alpha, \hat{O}_{\rm coll}^\dagger] \quad , \tag{2.29}$$

which are derived from the definition (2.8) provided that $(f_{\mu i}, f_{\mu i}^*)$ are analytic functions of $(\eta, \eta^*; \xi_\alpha, \xi_\alpha^*)$. By applying the expansion (2.11) to Eq. (2.29), we obtain the relations among the zeroth-order terms,

$$[\frac{\partial \hat{O}_{\rm coll}^\dagger}{\partial \xi_\alpha}] - \frac{\partial \hat{X}_\alpha^\dagger}{\partial \eta} = [\hat{X}_{\rm coll}^\dagger, \hat{X}_\alpha^\dagger] \quad ,$$

$$[\frac{\partial \hat{O}_{\rm coll}^\dagger}{\partial \xi_\alpha^*}] + \frac{\partial \hat{X}_\alpha}{\partial \eta} = [\hat{X}_\alpha, \hat{X}_{\rm coll}^\dagger] \quad . \tag{2.30}$$

With the aid of Eqs. (2.21) and (2.30), the first-order TDHF equation (2.16) can be reduced to

$$\delta\langle\phi_0| \{ i\sum_\alpha (\dot{\xi}_\alpha \hat{X}_\alpha^\dagger - \dot{\xi}_\alpha^* \hat{X}_\alpha) - [e^{-i\hat{G}}\hat{H}e^{i\hat{G}}, \sum_\alpha (\xi_\alpha \hat{X}_\alpha^\dagger - \xi_\alpha^* \hat{X}_\alpha)]$$

$$+i(\dot{\eta}\frac{\partial}{\partial\eta}+\dot{\eta}^*\frac{\partial}{\partial\eta^*})\sum_{\alpha}(\xi_\alpha\hat{X}_\alpha^\dagger-\xi_\alpha^*\hat{X}_\alpha)\}\,|\,\phi_0\rangle = 0\ , \tag{2.31}$$

where $\hat{H}'$ is defined by

$$e^{-i\hat{G}}\hat{H}'e^{i\hat{G}} \equiv e^{-i\hat{G}}\hat{H}e^{i\hat{G}}-\frac{\partial\,[H]}{\partial\eta^*}\hat{X}_{\mathrm{coll}}^\dagger-\frac{\partial\,[H]}{\partial\eta}\hat{X}_{\mathrm{coll}}\ . \tag{2.32}$$

Since $\hat{H}'$ satisfies the Hartree-Fock equation (2.27), it defines a set of single-particle states in the generalized moving frame and we call it hereafter the Hamiltonian in the generalized moving frame.[4),10)]

In order to obtain an explicit expression for the first-order canonical equations of motion (2.22) for the non-collective variables, we use the relations such as

$$[\frac{\partial^2 H}{\partial\xi_\alpha\partial\xi_\beta^*}] = \langle\phi_0|\,[\hat{X}_\beta,\,[e^{-i\hat{G}}\hat{H}e^{i\hat{G}},\hat{X}_\alpha^\dagger]\,]\,|\phi_0\rangle$$

$$+\langle\phi_0|\,[\,[\frac{\partial\hat{O}_\beta}{\partial\xi_\alpha}]\,,e^{-i\hat{G}}\hat{H}e^{i\hat{G}}]\,|\,\phi_0\rangle\ . \tag{2.33}$$

The second term on the right-hand side of Eq. (2.33) is rewritten, with the aid of the zeroth-order TDHF equation (2.15), as

$$\langle\phi_0|\,[\,[\frac{\partial\hat{O}_\beta}{\partial\xi_\alpha}]\,,e^{-i\hat{G}}\hat{H}e^{i\hat{G}}]\,|\,\phi_0\rangle = \langle\phi_0|\,[\,[\frac{\partial\hat{O}_\beta}{\partial\xi_\alpha}]\,,i(\dot{\eta}\hat{X}_{\mathrm{coll}}^\dagger-\dot{\eta}^*\hat{X}_{\mathrm{coll}})]\,|\,\phi_0\rangle\ , \tag{2.34}$$

because the operator $[\partial\hat{O}_\beta/\partial\xi_\alpha]$ is a one-body operator. With the aid of the relation,

$$[\frac{\partial\hat{O}_\beta}{\partial\xi_\alpha}] = [\hat{X}_\beta,\hat{X}_\alpha^\dagger] - [\frac{\partial\hat{O}_\alpha^\dagger}{\partial\xi_\beta^*}]\ , \tag{2.35}$$

similar to Eq. (2.30), the first term on the right-hand side of Eq. (2.34) can be expressed as

$$i\dot{\eta}\langle\phi_0|\,[\,[\frac{\partial\hat{O}_\beta}{\partial\xi_\alpha}]\,,\hat{X}_{\rm coll}^\dagger]\,|\phi_0\rangle$$

$$= i\dot{\eta}\,\{\langle\phi_0|\,[\,[\hat{X}_\beta,\hat{X}_\alpha^\dagger]\,,\hat{X}_{\rm coll}^\dagger]\,|\phi_0\rangle-\langle\phi_0|\,[\,[\frac{\partial\hat{O}_\alpha^\dagger}{\partial\xi_\beta^*}]\,,\hat{X}_{\rm coll}^\dagger]\,|\phi_0\rangle\}$$

$$= i\dot{\eta}\,\{\langle\phi_0|\,[\,(\,[\frac{\partial\hat{O}_{\rm coll}^\dagger}{\partial\xi_\beta^*}]+\frac{\partial\hat{X}_\beta}{\partial\eta}),\hat{X}_\alpha^\dagger]\,|\phi_0\rangle-\langle\phi_0|\,[\hat{X}_\beta,\,[\hat{X}_{\rm coll}^\dagger,\hat{X}_\alpha^\dagger]\,]\,|\phi_0\rangle$$

$$-\langle\phi_0|\,[\,[\frac{\partial\hat{O}_\alpha^\dagger}{\partial\xi_\beta^*}]\,,\hat{X}_{\rm coll}^\dagger]\,|\phi_0\rangle\}$$

$$=i\dot{\eta}\,\{\langle\phi_0|\,[\frac{\partial\hat{X}_\beta}{\partial\eta},\hat{X}_\alpha^\dagger]\,|\phi_0\rangle-\langle\phi_0|\,[\hat{X}_\beta,\,[\hat{X}_{\rm coll}^\dagger,\hat{X}_\alpha^\dagger]\,]\,|\phi_0\rangle$$

$$+\,[\frac{\partial}{\partial\xi_\beta^*}\langle\phi_0|\,[\hat{O}_{\rm coll}^\dagger,\hat{O}_\alpha^\dagger]\,|\phi_0\rangle]\,\}\quad, \qquad (2.36)$$

where we have used the Jacobi identity,

$$[\,[\hat{X}_\beta,\hat{X}_\alpha^\dagger]\,,\hat{X}_{\rm coll}^\dagger] = [\,[\hat{X}_\beta,\hat{X}_{\rm coll}^\dagger]\,,\hat{X}_\alpha^\dagger]-[\,[\hat{X}_\alpha^\dagger,\hat{X}_{\rm coll}^\dagger]\,,\hat{X}_\beta]\quad, \qquad (2.37)$$

and then used Eq. (2.30). In such a way, we finally obtain the following expression for Eq. (2.22),

$$i\dot{\xi}_\beta = \langle\phi_0|\,[\hat{X}_\beta,\,\{\,[e^{-i\hat{G}}\hat{H}'e^{i\hat{G}},\sum_\alpha(\xi_\alpha\hat{X}_\alpha^\dagger-\xi_\alpha^*\hat{X}_\alpha)]$$

$$-i\,(\dot{\eta}\frac{\partial}{\partial\eta}+\dot{\eta}^*\frac{\partial}{\partial\eta^*})\sum_\alpha(\xi_\alpha\hat{X}_\alpha^\dagger-\xi_\alpha^*\hat{X}_\alpha)\}\,]\,|\phi_0\rangle$$

$$+\sum_\alpha\xi_\alpha\,[\frac{\partial}{\partial\xi_\beta^*}\langle\phi_0|\,[\,(i\dot{\eta}\hat{O}_{\rm coll}^\dagger-i\dot{\eta}^*\hat{O}_{\rm coll}),\hat{O}_\alpha^\dagger]\,|\phi_0\rangle]$$

$$-\sum_\alpha\xi_\alpha^*\,[\frac{\partial}{\partial\xi_\beta^*}\langle\phi_0|\,[\,(i\dot{\eta}\hat{O}_{\rm coll}^\dagger-i\dot{\eta}^*\hat{O}_{\rm coll}),\hat{O}_\alpha]\,|\phi_0\rangle]\qquad \text{and c.c.}\;, \qquad (2.38)$$

where the Hamiltonian in the generalized moving frame $\hat{H}'$ also explicitly manifests itself.

With the purpose to find out the conditions under which the first-order TDHF equation (2.31) is equivalent to the canonical equations of motion (2.22), we take the variations of Eq.(2.31) toward the non-collective directions ($\hat{X}_\alpha, \hat{X}^\dagger_\alpha; \alpha=1,2,\cdots,MN-1$). We then obtain

$$i\dot{\xi}_\beta = \langle\phi_0|\,[\hat{X}_\beta,\ \{\,[e^{-i\hat{G}}\hat{H}e^{i\hat{G}}, \sum_\alpha(\xi_\alpha\hat{X}^\dagger_\alpha-\xi^*_\alpha\hat{X}_\alpha)]$$

$$-i(\dot{\eta}\frac{\partial}{\partial\eta}+\dot{\eta}^*\frac{\partial}{\partial\eta^*})\sum_\alpha(\xi_\alpha\hat{X}^\dagger_\alpha-\xi^*_\alpha\hat{X}_\alpha)\}\,]\,|\phi_0\rangle \qquad \text{and c.c. ,} \qquad (2.39)$$

provided that the weak boson-like commutation relations among the non-collective modes of motion

$$\langle\phi_0|\,[\hat{X}_\alpha,\hat{X}^\dagger_\beta]\,|\phi_0\rangle = \delta_{\alpha\beta}\ , \qquad \langle\phi_0|\,[\hat{X}_\alpha,\hat{X}_\beta]\,|\phi_0\rangle = 0\ , \qquad (2.40)$$

are satisfied.

By comparing Eq.(2.38) with Eq.(2.39), it becomes clear that the first-order TDHF equation (2.16) can be reduced to the canonical equations of motion (2.22) under Eq.(2.40) and a set of additional equations,

$$[\frac{\partial}{\partial\xi_\beta}\langle\phi_0|\,[\hat{O}_{\rm coll},\hat{O}^\dagger_\alpha]\,|\phi_0\rangle] = 0\ ,$$

$$[\frac{\partial}{\partial\xi_\beta}\langle\phi_0|\,[\hat{O}^\dagger_{\rm coll},\hat{O}^\dagger_\alpha]\,|\phi_0\rangle] = 0\ , \qquad \text{etc. .} \qquad (2.41)$$

Similar to the role of condition (2.24) in the case of the zeroth-order equations discussed in § 2.2, the condition (2.41) is not necessary for specifying the operator ($\hat{X}_\alpha, \hat{X}^\dagger_\alpha$), but plays an important role in specifying the higher-order operators ($[\partial\hat{O}_\alpha/\partial\xi_\beta]$, $[\partial\hat{O}^\dagger_\alpha/\partial\xi_\beta]$) in our expansion method, after having determined ($\hat{X}_\alpha, \hat{X}^\dagger_\alpha$).

In order to uniquely define the non-collective variables, we further suppose that the non-collective variables (ξ_α, ξ^*_α) are of the normal modes satisfying

$$[\frac{\partial^2 H}{\partial\xi_\alpha\partial\xi^*_\beta}] = [\frac{\partial^2 H}{\partial\xi_\alpha\partial\xi^*_\alpha}]\,\delta_{\alpha\beta}\ , \qquad [\frac{\partial^2 H}{\partial\xi_\alpha\partial\xi_\beta}] = [\frac{\partial^2 H}{\partial\xi^*_\alpha\partial\xi^*_\beta}] = 0\ , \tag{2.42a}$$

$$\xi_\alpha(t) = \xi_\alpha(0)e^{-i\omega_\alpha t}\ , \qquad \xi^*_\alpha(t) = \xi^*_\alpha(0)e^{i\omega_\alpha t}\ . \tag{2.42b}$$

The assumption of small-amplitude oscillation denoted by Eq.(2.42b) is consistent with that used in deriving Eq.(2.16) and is the same as those usually employed in getting the RPA equation from the TDHF equation.

By substituting Eq.(2.42b) for Eq.(2.39), we obtain

$$<\phi_0|\,[\hat{X}_\beta,[e^{-i\hat{C}}\hat{H}e^{i\hat{C}},\hat{X}^\dagger_\alpha]\,]\,|\phi_0> = \omega_\alpha\delta_{\alpha\beta}\ ,$$

$$<\phi_0|\,[\hat{X}_\beta,[e^{-i\hat{C}}\hat{H}e^{i\hat{C}},\hat{X}_\alpha]\,]\,|\phi_0> = 0\ , \tag{2.43}$$

under the condition

$$<\phi_0|\,[\hat{X}_\beta,i(\dot{\eta}\frac{\partial}{\partial\eta}+\dot{\eta}^*\frac{\partial}{\partial\eta^*})\hat{X}^\dagger_\alpha]\,|\phi_0> = 0\ ,$$

$$<\phi_0|\,[\hat{X}_\beta,i(\dot{\eta}\frac{\partial}{\partial\eta}+\dot{\eta}^*\frac{\partial}{\partial\eta^*})\hat{X}_\alpha]\,|\phi_0> = 0\ . \tag{2.44}$$

This implies that, under the condition (2.44), the one-body operator $(\hat{X}_\alpha,\hat{X}^\dagger_\alpha)$ may be described by the RPA-type normal modes of the Hamiltonian in the generalized moving frame $e^{-\hat{C}}\hat{H}e^{i\hat{C}}$ with intrinsic excitation energies ω_α.

Since the Hamiltonian in the generalized moving frame $e^{-i\hat{C}}\hat{H}e^{i\hat{C}}$ satisfies the Hartree-Fock equation (2.27), it does not contain any particle-hole (p-h) components $(a^\dagger_\mu b^\dagger_i, b_i a_\mu)$. Without any loss of generality, therefore, the solutions of Eq.(2.43) which call hereafter intrinsic excitation modes can be written in the form

$$\tilde{X}_\alpha^\dagger \equiv \sum_{\mu i} \{\psi_{\mu i}^\alpha(\eta,\eta^*) a_\mu^\dagger b_i^\dagger - \varphi_{\mu i}^\alpha(\eta,\eta^*) b_i a_\mu\} \quad . \tag{2.45}$$

This is in sharp contrast to the fact that the collective mode of motion $(\hat{X}_{\rm coll}, \hat{X}_{\rm coll}^\dagger)$ has been uniquely determined in the form (2.12), which has the particle-particle (p-p) and hole-hole (h-h) parts as well as the p-h part.

We call the condition (2.44) a separability condition between the collective and non-collective modes of motion hereafter.[12] The physical meaning of the condition (2.44) will be discussed in § 4.

2.4. *Intrinsic Hamiltonian for Intrinsic Excitation Modes*

Equation (2.43) shows that the Hamiltonian in the generalized moving frame $\hat{H}'$ plays a decisive role to specify the intrinsic excitation modes $(\tilde{X}_\alpha, \tilde{X}_\alpha^\dagger)$ and the intrinsic excitation energies ω_α. However, the operator $\hat{H}'$ generally cannot be the proper intrinsic Hamiltonian associated with the collective motion under consideration, because it contains a collective component within the RPA boson approximation, i.e.,

$$<\phi_0 | [\hat{X}_{\rm coll}, [e^{-i\hat{G}} \hat{H}' e^{i\hat{G}}, \hat{X}_{\rm coll}^\dagger]] | \phi_0> \neq 0 \ . \tag{2.46}$$

By contrast, the proper intrinsic Hamiltonian $\hat{H}_{\rm intr}$ should satisfy

$$<\phi_0 | [\hat{X}_{\rm coll}, [e^{-i\hat{G}} \hat{H}_{\rm intr} e^{i\hat{G}}, \hat{X}_{\rm coll}^\dagger]] | \phi_0> = 0 \ . \tag{2.47}$$

In this subsection, we derive the intrinsic Hamiltonian $\hat{H}_{\rm intr}$ within the RPA boson approximation.

Corresponding to the fermion pair operators $(a_\mu^\dagger b_i^\dagger, b_i a_\mu)$ in the RPA boson approximation, we introduce the boson operators

$$a_\mu^\dagger b_i^\dagger \rightarrow B_{\mu i}^\dagger \ , \qquad b_i a_\mu \rightarrow B_{\mu i} \ , \tag{2.48}$$

which satisfy the boson commutation relation

$$[B_{\mu i},B^{\dagger}_{\nu j}] = \delta_{\mu\nu}\delta_{ij} , \qquad [B_{\mu i},B_{\nu j}] = 0 . \tag{2.49}$$

For later convenience, we express the p-h part of collective mode of motion as

$$\tilde{X}_{\rm coll}^{\dagger} = \sum_{\mu i} \{\Psi_{\mu i}(\eta,\eta^*)a^{\dagger}_{\mu}b^{\dagger}_{i}-\Phi_{\mu i}(\eta,\eta^*)b_i a_{\mu}\} . \tag{2.50}$$

With the use of the boson operators (2.48), the intrinsic excitation modes $(\tilde{X}_{\alpha},\tilde{X}^{\dagger}_{\alpha})$ as well as $(\tilde{X}_{\rm coll},\tilde{X}_{\rm coll}^{\dagger})$ are represented as

$$\tilde{X}^{\dagger}_{\alpha} \rightarrow X^{\dagger}_{\alpha} = \sum_{\mu i} \{\psi^{\alpha}_{\mu i}B^{\dagger}_{\mu i}-\varphi^{\alpha}_{\mu i}B_{\mu i}\} ,$$

$$\tilde{X}_{\rm coll}^{\dagger} \rightarrow X^{\dagger}_{\rm coll} = \sum_{\mu i} \{\Psi_{\mu i}B^{\dagger}_{\mu i}-\Phi_{\mu i}B_{\mu i}\} . \tag{2.51}$$

The Hamiltonian in the generalized moving frame $\hat{H}'$ within the RPA boson approximation is then given by

$$e^{-i\hat{G}}\hat{H}'e^{i\hat{G}} \rightarrow H' = \sum_{\mu\nu ij} \{\mathcal{A}_{\mu i,\nu j}B^{\dagger}_{\mu i}B_{\nu j}-\frac{1}{2}\mathcal{B}_{\mu i,\nu j}B^{\dagger}_{\mu i}B^{\dagger}_{\nu j}-\frac{1}{2}\mathcal{B}^{*}_{\mu i,\nu j}B_{\mu i}B_{\nu j}\} , \tag{2.52}$$

where $\mathcal{A}_{\mu i,\nu j}$ and $\mathcal{B}_{\mu i,\nu j}$ are defined by

$$\mathcal{A}_{\mu i,\nu j} = \langle\phi_0|\ [b_i a_{\mu},[e^{-i\hat{G}}\hat{H}'e^{i\hat{G}},a^{\dagger}_{\nu}b^{\dagger}_{j}]]\ |\phi_0\rangle ,$$

$$\mathcal{B}_{\mu i,\nu j} = \langle\phi_0|\ [b_i a_{\mu},[e^{-i\hat{G}}\hat{H}'e^{i\hat{G}},b_j a_{\nu}]]\ |\phi_0\rangle . \tag{2.53}$$

In order to obtain the intrinsic Hamiltonian, we introduce the following projection operator expressed by a $2MN\times 2MN$ matrix,

$$P \equiv I-\begin{bmatrix}\Psi\\ \Phi\end{bmatrix}[\ \Psi^{\dagger}\ \ \Phi^{\dagger}\ ]\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}+\begin{bmatrix}\Phi^{*}\\ \Psi^{*}\end{bmatrix}[\ \Phi^{*\dagger}\ \ \Psi^{*\dagger}\ ]\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} . \tag{2.54}$$

Here I is the $2MN\times 2MN$ unit matrix, and 1 and 0 are the $MN\times MN$ unit and null matrices, respectively. Ψ, Φ and $\Psi^{\dagger},\Phi^{\dagger}$ are $MN\times 1$ column- and $1\times MN$ rowmatrices defined with the amplitudes of $(\Psi_{\mu i}, \Phi_{\nu j})$ (in Eq.(2.51)) in the following way,

$$\Psi = \begin{bmatrix} (\Psi_{\mu i})_1 \\ \vdots \\ \vdots \\ (\Phi_{\mu i})_{MN} \end{bmatrix}, \qquad \Phi = \begin{bmatrix} (\Phi_{\mu i})_1 \\ \vdots \\ \vdots \\ (\Phi_{\mu i})_{MN} \end{bmatrix},$$

$$\Psi^\dagger = [\ (\Psi^*_{\mu i})_1 \quad \cdots \quad (\Psi^*_{\mu i})_{MN}\ ], \qquad \Phi^\dagger = [\ (\Phi^*_{\mu i})_1 \quad \cdots \quad (\Phi^*_{\mu i})_{MN}\ ]. \tag{2.55}$$

Since the collective mode of motion $(\hat{X}_{\rm coll}, \hat{X}_{\rm coll}^\dagger)$ is determined in the SCC method so as to satisfy the orthonormality relation (2.33), we can easily prove the relation

$$P^2 = P\ . \tag{2.56}$$

With the aid of the condition (2.24), it is clear that the projection operator P satisfies

$$P\begin{bmatrix} \Psi_\alpha \\ \Phi_\alpha \end{bmatrix} = \begin{bmatrix} \Psi_\alpha \\ \Phi_\alpha \end{bmatrix}, \qquad P\begin{bmatrix} \Phi^*_\alpha \\ \Psi^*_\alpha \end{bmatrix} = \begin{bmatrix} \Phi^*_\alpha \\ \Psi^*_\alpha \end{bmatrix},$$

$$P\begin{bmatrix} \Psi \\ \Phi \end{bmatrix} = 0\ , \qquad P\begin{bmatrix} \Phi^* \\ \Psi^* \end{bmatrix} = 0\ , \tag{2.57}$$

where

$$\Psi_\alpha = \begin{bmatrix} (\psi^\alpha_{\mu i})_1 \\ \vdots \\ \vdots \\ (\psi^\alpha_{\mu i})_{MN} \end{bmatrix}, \qquad \Phi_\alpha = \begin{bmatrix} (\varphi^\alpha_{\mu i})_1 \\ \vdots \\ \vdots \\ (\varphi^\alpha_{\mu i})_{MN} \end{bmatrix}. \tag{2.58}$$

Equation (2.57) shows that the projection operator P plays a role to project out the collective mode of motion within the RPA boson approximation.

Now, we express the intrinsic Hamiltonian $H_{\rm intr}$ within the RPA boson approximation in the form

$$H_{\rm intr} = \sum_{\mu\nu ij} \{ \mathcal{A}'_{\mu i,\nu j} B^\dagger_{\mu i} B_{\nu j} - \frac{1}{2}\mathcal{B}'_{\mu i,\nu j} B^\dagger_{\mu i} B^\dagger_{\nu j} - \frac{1}{2}\mathcal{B}'^*_{\mu i,\nu j} B_{\mu i} B_{\nu j} \} \quad . \tag{2.59}$$

With the use of the projection operator P, then, we obtain $\mathcal{A}'_{\mu i,\nu j}$ and $\mathcal{B}'_{\mu i,\nu j}$ as follows,

$$\begin{bmatrix} \mathcal{A}' & \mathcal{B}' \\ -\mathcal{B}'^* & -\mathcal{A}'^* \end{bmatrix} = P\begin{bmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B}^* & -\mathcal{A}^* \end{bmatrix} P \ , \tag{2.60}$$

where the matrix elements of the $MN\times MN$ matrices $\mathcal{A}$, $\mathcal{B}$, $\mathcal{A}'$ and $\mathcal{B}'$ are given by

$$(\mathcal{A})_{\mu i,\nu j} \equiv \mathcal{A}_{\mu i,\nu j} \ , \qquad (\mathcal{B})_{\mu i,\nu j} \equiv \mathcal{B}_{\mu i,\nu j} \ ,$$
$$(\mathcal{A}')_{\mu i,\nu j} \equiv \mathcal{A}'_{\mu i,\nu j} \ , \qquad (\mathcal{B}')_{\mu i,\nu j} \equiv \mathcal{B}'_{\mu i,\nu j} \ , \tag{2.61}$$

and $\mathcal{A}^*, \mathcal{B}^*, \mathcal{A}'^*$ and $\mathcal{B}'^*$ are complex conjugate matrices of $\mathcal{A}$, $\mathcal{B}$, $\mathcal{A}'$ and $\mathcal{B}'$, respectively.

From Eq.(2.60), the intrinsic Hamiltonian $H_{\rm intr}$ within the RPA boson approximation is given by

$$\begin{aligned} H_{\rm intr} = H' &- \{ [H', X^\dagger_{\rm coll}] \} X_{\rm coll} + X^\dagger_{\rm coll} \{ [H', X_{\rm coll}] \} \\ &+ \{ [[X_{\rm coll}, H'], X^\dagger_{\rm coll}] \} \cdot X^\dagger_{\rm coll} X_{\rm coll} \\ &+ \frac{1}{2} \{ [[H', X^\dagger_{\rm coll}], X^\dagger_{\rm coll}] \} \cdot X_{\rm coll} X_{\rm coll} \\ &+ \frac{1}{2} \{ [[H', X_{\rm coll}], X_{\rm coll}] \} \cdot X^\dagger_{\rm coll} X^\dagger_{\rm coll} \ . \end{aligned} \tag{2.62}$$

The intrinsic excitation modes $(X_\alpha, X^\dagger_\alpha)$ as well as the intrinsic excitation energies ω_α are obtained by solving the following RPA equation with the intrinsic Hamiltonian,

$$[H_{\rm intr}, X^\dagger_\alpha] = \omega_\alpha X^\dagger_\alpha \ , \tag{2.63}$$

with the orthonormalized condition

$$[X_\alpha, X^\dagger_\beta] = \delta_{\alpha\beta} \ , \qquad [X_\alpha, X_\beta] = 0 \ . \tag{2.64}$$

We also obtain

$$[H_{\rm intr}, X^{\dagger}_{\rm coll}] = 0 \ , \tag{2.65}$$

which corresponds to Eq. (2.47) and demonstrates $(X_{\rm coll}, X^{\dagger}_{\rm coll})$ to be an eigenmode of $H_{\rm intr}$ with zero energy, implying that the condition (2.24) between the collective and non-collective modes of motion is automatically satisfied. Namely, the effects of the projection operator P in Eq. (2.60) is effectively expressed by the terms $(H_{\rm intr} - H')$ in the boson representation.

In such a way, Eqs. (2.63) and (2.64) enable us to determine the intrinsic excitation modes $(X_\alpha, X^{\dagger}_\alpha)$ and the intrinsic excitation energies ω_α under the separability condition (2.44).

§ 3. Geometrical Structure of Dynamical Collective Submanifold

Within the framework of the TDHF theory, in the previous section, we formulated the theory of dynamical collective submanifold. The essential concept introduced in the theory is the approximate integral surface embedded in the TDHF manifold. In this section, we investigate the geometrical properties of the approximate integral surface embedded in the TDHF manifold, by formulating the TDHF equation in the form of the canonical equations of motion in classical mechanics in the TDHF manifold.[13)]

3.1. *Canonical-Variable Representation of the TDHF Theory*

As was stated in § 2.1, the TDHF equation is given by Eq.(2.1). We first introduce a new set of time-dependent variables $(C_{\mu i}(t), C_{\mu i}^*(t))$ through a variable transformation

$$f_{\mu i} = f_{\mu i}(C_{\mu i}, C_{\mu i}^*) \ , \qquad f_{\mu i}^* = f_{\mu i}^*(C_{\mu i}, C_{\mu i}^*) \ . \tag{3.1}$$

The symplectic structure of the TDHF theory always enables us to choose the variables $(C_{\mu i}, C_{\mu i}^*)$ to be of the canonical-variable representation,[5)~8)] in which the TDHF equation (2.1) can be expressed as the canonical equations of motion in classical mechanics,

$$i\dot{C}_{\mu i} = \frac{\partial H}{\partial C_{\mu i}^*} \ , \qquad i\dot{C}_{\mu i}^* = -\frac{\partial H}{\partial C_{\mu i}} \ , \tag{3.2a}$$

i.e.,

$$\dot{q}_{\mu i} = \frac{\partial H}{\partial p_{\mu i}} \ , \qquad \dot{p}_{\mu i} = -\frac{\partial H}{\partial q_{\mu i}} \ ;$$

$$q_{\mu i} \equiv \frac{1}{\sqrt{2}}(C_{\mu i}^* + C_{\mu i}) \ , \qquad p_{\mu i} \equiv \frac{i}{\sqrt{2}}(C_{\mu i}^* - C_{\mu i}) \ , \tag{3.2b}$$

where the classical Hamiltonian H is given by Eq.(2.19).

The procedure of a choice of the canonical variables $(C_{\mu i}, C_{\mu i}^*)$ is following.[5] We adopt the new variables so as to satisfy

$$\omega \equiv \sum_{\mu i}\{<\phi_0| e^{-i\hat{F}}\frac{\partial}{\partial C_{\mu i}}e^{i\hat{F}}|\phi_0>dC_{\mu i}+<\phi_0| e^{-i\hat{F}}\frac{\partial}{\partial C_{\mu i}^*}e^{i\hat{F}}|\phi_0>dC_{\mu i}^*\}$$

$$= \frac{1}{2}\mathrm{Tr}\ \{C^\dagger\cdot dC-dC^\dagger\cdot C\} \quad , \tag{3.3}$$

where C and $C^\dagger$ denote an $M\times N$ matrix and its Hermitian conjugate, respectively, defined by

$$(C)_{\mu i} = C_{\mu i} \ , \qquad (C^\dagger)_{i\mu} = C_{\mu i}^* \ , \tag{3.4}$$

and the matrix notation on the right-hand side represents

$$\mathrm{Tr}\ \{C^\dagger\cdot dC-dC^\dagger\cdot C\} \equiv \sum_{\mu i}\{C_{\mu i}^* dC_{\mu i}-C_{\mu i}dC_{\mu i}^*\} \quad . \tag{3.5}$$

After direct calculations, we have

$$<\phi_0| e^{-i\hat{F}}\frac{\partial}{\partial C_{\mu i}}e^{i\hat{F}}|\phi_0> = \mathrm{Tr}\ \{F^\dagger\frac{\sin^2\sqrt{FF^\dagger}}{2FF^\dagger}\cdot\frac{\partial F}{\partial C_{\mu i}}-\frac{\partial F^\dagger}{\partial C_{\mu i}}\cdot\frac{\sin^2\sqrt{FF^\dagger}}{2FF^\dagger}F\} \quad , \tag{3.6}$$

where the matrices F and $F^\dagger$ are defined by

$$(F)_{\mu i} = f_{\mu i} \ , \qquad (F^\dagger)_{i\mu} = f_{\mu i}^* \ . \tag{3.7}$$

By substituting Eq.(3.6) into Eq.(3.3) and by using

$$dF = \sum_{\mu i}\{\frac{\partial F}{\partial C_{\mu i}}dC_{\mu i}+\frac{\partial F}{\partial C_{\mu i}^*}dC_{\mu i}^*\} \quad , \tag{3.8}$$

we obtain

$$\omega = \frac{1}{2}\mathrm{Tr}\ \{F^\dagger\frac{\sin^2\sqrt{FF^\dagger}}{FF^\dagger}\cdot dF\} -\frac{1}{2}\mathrm{Tr}\ \{dF^\dagger\cdot\frac{\sin^2\sqrt{FF^\dagger}}{FF^\dagger}F\}$$

$$= \frac{1}{2}\mathrm{Tr}\ \{\ (F^\dagger\frac{\sin\sqrt{FF^\dagger}}{\sqrt{FF^\dagger}})\cdot d(\frac{\sin\sqrt{FF^\dagger}}{\sqrt{FF^\dagger}}F)$$

$$-d(F^\dagger \frac{\sin\sqrt{FF^\dagger}}{\sqrt{FF^\dagger}}) \cdot (\frac{\sin\sqrt{FF^\dagger}}{\sqrt{FF^\dagger}}F)\}$$

$$= \frac{1}{2}\mathrm{Tr}\ \{C^\dagger \cdot dC - dC^\dagger \cdot C\} \quad . \tag{3.9}$$

This implies that the new variables $(C_{\mu i}, C_{\mu i}^*)$ given by

$$C = \frac{\sin\sqrt{FF^\dagger}}{\sqrt{FF^\dagger}}F \ , \qquad C^\dagger = F^\dagger \frac{\sin\sqrt{FF^\dagger}}{\sqrt{FF^\dagger}} \ , \tag{3.10}$$

satisfy the following canonical-variable condition,

$$<\phi_0 | \hat{O}_{\mu i}^\dagger | \phi_0> = \frac{1}{2}C_{\mu i}^* \ , \qquad <\phi_0 | \hat{O}_{\mu i} | \phi_0> = \frac{1}{2}C_{\mu i} \ , \tag{3.11}$$

where the operators $\hat{O}_{\mu i}^\dagger$ and $\hat{O}_{\mu i}$ are the local infinitesimal generators with respect to $C_{\mu i}$ and $C_{\mu i}^*$ respectively, defined by

$$\hat{O}_{\mu i}^\dagger \equiv e^{-i\hat{F}} \frac{\partial}{\partial C_{\mu i}} e^{i\hat{F}} \ , \qquad \hat{O}_{\mu i} \equiv -e^{-i\hat{F}} \frac{\partial}{\partial C_{\mu i}^*} e^{i\hat{F}} \ . \tag{3.12}$$

In order to see that the variables $(C_{\mu i}, C_{\mu i}^*)$ obtained by Eq.(3.10) really satisfy the canonical equations of motion (3.1), we first notice the following relations,

$$<\phi_0 | [\hat{O}_{\mu i}, \hat{O}_{\nu j}^\dagger] | \phi_0> = \delta_{\mu\nu}\delta_{ij} \ , \qquad <\phi_0 | [\hat{O}_{\mu i}, \hat{O}_{\nu j}] | \phi_0> = 0 \ , \tag{3.13}$$

which is obtained from Eq.(3.11) provided that the TDHF manifold is a complex analytic manifold satisfying the integrability condition[4),9),10)]

$$(\frac{\partial^2}{\partial C_{\mu i}^* \partial C_{\nu j}} - \frac{\partial^2}{\partial C_{\nu j} \partial C_{\mu i}^*}) e^{i\hat{F}} | \phi_0> = 0 \ ,$$

$$(\frac{\partial^2}{\partial C_{\mu i} \partial C_{\nu j}} - \frac{\partial^2}{\partial C_{\nu j} \partial C_{\mu i}}) e^{i\hat{F}} | \phi_0> = 0 \ . \tag{3.14}$$

Now, the TDHF equation (2.1) can be written in terms of the variables $(C_{\mu i}(t), C_{\mu i}^*(t))$ in the form

$$\delta\langle\phi_0|\{i\sum_{\mu i}(\dot{C}_{\mu i}\hat{O}^\dagger_{\mu i}-\dot{C}^*_{\mu i}\hat{O}_{\mu i})-e^{-i\hat{F}}\hat{H}e^{i\hat{F}}\}|\phi_0\rangle = 0\ . \tag{3.15}$$

By taking $|\delta\phi_0\rangle\propto:\hat{O}^\dagger_{\mu i}:|\phi_0\rangle$ and $:\hat{O}_{\mu i}:|\phi_0\rangle$ and by using the canonical-variable condition (3.13), Eq.(3.15) is simply reduced to the canonical equations of motion (3.2a).

3.2. *Geometrical Properties of the Self-consistent Collective Coordinate Method*

The solution of Eq.(3.2a) generally give trajectories in the $2MN$-dimensional TDHF manifold denoted by M^{2MN}: $\{C_{\mu i},C^*_{\mu i}\}$. If a group of many TDHF trajectories gathering together around the collective submanifold Σ^2: $\{\eta,\eta^*\}$ at an instant can be always confined in a small domain of the submanifold for a long enough time, we may naturally say that the system under consideration displays large-amplitude collective motion along an optimized trajectory, i.e., the SCC trajectory on the surface Σ^2, which is a representative of the group of TDHF trajectories.

Under such an assumption, we consider a general variable transformation from the canonical variables $(C_{\mu i},C^*_{\mu i})$ to a set of variables $(\eta,\eta^*,\xi_\alpha,\xi^*_\alpha;\alpha=1,2,\cdots,MN-1)$ including the collective variables (η,η^*),

$$C_{\mu i} = C_{\mu i}(\eta,\eta^*;\xi_\alpha,\xi^*_\alpha)\ ,\qquad C^*_{\mu i} = C^*_{\mu i}(\eta,\eta^*;\xi_\alpha,\xi^*_\alpha)\ , \tag{3.16}$$

where $(\xi_\alpha,\xi^*_\alpha;\alpha=1,2,\cdots,MN-1)$ are the non-collective variables introduced in § 2. If the transformation were an exact canonical transformation satisfying

$$i\dot{\eta} = \frac{\partial H}{\partial\eta^*}\ ,\qquad i\dot{\eta}^* = -\frac{\partial H}{\partial\eta}\ ,$$

$$i\dot{\xi}_\alpha = \frac{\partial H}{\partial\xi^*_\alpha}\ ,\qquad i\dot{\xi}^*_\alpha = -\frac{\partial H}{\partial\xi_\alpha}\ , \tag{3.17}$$

the transformation would have to satisfy the Lagrangian bracket[17]

$$\sum_{\mu i}\left\{\frac{\partial C_{\mu i}^*}{\partial \eta^*}\frac{\partial C_{\mu i}}{\partial \eta}-\frac{\partial C_{\mu i}^*}{\partial \eta}\frac{\partial C_{\mu i}}{\partial \eta^*}\right\} = 1 \ , \tag{3.18a}$$

$$\sum_{\mu i}\left\{\frac{\partial C_{\mu i}^*}{\partial \eta^*}\frac{\partial C_{\mu i}}{\partial \xi_\alpha}-\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}\frac{\partial C_{\mu i}}{\partial \eta^*}\right\} = \sum_{\mu i}\left\{\frac{\partial C_{\mu i}^*}{\partial \eta}\frac{\partial C_{\mu i}}{\partial \xi_\alpha}-\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}\frac{\partial C_{\mu i}}{\partial \eta}\right\} = 0 \ , \tag{3.18b}$$

$$\sum_{\mu i}\left\{\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha^*}\frac{\partial C_{\mu i}}{\partial \xi_\beta}-\frac{\partial C_{\mu i}^*}{\partial \xi_\beta}\frac{\partial C_{\mu i}}{\partial \xi_\alpha^*}\right\} = \delta_{\alpha\beta} \ , \qquad \sum_{\mu i}\left\{\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}\frac{\partial C_{\mu i}}{\partial \xi_\beta}-\frac{\partial C_{\mu i}^*}{\partial \xi_\beta}\frac{\partial C_{\mu i}}{\partial \xi_\alpha}\right\} = 0 \ . \tag{3.18c}$$

Our theory does not require such an exact canonical transformation, but demand that the non-collective variables $(\xi_\alpha, \xi_\alpha^*)$ describe small-amplitude motion in a small domain of the neighborhood of the collective surface Σ^2. Similar to the expansion introduced in § 2.1, we may then introduce a Taylor expansion of $(C_{\mu i}, C_{\mu i}^*)$ with respect to $(\xi_\alpha, \xi_\alpha^*)$ around a surface $(\eta, \eta^*; \xi_\alpha = \xi_\alpha^* = 0)$,

$$C_{\mu i} = [C_{\mu i}] + \sum_\alpha \left\{ \xi_\alpha \left[\frac{\partial C_{\mu i}}{\partial \xi_\alpha}\right] + \xi_\alpha^* \left[\frac{\partial C_{\mu i}}{\partial \xi_\alpha^*}\right] \right\} + \cdots \ , \tag{3.19}$$

where the symbol $[f]$ for any function $f(\eta, \eta^*; \xi_\alpha, \xi_\alpha^*)$ is defined by Eq. (2.9). In the same way, the Taylor expansions of H and $\partial H/\partial C_{\mu i}^*$ are expressed as

$$H = [H] + \sum_\alpha \left\{ \xi_\alpha \left[\frac{\partial H}{\partial \xi_\alpha}\right] + \xi_\alpha^* \left[\frac{\partial H}{\partial \xi_\alpha^*}\right] \right\} + \frac{1}{2}\sum_{\alpha\beta} \left\{ \xi_\alpha \xi_\beta \left[\frac{\partial^2 H}{\partial \xi_\alpha \partial \xi_\beta}\right] \right.$$

$$\left. + 2\xi_\alpha \xi_\beta^* \left[\frac{\partial^2 H}{\partial \xi_\alpha \partial \xi_\beta^*}\right] + \xi_\alpha^* \xi_\beta^* \left[\frac{\partial^2 H}{\partial \xi_\alpha^* \partial \xi_\beta^*}\right] \right\} + \cdots \ , \tag{3.20a}$$

$$\frac{\partial H}{\partial C_{\mu i}^*} = \left[\frac{\partial H}{\partial C_{\mu i}^*}\right] + \sum_\alpha \left\{ \xi_\alpha \left[\frac{\partial^2 H}{\partial \xi_\alpha \partial C_{\mu i}^*}\right] + \xi_\alpha^* \left[\frac{\partial^2 H}{\partial \xi_\alpha^* \partial C_{\mu i}^*}\right] \right\} + \cdots \ . \tag{3.20b}$$

By applying the Taylor expansion for Eq. (3.1a), we obtain an expansion form of the canonical equations of motion with respect to $(\xi_\alpha, \xi_\alpha^*)$. The zeroth-order equation is written as

$$i\frac{d}{dt}[C_{\mu i}] = \left[\frac{\partial H}{\partial C_{\mu i}^*}\right] \ , \qquad i\frac{d}{dt}[C_{\mu i}^*] = -\left[\frac{\partial H}{\partial C_{\mu i}}\right] \ , \tag{3.21}$$

and the first-order equation is given by

$$i\frac{d}{dt}\sum_{\alpha}\{\xi_\alpha[\frac{\partial C_{\mu i}}{\partial \xi_\alpha}]+\xi_\alpha^*[\frac{\partial C_{\mu i}}{\partial \xi_\alpha^*}]\} = \sum_{\alpha}\{\xi_\alpha[\frac{\partial^2 H}{\partial \xi_\alpha \partial C_{\mu i}^*}]+\xi_\alpha^*[\frac{\partial^2 H}{\partial \xi_\alpha^* \partial C_{\mu i}^*}]\} ,$$

$$i\frac{d}{dt}\sum_{\alpha}\{\xi_\alpha[\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}]+\xi_\alpha^*[\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha^*}]\} = -\sum_{\alpha}\{\xi_\alpha[\frac{\partial^2 H}{\partial \xi_\alpha \partial C_{\mu i}}]+\xi_\alpha^*[\frac{\partial^2 H}{\partial \xi_\alpha^* \partial C_{\mu i}}]\} , \quad (3.22)$$

where the order of $(\dot{\xi}_\alpha, \dot{\xi}_\alpha^*)$ is assumed to be the same as that of $(\xi_\alpha, \xi_\alpha^*)$ in accordance with the small-amplitude assumption on the non-collective motion.

The time-dependence of the variables $(\eta, \eta^*; \xi_\alpha, \xi_\alpha^*)$ must be determined in terms of the canonical equations of motion (2.21) and (2.22) under the maximal-decoupling condition (2.20). Thus, the consistency conditions under which Eqs.(3.21) and (3.22) reduce to Eqs.(2.21), (2.22) and (2.20) play a role to specify the collective submanifold as well as the non-collective modes under consideration, similar to the discussion in § 2.

First, we discuss the consistency conditions under which Eq.(3.21) reduce to Eqs.(2.20) and (2.21). The consistency conditions are obtained in the following way. Eq.(3.21) can be expressed as

$$i\dot{\eta}[\frac{\partial C_{\mu i}}{\partial \eta}]+i\dot{\eta}^*[\frac{\partial C_{\mu i}}{\partial \eta^*}] = [\frac{\partial H}{\partial C_{\mu i}^*}] , \qquad \text{and c.c. .} \quad (3.23)$$

With the aid of Eq.(3.23), the right-hand side of Eq.(2.21) can be rewritten as

$$\frac{\partial [H]}{\partial \eta^*} = \sum_{\mu i}\{[\frac{\partial C_{\mu i}}{\partial \eta^*}][\frac{\partial H}{\partial C_{\mu i}}]+[\frac{\partial C_{\mu i}^*}{\partial \eta^*}][\frac{\partial H}{\partial C_{\mu i}^*}]\}$$

$$= i\dot{\eta}\sum_{\mu i}\{[\frac{\partial C_{\mu i}^*}{\partial \eta^*}][\frac{\partial C_{\mu i}}{\partial \eta}]-[\frac{\partial C_{\mu i}^*}{\partial \eta}][\frac{\partial C_{\mu i}}{\partial \eta^*}]\} , \text{ and c.c. .} \quad (3.24)$$

Equation (3.24) can be reduced to Eq.(2.21) under the following condition,

$$\sum_{\mu i}\{[\frac{\partial C_{\mu i}^*}{\partial \eta^*}][\frac{\partial C_{\mu i}}{\partial \eta}]-[\frac{\partial C_{\mu i}^*}{\partial \eta}][\frac{\partial C_{\mu i}}{\partial \eta^*}]\} = 1 , \quad (3.25)$$

which corresponds to the zeroth-order term of Eq. (3.18a). With the aid of Eq. (3.23), the left-hand side of Eq. (2.20) can also be expressed as

$$
\begin{aligned}
[\frac{\partial H}{\partial \xi_\alpha}] &= \sum_{\mu i} \{ [\frac{\partial C_{\mu i}}{\partial \xi_\alpha}] [\frac{\partial H}{\partial C_{\mu i}}] + [\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}] [\frac{\partial H}{\partial C_{\mu i}^*}] \} \\
&= i\dot{\eta} \sum_{\mu i} \{ [\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}] [\frac{\partial C_{\mu i}}{\partial \eta}] - [\frac{\partial C_{\mu i}^*}{\partial \eta}] [\frac{\partial C_{\mu i}}{\partial \xi_\alpha}] \} \\
&+ i\dot{\eta}^* \sum_{\mu i} \{ [\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}] [\frac{\partial C_{\mu i}}{\partial \eta^*}] - [\frac{\partial C_{\mu i}^*}{\partial \eta^*}] [\frac{\partial C_{\mu i}}{\partial \xi_\alpha}] \} \quad . \qquad (3.26)
\end{aligned}
$$

It is thus clear from the expression (3.26) that Eq. (3.22) can be reduced to the maximal-decoupling condition (2.20) under the following condition,

$$
\begin{aligned}
&\sum_{\mu i} \{ [\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}] [\frac{\partial C_{\mu i}}{\partial \eta}] - [\frac{\partial C_{\mu i}^*}{\partial \eta}] [\frac{\partial C_{\mu i}}{\partial \xi_\alpha}] \} = 0 \ , \\
&\sum_{\mu i} \{ [\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}] [\frac{\partial C_{\mu i}}{\partial \eta^*}] - [\frac{\partial C_{\mu i}^*}{\partial \eta^*}] [\frac{\partial C_{\mu i}}{\partial \xi_\alpha}] \} = 0 \ , \qquad (3.27)
\end{aligned}
$$

which corresponds to the zeroth-order term of Eq. (3.18b).

In the SCC method, the collective variables (η, η^*) which satisfy the condition (3.25) have been chosen through

$$
\begin{aligned}
&\sum_{\mu i} \{ [C_{\mu i}^*] [\frac{\partial C_{\mu i}}{\partial \eta}] - [\frac{\partial C_{\mu i}^*}{\partial \eta}] [C_{\mu i}] \} = \eta^* \ , \\
&\sum_{\mu i} \{ [\frac{\partial C_{\mu i}^*}{\partial \eta^*}] [C_{\mu i}] - [C_{\mu i}^*] [\frac{\partial C_{\mu i}}{\partial \eta^*}] \} = \eta \ , \qquad (3.28)
\end{aligned}
$$

which is equivalent to Eq. (2.25). With the use of the canonical equations of collective motion (2.21), on the other hand, Eq. (3.23) is rewritten as

$$
[\frac{\partial H}{\partial C_{\mu i}^*}] - \frac{\partial [H]}{\partial \eta^*} [\frac{\partial C_{\mu i}}{\partial \eta}] + \frac{\partial [H]}{\partial \eta} [\frac{\partial C_{\mu i}}{\partial \eta^*}] = 0 \ ,
$$

$$\left[\frac{\partial H}{\partial C_{\mu i}}\right] - \frac{\partial [H]}{\partial \eta}\left[\frac{\partial C_{\mu i}^*}{\partial \eta^*}\right] + \frac{\partial [H]}{\partial \eta^*}\left[\frac{\partial C_{\mu i}^*}{\partial \eta}\right] = 0 \ . \tag{3.29}$$

Both Eqs.(3.28) and (3.29), which are the basic equations of the SCC method, play a role to define the collective submanifold Σ^2 in the TDHF manifold M^{2MN}, while Eq.(2.21) describes a time-evolution of the collective motion on the two-dimensional symplectic manifold M^2: $\{\eta,\eta^*\}$.[5),18)] (See Fig. 1.)

3.3. *Local Canonical-Variable Approximation for Non-Collective Modes of Motion*

Next, we make clear the consistency conditions under which Eq.(3.22) reduces to Eq.(2.22). The consistency conditions are obtained in the following way. Eq.(3.22) can be rewritten as

$$i\sum_\alpha \{\dot{\xi}_\alpha\left[\frac{\partial C_{\mu i}}{\partial \xi_\alpha}\right] + \dot{\xi}_\alpha^*\left[\frac{\partial C_{\mu i}}{\partial \xi_\alpha^*}\right]\} + i\,(\dot{\eta}\frac{\partial}{\partial \eta} + \dot{\eta}^*\frac{\partial}{\partial \eta^*})\sum_\alpha \{\xi_\alpha\left[\frac{\partial C_{\mu i}}{\partial \xi_\alpha}\right] + \xi_\alpha^*\left[\frac{\partial C_{\mu i}}{\partial \xi_\alpha^*}\right]\}$$

$$= \sum_\alpha \{\xi_\alpha\left[\frac{\partial^2 H}{\partial \xi_\alpha \partial C_{\mu i}^*}\right] + \xi_\alpha^*\left[\frac{\partial^2 H}{\partial \xi_\alpha^* \partial C_{\mu i}^*}\right]\} \ . \tag{3.30}$$

Multiplying Eq.(3.30) by $[\partial C_{\mu i}^*/\partial \xi_\beta^*]$ and its complex conjugate by $[\partial C_{\mu i}/\partial \xi_\beta^*]$, we obtain a relation

$$i\sum_\alpha \dot{\xi}_\alpha \sum_{\mu i}\{\left[\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}\right]\left[\frac{\partial C_{\mu i}}{\partial \xi_\alpha}\right] - \left[\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}\right]\left[\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}\right]\}$$

$$+ i\sum_\alpha \dot{\xi}_\alpha^* \sum_{\mu i}\{\left[\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}\right]\left[\frac{\partial C_{\mu i}}{\partial \xi_\alpha^*}\right] - \left[\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha^*}\right]\left[\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}\right]\}$$

$$= \sum_\alpha \xi_\alpha \sum_{\mu i}\{\left[\frac{\partial^2 H}{\partial \xi_\alpha \partial C_{\mu i}^*}\right]\left[\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}\right] + \left[\frac{\partial^2 H}{\partial \xi_\alpha \partial C_{\mu i}}\right]\left[\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}\right]\}$$

$$+ \sum_\alpha \xi_\alpha^* \sum_{\mu i}\{\left[\frac{\partial^2 H}{\partial \xi_\alpha^* \partial C_{\mu i}^*}\right]\left[\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}\right] + \left[\frac{\partial^2 H}{\partial \xi_\alpha^* \partial C_{\mu i}}\right]\left[\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}\right]\}$$

$$+i\sum_{\alpha}\xi_\alpha \sum_{\mu i}\{ [\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}] \cdot \frac{d}{dt}[\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}] - [\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}] \cdot \frac{d}{dt}[\frac{\partial C_{\mu i}}{\partial \xi_\alpha}] \}$$

$$+i\sum_{\alpha}\xi_\alpha^* \sum_{\mu i}\{ [\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}] \cdot \frac{d}{dt}[\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha^*}] - [\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}] \cdot \frac{d}{dt}[\frac{\partial C_{\mu i}}{\partial \xi_\alpha^*}] \} \qquad (3.31)$$

In order to obtain an explicit expression for Eq. (2.22), on the other hand, we use the relations such as

$$[\frac{\partial^2 H}{\partial \xi_\alpha \partial \xi_\beta^*}] = \sum_{\mu i}\{ [\frac{\partial^2 H}{\partial \xi_\alpha \partial C_{\mu i}^*}] [\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}] + [\frac{\partial^2 H}{\partial \xi_\alpha \partial C_{\mu i}}] [\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}]$$

$$+ [\frac{\partial H}{\partial C_{\mu i}^*}] [\frac{\partial^2 C_{\mu i}^*}{\partial \xi_\alpha \partial \xi_\beta^*}] + [\frac{\partial H}{\partial C_{\mu i}}] [\frac{\partial^2 C_{\mu i}}{\partial \xi_\alpha \partial \xi_\beta^*}] \} \qquad (3.32)$$

The third and fourth terms on the right-hand side of Eq. (3.32) are rewritten, with the aid of Eq. (3.23), as

$$\sum_{\mu i}\{ [\frac{\partial H}{\partial C_{\mu i}^*}] [\frac{\partial^2 C_{\mu i}^*}{\partial \xi_\alpha \partial \xi_\beta^*}] + [\frac{\partial H}{\partial C_{\mu i}}] [\frac{\partial^2 C_{\mu i}}{\partial \xi_\alpha \partial \xi_\beta^*}] \}$$

$$= i\dot{\eta}\sum_{\mu i}\{ [\frac{\partial C_{\mu i}}{\partial \eta}] [\frac{\partial^2 C_{\mu i}^*}{\partial \xi_\alpha \partial \xi_\beta^*}] - [\frac{\partial C_{\mu i}^*}{\partial \eta}] [\frac{\partial^2 C_{\mu i}}{\partial \xi_\alpha \partial \xi_\beta^*}] \}$$

$$+ i\dot{\eta}^*\sum_{\mu i}\{ [\frac{\partial C_{\mu i}}{\partial \eta^*}] [\frac{\partial^2 C_{\mu i}^*}{\partial \xi_\alpha \partial \xi_\beta^*}] - [\frac{\partial C_{\mu i}^*}{\partial \eta^*}] [\frac{\partial^2 C_{\mu i}}{\partial \xi_\alpha \partial \xi_\beta^*}] \} \qquad (3.33)$$

The first term on the right-hand side of Eq. (3.33) can be expressed as

$$i\dot{\eta}\sum_{\mu i}\{ [\frac{\partial C_{\mu i}}{\partial \eta}] [\frac{\partial^2 C_{\mu i}^*}{\partial \xi_\alpha \partial \xi_\beta^*}] - [\frac{\partial C_{\mu i}^*}{\partial \eta}] [\frac{\partial^2 C_{\mu i}}{\partial \xi_\alpha \partial \xi_\beta^*}] \}$$

$$= i\dot{\eta}\sum_{\mu i}\{ [\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}] \cdot \frac{\partial}{\partial \eta}[\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}] - [\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}] \cdot \frac{\partial}{\partial \eta}[\frac{\partial C_{\mu i}}{\partial \xi_\alpha}]$$

$$+ [\frac{\partial}{\partial \xi_\alpha}(\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}\frac{\partial C_{\mu i}}{\partial \eta} - \frac{\partial C_{\mu i}^*}{\partial \eta}\frac{\partial C_{\mu i}}{\partial \xi_\beta^*})] \} \quad . \qquad (3.34)$$

In such a way, we finally obtain the following expression for Eq. (2.22),

$$
\begin{aligned}
i\dot{\xi}_\beta &= \sum_\alpha \{ \xi_\alpha [\frac{\partial^2 H}{\partial \xi_\alpha \partial \xi_\beta^*}] + \xi_\alpha^* [\frac{\partial^2 H}{\partial \xi_\alpha^* \partial \xi_\beta^*}] \} \\
&= \sum_\alpha \xi_\alpha \sum_{\mu i} \{ [\frac{\partial^2 H}{\partial \xi_\alpha \partial C_{\mu i}^*}] [\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}] + [\frac{\partial^2 H}{\partial \xi_\alpha \partial C_{\mu i}}] [\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}] \} \\
&+ \sum_\alpha \xi_\alpha^* \sum_{\mu i} \{ [\frac{\partial^2 H}{\partial \xi_\alpha^* \partial C_{\mu i}^*}] [\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}] + [\frac{\partial^2 H}{\partial \xi_\alpha^* \partial C_{\mu i}}] [\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}] \} \\
&+ i \sum_\alpha \xi_\alpha \sum_{\mu i} \{ [\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}] \cdot \frac{d}{dt} [\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}] - [\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}] \cdot \frac{d}{dt} [\frac{\partial C_{\mu i}}{\partial \xi_\alpha}] \} \\
&+ i \sum_\alpha \xi_\alpha^* \sum_{\mu i} \{ [\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}] \cdot \frac{d}{dt} [\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha^*}] - [\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}] \cdot \frac{d}{dt} [\frac{\partial C_{\mu i}}{\partial \xi_\alpha^*}] \} \\
&+ i\dot{\eta} \sum_\alpha \sum_{\mu i} \{ \xi_\alpha [\frac{\partial}{\partial \xi_\alpha} (\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*} \frac{\partial C_{\mu i}}{\partial \eta} - \frac{\partial C_{\mu i}^*}{\partial \eta} \frac{\partial C_{\mu i}}{\partial \xi_\beta^*})] \\
&\qquad + \xi_\alpha^* [\frac{\partial}{\partial \xi_\alpha^*} (\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*} \frac{\partial C_{\mu i}}{\partial \eta} - \frac{\partial C_{\mu i}^*}{\partial \eta} \frac{\partial C_{\mu i}}{\partial \xi_\beta^*})] \} \\
&+ i\dot{\eta}^* \sum_\alpha \sum_{\mu i} \{ \xi_\alpha [\frac{\partial}{\partial \xi_\alpha} (\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*} \frac{\partial C_{\mu i}}{\partial \eta^*} - \frac{\partial C_{\mu i}^*}{\partial \eta^*} \frac{\partial C_{\mu i}}{\partial \xi_\beta^*})] \\
&\qquad + \xi_\alpha^* [\frac{\partial}{\partial \xi_\alpha^*} (\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*} \frac{\partial C_{\mu i}}{\partial \eta^*} - \frac{\partial C_{\mu i}^*}{\partial \eta^*} \frac{\partial C_{\mu i}}{\partial \xi_\beta^*})] \} \quad . \qquad (3.35)
\end{aligned}
$$

By comparing Eq.(3.31) with Eq.(3.35), it can be shown that the first-order canonical equations of motion (3.22) can be reduced to the canonical equations of non-collective motion (2.22) under the following conditions,

$$
\begin{aligned}
&\sum_{\mu i} \{ [\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha^*}] [\frac{\partial C_{\mu i}}{\partial \xi_\beta}] - [\frac{\partial C_{\mu i}^*}{\partial \xi_\beta}] [\frac{\partial C_{\mu i}}{\partial \xi_\alpha^*}] \} = \delta_{\alpha\beta} \; , \\
&\sum_{\mu i} \{ [\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}] [\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}] - [\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}] [\frac{\partial C_{\mu i}}{\partial \xi_\alpha^*}] \} = 0 \; , \qquad (3.36)
\end{aligned}
$$

and

$$[\frac{\partial}{\partial \xi_\alpha}(\frac{\partial C^*_{\mu i}}{\partial \xi^*_\beta}\frac{\partial C_{\mu i}}{\partial \eta}-\frac{\partial C^*_{\mu i}}{\partial \eta}\frac{\partial C_{\mu i}}{\partial \xi^*_\beta})] = 0 \ ,$$

$$[\frac{\partial}{\partial \xi_\alpha}(\frac{\partial C^*_{\mu i}}{\partial \xi^*_\beta}\frac{\partial C_{\mu i}}{\partial \eta^*}-\frac{\partial C^*_{\mu i}}{\partial \eta^*}\frac{\partial C_{\mu i}}{\partial \xi^*_\beta})] = 0 \ , \qquad \text{etc.} \ , \qquad (3.37)$$

where Eq. (3.36) just corresponds to the zeroth-order term of Eq. (3.18c) and Eq. (3.37) to the first-order term of Eq. (3.18b) in our expansion. Under the conditions (3.36) and (3.37), both Eqs. (3.22) and (2.22) are reduced to the equation given by

$$\begin{aligned} i\dot{\xi}_\beta = & \sum_\alpha \xi_\alpha \sum_{\mu i}\{[\frac{\partial^2 H}{\partial \xi_\alpha \partial C^*_{\mu i}}][\frac{\partial C^*_{\mu i}}{\partial \xi^*_\beta}] + [\frac{\partial^2 H}{\partial \xi_\alpha \partial C_{\mu i}}][\frac{\partial C_{\mu i}}{\partial \xi^*_\beta}]\} \\ & + \sum_\alpha \xi^*_\alpha \sum_{\mu i}\{[\frac{\partial^2 H}{\partial \xi^*_\alpha \partial C^*_{\mu i}}][\frac{\partial C^*_{\mu i}}{\partial \xi^*_\beta}] + [\frac{\partial^2 H}{\partial \xi^*_\alpha \partial C_{\mu i}}][\frac{\partial C_{\mu i}}{\partial \xi^*_\beta}]\} \\ & + i\sum_\alpha \xi_\alpha \sum_{\mu i}\{[\frac{\partial C_{\mu i}}{\partial \xi^*_\beta}]\cdot\frac{d}{dt}[\frac{\partial C^*_{\mu i}}{\partial \xi_\alpha}] - [\frac{\partial C^*_{\mu i}}{\partial \xi^*_\beta}]\cdot\frac{d}{dt}[\frac{\partial C_{\mu i}}{\partial \xi_\alpha}]\} \\ & + i\sum_\alpha \xi^*_\alpha \sum_{\mu i}\{[\frac{\partial C_{\mu i}}{\partial \xi^*_\beta}]\cdot\frac{d}{dt}[\frac{\partial C^*_{\mu i}}{\partial \xi^*_\alpha}] - [\frac{\partial C^*_{\mu i}}{\partial \xi^*_\beta}]\cdot\frac{d}{dt}[\frac{\partial C_{\mu i}}{\partial \xi^*_\alpha}]\} \ . \end{aligned} \qquad (3.38)$$

Equation (3.37), which just corresponds to Eq. (2.41), is not necessary for our expansion, but may be necessary in treating the higher-order derivative terms like $[\partial^2 C_{\mu i}/\partial \xi_\alpha \partial \xi^*_\beta]$.

Here we further assume the normal-mode condition (2.42) for the non-collective variables $(\xi_\alpha, \xi^*_\alpha)$. By substituting Eq. (2.42) for Eq. (3.38), we obtain

$$\sum_{\mu i}\{[\frac{\partial^2 H}{\partial \xi_\alpha \partial C^*_{\mu i}}][\frac{\partial C^*_{\mu i}}{\partial \xi^*_\beta}] + [\frac{\partial^2 H}{\partial \xi_\alpha \partial C_{\mu i}}][\frac{\partial C_{\mu i}}{\partial \xi^*_\beta}]\} = \omega_\alpha \delta_{\alpha\beta} \ ,$$

$$\sum_{\mu i}\{[\frac{\partial^2 H}{\partial \xi^*_\alpha \partial C^*_{\mu i}}][\frac{\partial C^*_{\mu i}}{\partial \xi^*_\beta}] + [\frac{\partial^2 H}{\partial \xi^*_\alpha \partial C_{\mu i}}][\frac{\partial C_{\mu i}}{\partial \xi^*_\beta}]\} = 0 \ , \qquad (3.39)$$

under the condition

$$\sum_{\mu i}\{[\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}]\cdot\frac{d}{dt}[\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha}]-[\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}]\cdot\frac{d}{dt}[\frac{\partial C_{\mu i}}{\partial \xi_\alpha}]\} = 0 ,$$

$$\sum_{\mu i}\{[\frac{\partial C_{\mu i}}{\partial \xi_\beta^*}]\cdot\frac{d}{dt}[\frac{\partial C_{\mu i}^*}{\partial \xi_\alpha^*}]-[\frac{\partial C_{\mu i}^*}{\partial \xi_\beta^*}]\cdot\frac{d}{dt}[\frac{\partial C_{\mu i}}{\partial \xi_\alpha^*}]\} = 0 . \tag{3.40}$$

With the aid of Eq.(3.36), Eq.(3.39) can be rewritten as the RPA-type equation

$$\sum_{\nu j}\begin{bmatrix} [\partial^2 H/\partial C_{\mu i}^*\partial C_{\nu j}] & [\partial^2 H/\partial C_{\mu i}^*\partial C_{\nu j}^*] \\ -[\partial^2 H/\partial C_{\mu i}\partial C_{\nu j}] & -[\partial^2 H/\partial C_{\mu i}\partial C_{\nu j}^*] \end{bmatrix}\begin{bmatrix} [\partial C_{\nu j}/\partial \xi_\alpha] \\ [\partial C_{\nu j}^*/\partial \xi_\alpha] \end{bmatrix}$$

$$= \omega_\alpha\begin{bmatrix} [\partial C_{\mu i}/\partial \xi_\alpha] \\ [\partial C_{\mu i}^*/\partial \xi_\alpha] \end{bmatrix} . \tag{3.41}$$

Needless to say, Eq.(3.41) guarantees Eq.(2.42a) under the condition (3.40). By solving Eq.(3.41) with the conditions (3.27) and (3.36) under the condition (3.40), thus, we can obtain the intrinsic excitation energies ω_α and the vectors $([\partial C_{\mu i}/\partial \xi_\alpha], [\partial C_{\mu i}/\partial \xi_\alpha^*])$ of the TDHF manifold M^{2MN}, which specify directions of the non-collective modes of motion described by $(\xi_\alpha, \xi_\alpha^*)$.

The condition (3.40) is supposed to be an important condition for characterizing the local non-collective normal modes $(\xi_\alpha, \xi_\alpha^*)$ with the small-amplitude assumption (2.42b), in a consistent manner with the large-amplitude collective motion described by (η, η^*). Also the condition (3.27) demands that the non-collective directions should be orthogonal to the collective direction $([\partial C_{\mu i}/\partial \eta], [\partial C_{\mu i}^*/\partial \eta])$, which is defined by a tangential vector at each point on the submanifold Σ^2. Since the non-collective directions $([\partial C_{\mu i}/\partial \xi_\alpha], [\partial C_{\mu i}^*/\partial \xi_\alpha])$ are determined by the local RPA equation (3.41), the condition (3.27) should be regarded as a local condition on Σ^2. This fact implies that the concept of non-collective modes of motion can be considered as a local concept valid only in the neighborhood of the respective point on Σ^2.

It is now clear that a set of basic equations, which specifies the global collective mode (η,η^*) and the local canonical-variable approximation for the non-collective modes $(\xi_\alpha,\xi_\alpha^*)$, consists of Eqs. (3.29) and (3.41) and the conditions (3.28), (3.36), (3.39) and (3.40).[13]

§ 4. Validity of Concept of Dynamical Collective Submanifold

In the previous two sections, we have treated the collective variables (η, η^*) as the global canonical-variables and the non-collective variables $(\xi_\alpha, \xi_\alpha^*)$ as the local canonical-variables. It has been clarified that the collective mode of motion $(\hat{X}_{coll}, \hat{X}_{coll}^\dagger)$ and the RPA-type non-collective modes of motion $(\hat{X}_\alpha, \hat{X}_\alpha^\dagger)$ are determined by Eqs. (2.27) and (2.63) with the conditions (2.24), (2.25) and (2.40), and the separability condition (2.44) between the collective and non-collective modes of motion plays an important role for validity of such a treatment. In this section, we discuss the physical meaning of both the separability condition (2.44) (or (3.40)) between the collective and non-collective modes of motion and the stability condition of the collective submanifold, which depends on whether the intrinsic excitation energies ω_α are real or imaginary.[11),13)]

4.1. *Stability of Collective Submanifold and Approximate Integral Surface*

It is well known that the stability of a Hartree-Fock state is determined by whether RPA eigenvalues are real or imaginary.[19),20),4)] In the same way, we determine the stability of the collective submanifold Σ^2 according as whether the intrinsic excitation energies ω_α are real or imaginary. In order to explicitly obtain the intrinsic excitation energies ω_α and the non-collective modes of motion, one has to solve the RPA equation (2.63) (or the RPA equation (3.41) under the constrained condition (3.27)). The condition (2.40) (or (3.36)) thus serves to orthonormalize the non-collective modes of motion. Equation (2.63) shows that the intrinsic excitation energies ω_α depend on the local point (η, η^*) of the submanifold Σ^2. Consequently, the orthonormalized condition (2.40) should also be considered as a local condition like the condition (2.24) (or (3.27)). We may therefore say that the collective submanifold Σ^2 which satisfies the maximal-decoupling condition is stable provided that all of the RPA equation (2.63) have real eigenvalues at each point on it, i.e.,

$$(\omega_\alpha(\eta,\eta^*))^2 > 0 \ , \qquad\qquad \text{for } {}^\forall\alpha \ . \tag{4.1}$$

If the condition (4.1) is satisfied and the intrinsic excitation energies ω_α take relatively large values, the restoring forces toward the non-collective directions perpendicular to the collective submanifold Σ^2 become large. Consequently, the group of TDHF trajectories in the small domain of the neighborhood of Σ^2 is always confined in the small domain due to the large restoring forces. In this case, a large amount of TDHF trajectories are expected to accumulate around Σ^2 and the collectivity of Σ^2 increases to a fairly large extent. It is thus reasonable to introduce the concept of an approximate integral surface for Σ^2.

If the condition (4.1) is satisfied but the intrinsic excitation energies ω_α are relatively small, the TDHF trajectories starting from the small domain at an initial instant can travel toward the non-collective directions with a fairly large amount of deviations from Σ^2 because of the small restoring forces. In this case, we cannot expect a large accumulation of the TDHF trajectories around Σ^2. Furthermore, the small-amplitude assumption in Eq.(2.42) for the non-collective degrees of freedom is no more valid, and the extraction of the collective submanifold by the SCC method does not have a definite sense in comparison with the case of large $\omega(\eta,\eta^*)$. However, it has a sense that time-averaged property or phase-space averaged property of the group of TDHF trajectories is represented on the submanifold Σ^2.

If the condition (4.1) is not fulfilled, there does not exist any restoring forces toward the non-collective directions, and the TDHF trajectories starting from the small domain in the neighborhood of Σ^2 at an initial instant prefer to escape the small domain, having lost their initial focussing for a long enough time. In this case, the SCC trajectory is running on an unstable ridge-line and we cannot expect any other trajectories accumulating to the submanifold Σ^2.

4.2. *Separability Condition between Collective and Non-collective Modes of Motion*

Next we discuss the physical meaning of the condition (2.44) (or (3.40)). In the SCC method, we have determined the diffeomorphic mapping $M^2 \rightarrow \Sigma^2$ and the collective Hamiltonian by freezing the non-collective degrees of freedom. Except for the ideal case where Σ^2 is identified with the exact invariant surface, the collective and non-collective modes of motion are not independent of each other and the concept of the intrinsic motion decoupled from the collective one does not strictly hold.

The quantities $(\partial\hat{X}^\dagger_\alpha/\partial\eta, \partial\hat{X}^\dagger_\alpha/\partial\eta^*)$, which appeared in the condition (2.44), express non-locality of the non-collective modes of motion $(\hat{X}_\alpha, \hat{X}^\dagger_\alpha)$ on the submanifold Σ^2. In the strict sense, the condition (2.44) requires that non-local dependence of the non-collective modes of motion, e.g. $\partial\hat{X}^\dagger_\alpha/\partial\eta$, should be zero. This non-locality is a manifestation of the coupling between the collective and non-collective degrees of freedom. The large non-local effect does not allow us to derive the RPA equation (2.43) (or (3.41)). If there holds the condition (2.44) which is essential to introduce the local concept for non-collective degrees of freedom, we can get the local RPA equation (2.43) which determines the local intrinsic excitation modes compatible with the global collective motion governed by Eq.(2.21). Namely, we can regard the non-collective modes of motion as the local intrinsic excitation modes under the condition (2.44).

From the above discussion, it is clear that the local intrinsic excitation modes are definable as long as the condition (2.44) is satisfied. In actual case, however, the condition (2.44) is too stringent to hold. If the non-local effects are of less importance than the local effects, the concept of the local intrinsic excitation modes may still be justified. It is thus reasonable to apply the following condition in place of (2.44) or (3.40),

$$|\langle\phi_0|\,[\hat{X}_\beta, i(\dot{\eta}\frac{\partial}{\partial\eta}+\dot{\eta}^*\frac{\partial}{\partial\eta^*})\hat{X}^\dagger_\alpha]\,|\phi_0\rangle| \ll |\omega_\alpha| \ ,$$

$$|\langle\phi_0|\,[\hat{X}_\beta, i(\dot{\eta}\frac{\partial}{\partial\eta}+\dot{\eta}^*\frac{\partial}{\partial\eta^*})\hat{X}_\alpha]\,|\phi_0\rangle| \ll |\omega_\alpha| \ , \tag{4.2}$$

or

$$|\sum_{\mu i}\{(\frac{\partial C_{\mu i}}{\partial\xi^*_\beta})\cdot\frac{d}{dt}(\frac{\partial C^*_{\mu i}}{\partial\xi_\alpha})-(\frac{\partial C^*_{\mu i}}{\partial\xi^*_\beta})\cdot\frac{d}{dt}(\frac{\partial C_{\mu i}}{\partial\xi_\alpha})\}| \ll |\omega_\alpha| \ ,$$

$$|\sum_{\mu i}\{(\frac{\partial C_{\mu i}}{\partial\xi^*_\beta})\cdot\frac{d}{dt}(\frac{\partial C^*_{\mu i}}{\partial\xi^*_\alpha})-(\frac{\partial C^*_{\mu i}}{\partial\xi^*_\beta})\cdot\frac{d}{dt}(\frac{\partial C_{\mu i}}{\partial\xi^*_\alpha})\}| \ll |\omega_\alpha| \ . \tag{4.3}$$

The condition (4.2) or (4.3) can then be regarded as an approximate separability condition for dividing the collective and non-collective modes of motion.[10)~12)] If the condition (4.2) is not satisfied, one can no longer freeze the corresponding non-collective degrees of freedom, and cannot describe the trajectory under consideration solely by a single pair of collective variables (η, η^*).

4.3. *Collective, Dissipative and Stochastic Behaviors of TDHF Trajectories*

In the previous two subsections, we have discussed various properties of the TDHF manifold M^{2MN} in proximity to the collective submanifold Σ^2 defined by the SCC method with the aid of the stability and separability conditions. By means of these conditions, the optimum collective submanifold Σ^2 may be characterized by the following three regions,

$$\begin{aligned} &\text{Region I} \ ; \ \omega_\alpha^2(\eta,\eta^*) > 0 && \text{and} \quad 0 \leqq I_\alpha^\beta(\eta,\eta^*) < 1 \ , \\ &\text{Region II} \ ; \ \omega_\alpha^2(\eta,\eta^*) > 0 && \text{and} \quad 1 \leqq I_\alpha^\beta(\eta,\eta^*) \ , \\ &\text{Region III} \ ; \ \omega_\alpha^2(\eta,\eta^*) \leqq 0 \ , && \end{aligned} \tag{4.4}$$

where the non-local effect $I_\alpha^\beta(\eta,\eta^*)$ is defined by

$$I_\alpha^\beta(\eta,\eta^*) \equiv \left| <\phi_0 | \, [\hat{X}_\beta, i(\dot{\eta}\frac{\partial}{\partial\eta}+\dot{\eta}^*\frac{\partial}{\partial\eta^*})\hat{X}_\alpha^\dagger] \, | \phi_0>/\omega_\alpha \right| \ . \tag{4.5}$$

According to the discussion in the previous two subsections, the distinctive feature of the TDHF manifold M^{2MN} in proximity to Σ^2 may be represented by three different characteristic trajectories illustrated in Fig. 2.[11] In Region I, the trajectory starting from a neighborhood of the submanifold Σ^2 may be bound in close proximity to it. We may thus expect that the small domain in the neighborhood of Σ^2 consists of only approximate invariant tori like the KAM tori.[21] In Region II, the trajectory found near Σ^2 at a certain instant may oscillate toward the non-collective directions perpendicular to the submanifold Σ^2 with a fairly large amount of deviation from Σ^2. This implies that the collectivity of the system is still expected to survive even there are significant dissipative effects. In Region III, the trajectory occasionally travelling near Σ^2 does not come back in its neighborhood. This implies that the collective motion under consideration has dissappeared and the system shows a stochastic behavior.

§ 5. Application to Three-Level SU(3) Model

5.1. *Model Hamiltonian and the TDHF Theory*

In the preceding section, we have seen that the submanifold Σ^2 extracted by the SCC method is classified into three different physical regions by means of the stability and separability conditions, and the TDHF trajectories in three regions show collective, dissipative and stochastic behaviors, respectively. In order to justify the statement in § 5, in this section, we apply our theory to a modification of the SU(3) model used by Li, Klein and Dreizler.[11),14)]

The Hamiltonian is given by

$$\hat{H} = \varepsilon_0\hat{K}_{00}+\varepsilon_1\hat{K}_{11}+\varepsilon_2\hat{K}_{22}+\frac{V_1}{2}(\hat{K}_{10}\hat{K}_{10}+\text{h.c.})+\frac{V_2}{2}(\hat{K}_{20}\hat{K}_{20}+\text{h.c.}) \quad . \tag{5.1}$$

There are three levels with energies $\varepsilon_0<\varepsilon_1<\varepsilon_2$ and each level has N-fold degeneracy. The fermion pair operators $\hat{K}_{\alpha\beta}$ are defined by

$$\hat{K}_{\alpha\beta} \equiv \sum_{m=1}^{N} C^{\dagger}_{\alpha m}C_{\beta m} \;\; ; \;\; \alpha,\beta = 0,1,2 \;, \tag{5.2}$$

which satisfy the following commutation relations,

$$[\hat{K}_{\alpha\beta},\hat{K}_{\gamma\delta}] = \delta_{\beta\gamma}\hat{K}_{\alpha\delta}-\delta_{\alpha\delta}\hat{K}_{\gamma\beta} \quad . \tag{5.3}$$

We will hereafter consider a system with N particles and the lowest-energy state $|\phi_0\rangle$ without interaction i.e., V_1=V_2=0 is given by

$$|\phi_0\rangle = \prod_{m=1}^{N} c^{\dagger}_{0m}|0\rangle \quad , \tag{5.4}$$

where $|0\rangle$ denotes the vacuum of the fermion operators $c^{\dagger}_{\alpha m}$ and $c_{\alpha m}$. In this case we have two types of particle-hole creation operators $(\hat{K}_{10},\hat{K}_{20})$. (See Fig. 3.)

Now, we apply the TDHF theory to the present system. The TDHF single Slater determinant in this case is

$$|\phi(t)\rangle = e^{i\hat{F}}|\phi_0\rangle \ ; \qquad \hat{F} = \{F_1(t)\hat{K}_{10}+F_2(t)\hat{K}_{20}\} + \text{h.c.} \ , \tag{5.5}$$

where parameters F_1 and F_2 are complex and time-dependent. Instead of the parameters F_1 and F_2, we will use the canonical variables $(C_1, C_1^*; C_2, C_2^*)$ which are related to the original parameters through[22)]

$$F_k = \frac{-iC_k}{\sqrt{C_1^*C_1+C_2^*C_2}}\sin^{-1}\sqrt{(C_1^*C_1+C_2^*C_2)/N} \ , \tag{5.6a}$$

i.e.,

$$C_k = \frac{i\sqrt{N}F_k}{\sqrt{F_1^*F_1+F_2^*F_2}}\sin^{-1}\sqrt{F_1^*F_1+F_2^*F_2} \ ; \qquad k = 1,2 \ . \tag{5.6b}$$

Expression (5.6) guarantees that the local infinitesimal generators, defined by

$$\hat{O}_k^\dagger \equiv e^{-i\hat{F}}\frac{\partial}{\partial C_k}e^{i\hat{F}} \ , \qquad \hat{O}_k \equiv -e^{-i\hat{F}}\frac{\partial}{\partial C_k^*}e^{i\hat{F}} \ , \tag{5.7}$$

satisfy the canonical-variable condition

$$\langle\phi_0|\hat{O}_k^\dagger|\phi_0\rangle = \frac{1}{2}C_k^* \ , \qquad \langle\phi_0|\hat{O}_k|\phi_0\rangle = \frac{1}{2}C_k \ . \tag{5.8}$$

The weak boson-like commutation relation

$$\langle\phi_0|[\hat{O}_k,\hat{O}_l^\dagger]|\phi_0\rangle = \delta_{kl} \ , \qquad \langle\phi_0|[\hat{O}_k,\hat{O}_l]|\phi_0\rangle = 0 \tag{5.9}$$

is obtained from the canonical-variable condition (5.8). With the aid of Eq.(5.9), the TDHF equation (2.1), i.e.

$$\delta\langle\phi_0|\{i\sum_{k=1}^{2}(\dot{C}_k\hat{O}_k^\dagger-\dot{C}_k^*\hat{O}_k)-e^{-i\hat{F}}\hat{H}e^{i\hat{F}}\}|\phi_0\rangle = 0 \tag{5.10}$$

is simply reduced to the classical canonical equations of motion

$$i\dot{C}_k = \frac{\partial H}{\partial C_k^*} , \qquad i\dot{C}_k^* = -\frac{\partial H}{\partial C_k} , \tag{5.11}$$

where

$$\begin{aligned} H &= <\phi(t)|\hat{H}|\phi(t)> - <\phi_0|\hat{H}|\phi_0> \\ &= (\varepsilon_1-\varepsilon_0)C_1^*C_1 + (\varepsilon_2-\varepsilon_0)C_2^*C_2 \\ &\quad + \frac{V_1}{2}(N-1)(C_1^*C_1^* + C_1C_1)\{1-(C_1^*C_1+C_2^*C_2)/N\} \\ &\quad + \frac{V_2}{2}(N-1)(C_2^*C_2^* + C_2C_2)\{1-(C_1^*C_1+C_2^*C_2)/N\} \end{aligned} \tag{5.12}$$

Consequently, the TDHF equation (5.10), i.e. Eq.(5.11) determines a trajectory in a four-dimensional TDHF manifold (phase space) M^4 given by

$$\{p_1, q_1, p_2, q_2\} \; ; \qquad p_k \equiv \frac{i}{\sqrt{2}}(C_k^* - C_k) \; , \; q_k \equiv \frac{1}{\sqrt{2}}(C_k^* + C_k) \; . \tag{5.13}$$

5.2. *Application of the Theory*

According to our theory, we should be firstly interested in a certain trajectory which is approximately bound on a two-dimensional submanifold Σ^2 in the four-dimensional TDHF manifold M^4. Namely, we pay attention to the trajectory which is describable by a single pair of collective variables (η, η^*).

Aiming at extracting the submanifold Σ^2, we consider a general variable transformation

$$C_k = C_k(\eta,\eta^*;\xi,\xi^*) \; , \quad C_k^* = C_k^*(\eta,\eta^*;\xi,\xi^*) \; ; \qquad k = 1,2 \; . \tag{5.14}$$

As was discussed in § 2 and § 3, the SCC method enables us to determine the functional forms of $([C_k], [C_k^*]; k=1,2)$, and a set of basic equations of it are given by Eqs.(2.25) and (2.27), i.e.

$$\delta\langle\phi_0|\{e^{-i\hat{G}}\hat{H}e^{i\hat{G}}-\frac{\partial[H]}{\partial\eta^*}\hat{X}_{\rm coll}^\dagger-\frac{\partial[H]}{\partial\eta}\hat{X}_{\rm coll}\}|\phi_0\rangle = 0 \tag{5.15}$$

and

$$\langle\phi_0|\hat{X}_{\rm coll}^\dagger|\phi_0\rangle = \frac{1}{2}\eta^* , \qquad \langle\phi_0|\hat{X}_{\rm coll}|\phi_0\rangle = \frac{1}{2}\eta , \tag{5.16}$$

where the one-body operator $\hat{G}$ is given by

$$\hat{G} = [\hat{F}] = \{[F_1]\hat{K}_{10}+[F_2]\hat{K}_{20}\}+\text{h.c.} \ . \tag{5.17}$$

Solving Eqs.(5.16) and (5.17), namely, it can define a diffeomorphic mapping

$$M^2: \{\eta,\eta^*\} \rightarrow \Sigma^2: \{[C_k], [C_k^*] ;k=1,2\} , \tag{5.18}$$

which specifies the submanifold Σ^2 in the TDHF manifold M^4.

In the general case, we are forced to employ a perturbative treatment for determining functional forms of $([C_k], [C_k^*])$ with respect to (η,η^*). (See Appendix.) But our objective is to discuss the collectivity of the submanifold Σ^2 extracted by the SCC method, so it is preferable to deal with an exact solution of the SCC method. Namely, we will consider a special submanifold Σ^2 whose exact mapping functions are easily obtained analytically.

To this end, we will concentrate ourselves to a trajectory which starts initially from the $\{p_1,q_1\}$-submanifold of the four-dimensional manifold in Eq.(5.13). That is the trajectory whose small-amplitude limit can be described by only p_1 and q_1 degrees of freedom. In order to characterize the corresponding submanifold Σ^2, we choose the following initial boundary condition for $\hat{G}$ in Eq.(5.17),

$$i\hat{G}(\eta,\eta^*) \xrightarrow[(\eta,\eta^*)\to 0]{} \frac{1}{\sqrt{N}}(\eta\hat{K}_{10}-\eta^*\hat{K}_{01}) , \tag{5.19}$$

which adjusts the submanifold Σ^2 in such a way that the trajectory lies on it in its small-amplitude limit. With the boundary condition (5.19), the SCC method leads us to the following result,

$$[C_1] = \eta \ , \quad [C_1^\dagger] = \eta^* \ , \quad [C_2] = [C_2^\dagger] = 0 \ , \tag{5.20a}$$

i.e.,

$$[p_1] = p \ , \quad [q_1] = q \ , \quad [p_2] = [q_2] = 0 \ ;$$

$$p \equiv \frac{i}{\sqrt{2}}(\eta^* - \eta) \ , \quad q \equiv \frac{1}{\sqrt{2}}(\eta^* + \eta) \ , \tag{5.20b}$$

which demonstrates that the collective variables (η, η^*) describing the submanifold Σ^2 are simply related with the variables (p_1, q_1).

The canonical equations of collective motion are given by Eq.(2.21) with

$$[H] = (\varepsilon_1 - \varepsilon_0)\eta^*\eta + \frac{V_1}{2}(N-1)(1-\eta^*\eta/N) \ , \tag{5.21}$$

which is obtained from Eqs.(5.12) and (5.20). The SCC trajectories are embedded into the four-dimensional TDHF manifold M^4: $\{C_k, C_k^*; k=1,2\}$ through the mapping in Eq.(5.18).

With the aid of Eqs.(5.6a) and (5.20), the analytic expression of $\hat{G}(\eta, \eta^*)$ is given by

$$i\hat{G}(\eta, \eta^*) = \frac{1}{\sqrt{\eta^*\eta}}(\eta\hat{K}_{10} - \eta^*\hat{K}_{01})\sin^{-1}\sqrt{\eta^*\eta/N} \ , \tag{5.22}$$

whose lowest-order term with respect to (η, η^*) just coincides with the initial boundary condition (5.19).

By using an explicit expression of $\hat{G}(\eta, \eta^*)$ in Eq.(5.22), we can get an analytic expression of the collective mode of motion as follows;

$$\hat{X}_{\mathrm{col}}{}_1^\dagger = \frac{1}{2\sqrt{N}}\left(\sqrt{1-\eta^*\eta/N} + \frac{1}{\sqrt{1-\eta^*\eta/N}}\right)\hat{K}_{10}$$

$$+ \frac{1}{2\sqrt{N}}\left(\sqrt{1-\eta^*\eta/N} - \frac{1}{\sqrt{1-\eta^*\eta/N}}\right)\frac{\eta^*}{\eta}\hat{K}_{01} - \frac{\eta^*}{2N}(\hat{K}_{11} - \hat{K}_{00}) \ , \tag{5.23}$$

which obviously satisfies Eq.(5.16).

In such a way, we have obtained the collective submanifold Σ^2 analytically in the case of the Hamiltonian given by Eq.(5.1). Next, we must discuss the intrinsic excitation energy and the intrinsic excitation mode within the RPA boson approximation.

In order to obtain the intrinsic Hamiltonian $H_{\rm intr}$ within the RPA boson approximation defined by Eq.(2.59), we introduce the boson operators

$$\hat{K}_{k0} \rightarrow B_k^\dagger \ , \qquad \hat{K}_{0k} \rightarrow B_k \ ; \qquad\qquad k = 1,2 \ , \tag{5.24}$$

which satisfy the boson commutation relation

$$[B_k, B_l^\dagger] = N\delta_{kl} \ , \qquad [B_k, B_l] = 0 \ . \tag{5.25}$$

With the use of the boson operators (5.24), the p-h part of collective mode of motion is represented as

$$X_{\rm coll}^\dagger = \frac{1}{2\sqrt{N}}\left(\sqrt{1-\eta^*\eta/N}+\frac{1}{\sqrt{1-\eta^*\eta/N}}\right)B_1^\dagger$$

$$+\frac{1}{2\sqrt{N}}\left(\sqrt{1-\eta^*\eta/N}-\frac{1}{\sqrt{1-\eta^*\eta/N}}\right)\frac{\eta^*}{\eta}B_1 \ , \tag{5.26}$$

and the Hamiltonian in the generalized moving frame $\hat{H}'$ within the RPA boson approximation (2.52) is given by

$$H' = \frac{1}{N}\{\varepsilon_1-\varepsilon_0-\frac{V_1}{N}(N-1)(\eta^*\eta^*+\eta\eta)(4-\eta^*\eta/N)\}\ B_1^\dagger B_1$$

$$+\frac{1}{N}\{\varepsilon_2-\varepsilon_0-\frac{V_1}{2N}(N-1)(\eta^*\eta^*+\eta\eta)\}\ B_2^\dagger B_2$$

$$+\frac{V_1}{2N}(N-1)\{\ \{(1-\eta^*\eta/N)^2+\frac{\eta\eta\eta\eta}{N^2}\}\ B_1^\dagger B_1^\dagger+{\rm h.c.}\}$$

$$+\frac{V_2}{2N}(N-1)(1-\eta^*\eta/N)\ \{B_2^\dagger B_2^\dagger+{\rm h.c.}\} \ . \tag{5.27}$$

From Eqs. (5.26) and (5.27), we obtain the intrinsic Hamiltonian $H_{\rm intr}$ as follows;

$$H_{\rm intr} = \frac{1}{N}\{\varepsilon_2-\varepsilon_0-\frac{V_1}{2N}(N-1)(\eta^*\eta^*+\eta\eta)\}B_2^\dagger B_2$$

$$+\frac{V_2}{2N}(N-1)(1-\eta^*\eta/N)\{B_2^\dagger B_2^\dagger+{\rm h.c}\} \quad . \tag{5.28}$$

We are now in a position to obtain explicit forms of the intrinsic excitation energy $\omega(\eta,\eta^*)$ and the intrinsic excitation mode $(X_{\rm intr},X_{\rm intr}^\dagger)$. For this aim, the intrinsic excitation mode $(X_{\rm intr},X_{\rm intr}^\dagger)$ is expressed as

$$X_{\rm intr}^\dagger = \psi(\eta,\eta^*)B_2^\dagger-\varphi(\eta,\eta^*)B_2 \ . \tag{5.29}$$

Solving the RPA equation

$$[H_{\rm intr},X_{\rm intr}^\dagger] = \omega X_{\rm intr}^\dagger \tag{5.30}$$

with the normalization condition

$$[X_{\rm intr},X_{\rm intr}^\dagger] = \psi^*(\eta,\eta^*)\psi(\eta,\eta^*)-\varphi^*(\eta,\eta^*)\varphi(\eta,\eta^*) = 1 \ , \tag{5.31}$$

then, we get the intrinsic excitation energy $\omega(\eta,\eta^*)$ and the correlation amplitudes $\psi(\eta,\eta^*)$ and $\varphi(\eta,\eta^*)$ locally at each point of the submanifold Σ^2 as follows;

$$\omega^2(\eta,\eta^*) = \{\varepsilon_2-\varepsilon_0-\frac{V_1}{2N}(N-1)(\eta^*\eta^*+\eta\eta)\}^2-\{V_2(N-1)(1-\eta^*\eta/N)\}^2 \ , \tag{5.32a}$$

$$\psi(\eta,\eta^*) = \sqrt{\frac{\varepsilon_2-\varepsilon_0-\frac{V_1}{2N}(N-1)(\eta^*\eta^*+\eta\eta)+\omega(\eta,\eta^*)}{2N\omega(\eta,\eta^*)}} \ ,$$

$$\varphi(\eta,\eta^*) = \frac{-(N-1)V_2(1-\eta^*\eta/N)}{\sqrt{2N\omega(\eta,\eta^*)\{\varepsilon_2-\varepsilon_0-\frac{V_1}{2N}(N-1)(\eta^*\eta^*+\eta\eta)+\omega(\eta,\eta^*)\}}} \ , \tag{5.32b}$$

Equation (5.32) shows that the microscopic structure of the intrinsic excitation mode varies depending on the position (η,η^*) in the extracted submanifold Σ^2, and the submanifold Σ^2 is divided between the stable region and the unstable region by Eq. (5.32a).

At the end of this subsection, we will get the explicit form of the non-local effect $I(\eta,\eta^*)$ defined by Eq. (4.5). From Eqs. (5.21) and (5.32) we can obtain the explicit form of the non-local effect $I(\eta,\eta^*)$ easily as

$$I(\eta,\eta^*) \equiv |\langle\phi_0|\, [\hat{X}_{\rm intr}, i\frac{d}{dt}\hat{X}_{\rm intr}^\dagger]\, |\phi_0\rangle/\omega(\eta,\eta^*)|$$

$$= \frac{1}{\omega^{5/2}(\eta,\eta^*)} \frac{V_1V_2}{N}(N-1)^2\,\{\varepsilon_1+\varepsilon_2-2\varepsilon_0-\frac{V_1}{N}(N-1)(\eta^*\eta^*+\eta\eta)\}$$

$$\cdot\, |\eta^*\eta^*-\eta\eta|\,(1-\eta^*\eta/N)\ . \qquad (5.33)$$

Equations (5.32a) and (5.33) classify the submanifold Σ^2 into the three different regions, Region I, II and III.

5.3. *Numerical Results*

In the previous subsection, we have obtained the explicit forms of the stability and separability conditions in the case of the three-level SU(3) model. Now, we show a division of the submanifold Σ^2 into the three regions, Region I, II and III concretely, and next we clarify a justification of the discussion in § 4.3, i.e., the collective, dissipative and stochastic behaviors of the TDHF trajectories starting from the neighborhood of Σ^2 in Region I, II and III, respectively.

By solving the canonical equations of motion (2.21) with the collective Hamiltonian (5.21), in Fig. 4, we show various SCC trajectories with different total-energy value E. In this thesis, all the numerical calculations are carried out by using the following parameters; $N=10, \varepsilon_0=-1, \varepsilon_1=0, \varepsilon_2=1, V_1=-1/15$ and $V_2=-1/3$. Also a large unstable region with $\omega^2\leqq 0$ is indicated in Fig. 4.

The result is explained as follows; It is easily seen from the functional form of the intrinsic Hamiltonian (5.28) that the effects of the ground-state correlation for the intrinsic excitation mode become important in the small-amplitude region ($\eta^*\eta \ll N$) of the SCC trajectory and become of less importance in the large-amplitude region ($\eta^*\eta \sim N$) due to the Pauli-blocking effect. Since we are adopting fairly large value for $V_2=-1/3$ in the case, we can get rather large unstable region in the center (,i.e. the small-amplitude region) of {p,q} space.

In Fig. 5, we present a contour map of the non-local effect $I(\eta,\eta^*)$. As is easily seen from Eq.(5.33), the non-local effect I becomes zero at $1/2(p^2+q^2)=\eta^*\eta=N$. The expression of $[H]$ in Eq.(5.21) shows that the curve in the {p,q} space with $I=0$ due to $\eta^*\eta=N$ just coincides with the SCC trajectory with $E=(\varepsilon_1-\varepsilon_0)N=10$. It is also from Eq.(5.33) that the non-local effect I takes zero value at another situation where $\eta^*\eta^*-\eta\eta=-2ipq=0$, i.e. at straight lines with $p=0$ and with $q=0$. Since I contains $\omega(\eta,\eta^*)$ in the denominator, it takes infinity at the boundary curve with $\omega=0$ of the unstable region.

It is easily seen from Figs. 4 and 5 that the SCC trajectory with E=8,5 and 1 lie in Region Ⅰ, Ⅱ and Ⅲ, respectively. Now, we will justify the discussion in § 4.3 by calculating various TDHF trajectory starting from the neighborhood of Σ^2. For this aim, we calculated each trajectories governed by Eq.(5.11) with the following initial condition,

$$q_1 = q = 0\ ,\quad p_1 = p = p_0\ ,\quad q_2 = p_2 = 0.1 \ll p_0\ ,\quad \text{at } t = 0\ , \qquad (5.34)$$

where p_0 is chosen in such a way that the system has a given total energy E. As is seen from Eq.(5.20b), the calculated TDHF trajectory starts from the point which is in close proximity to a point $(q_1=q=0, p_1=p=p_0, q_2=0, p_2=0)$ on the submanifold Σ^2. In order to visualize properties of the trajectory, in Fig. 6, we illustrate the Poincaré-section maps by plotting intersection points of the trajectory on the (p_1, q_1)- and (p_2, q_2)-planes with conditions $(p_2>0, q_2=0)$ and $(p_1>0, q_1=0)$, respectively.[23),24)] For the same purpose, in Fig. 7, we illustrate the trajectory in a three-dimensional coordinate space $\{X, Y, Z\}$ by using double-circles defined as

$$X = (R_1+R_2\cos\theta_2)\cos\theta_1 \ ,$$

$$Y = (R_1+R_2\cos\theta_2)\sin\theta_1 \ ,$$

$$Z = R_2\sin\theta_2 \ , \tag{5.35}$$

where coordinates of two circles (R_1, θ_1) and (R_2, θ_2) are chosen as[25)]

$$R_1 = p_1^2+q_1^2 \ , \qquad \cos\theta_1 = \frac{q_1}{R_1} \ , \qquad \sin\theta_1 = \frac{p_1}{R_1} \ ,$$

$$R_2 = p_2^2+q_2^2 \ , \qquad \cos\theta_2 = \frac{q_2}{R_2} \ , \qquad \sin\theta_2 = \frac{p_2}{R_2} \ . \tag{5.36}$$

(See Fig. 8). As is seen from Eqs.(5.35) and (5.36), in the three-dimensional coordinate space, the SCC trajectory lies on the (X, Y)-plane and shows the same structure as that in the $\{p, q\}$ space in Fig. 4.

In Figs. 6(a) and 7(a), the case with E=8 where the corresponding SCC trajectory lies on Region I is studied. By comparing Fig. 4 with Fig. 6(a), the trajectory starting from $p_2=q_2=0.1$ at t=0 with E=8 shows almost the same geometrical structure as that of the SCC trajectory in the $\{p, q\}$ space. The Poincaré-section map on the (p_2, q_2)-plane indicates that the non-collective mode of motion in the $\{p_2, q_2\}$ space can be successfully described by the small-amplitude oscillation, i.e. the RPA-type equation in Eq.(5.30) around the submanifold Σ^2: $\{p_1=p, q_1=q, p_2=0, q_2=0\}$. In Fig. 7(a), the trajectory with E=8 is illustrated in the $\{X, Y, Z\}$ space. In the case of Region I , the trajectory is essentially expressed by a thin torus because the non-collective degrees of freedom p_2 and q_2 are always taking small values. The effect of the non-collective degrees of freedom plays a role to enlarge the thickness of the torus. According to the numerical results in Figs. 6(a) and 7(a), in Region I the TDHF trajectories travelling near Σ^2 may be expected to remain around Σ^2, not strongly dependent on small changes of the initial condition. Therefore, it is natural to call the Region I a collective region.

In Fig. 6(b), we investigate the trajectory with E=5 where the corresponding SCC trajectory is running in Region II. In this case, the Poincaré-section map of the trajectory on the (p_1, q_1)-plane with p_2=q_2=0.1 at t=0 is very similar to the SCC trajectory in Fig. 4. However, the Poincaré-section map on the (p_2, q_2)-plane shows that the trajectory under consideration intrudes into the $\{p_2, q_2\}$ space cosiderably and the motion in $\{p_2, q_2\}$ space cannot be described by the RPA-type equation around p_2=q_2=0. This situation is also well visualized in Fig. 7(b). In this case, the collectivity of the system in Region II is still expected to survive even there are significant dissipative effects. Therefore, we may call Region II a dissipative region.

In Fig. 6(c), the Poincaré-section map of the trajectory with E=1 is indicated. The corresponding SCC trajectory with E=1 belongs to the case of Region III and travels on the unstable ridge-line of the Hamiltonian. The Poincaré-section map in Fig. 6(c) shows that the trajectory starting from the point, i.e. p_2=q_2=0.1 at t=0 in close proximity to Σ^2 is not confined in the small-amplitude region of $\{p_2, q_2\}$ space but extends to a large-amplitude region ($q_2 \doteqdot 2.7$) with stochastic behavior. In Fig. 7(c), the trajectory is represented in the $\{X, Y, Z\}$ space. As is seen in Fig. 7(c), the trajectory shows a chaotic behavior without confined near the (X, Y)-plane corresponding to the submanifold Σ^2. In the case of Region III, therefore, we can expect neither accumulation of the trajectories nor collectivity. Consequently, Region Ⅲ is naturally called a stochastic region or a complete dissipative region.

§ 6. Conclusion

We have clarified the concept of optimum dynamical collective submanifold within the framework of the TDHF theory. It has been shown that the submanifold has to satisfy the stability condition and the separability condition so as to be really an approximate integral surface. With the aid of the two dynamical conditions, we have also clarified that transition mechanism among collective, dissipative and stochastic motions in large-amplitude motion can be well understood.

Since the theory has shown such an ability in clarifying the microscopic mechanism in nuclear collective dynamics, it will give an interesting subject to apply the theory to the realistic problems,[26] such as large-amplitude collective motion of soft nuclei, high-spin states, heavy-ion reactions, fissions, etc..

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Appendix

Here we give a perturbative treatment of solving the basic equations (2.25) and (2.27) of the SCC method self-consistently with a specific boundary condition appropriate for the collective motion under consideration.[10)]

The basic equations of the SCC method are the invariance principle of the time-dependent Schrödinger equation

$$\delta\langle\phi_0|\{e^{-i\hat{G}}\hat{H}e^{i\hat{G}}-\frac{\partial[H]}{\partial\eta^*}\hat{X}_{coll}^\dagger-\frac{\partial[H]}{\partial\eta}\hat{X}_{coll}\}|\phi_0\rangle=0 \tag{A.1}$$

and the condition

$$\langle\phi_0|\hat{X}_{coll}^\dagger|\phi_0\rangle=\frac{1}{2}\eta^*\ ,\qquad \langle\phi_0|\hat{X}_{coll}|\phi_0\rangle=\frac{1}{2}\eta\ , \tag{A.2}$$

where the one-body operator $\hat{G}(\eta,\eta^*)$ and the collective Hamiltonian $[H]$ are given by

$$\hat{G}(\eta,\eta^*)=\sum_{\mu i}\{[f_{\mu i}]a_\mu^\dagger b_i^\dagger+[f_{\mu i}^*]b_i a_\mu\}\ , \tag{A.3}$$

$$[H]=\langle\phi_0|e^{-i\hat{G}}\hat{H}e^{i\hat{G}}|\phi_0\rangle-\langle\phi_0|\hat{H}|\phi_0\rangle\ . \tag{A.4}$$

In order to choose a solution $\hat{G}(\eta,\eta^*)$ appropriate for the collective motion under consideration, it is rather convenient to use the complete set of the RPA eigenmodes $(\hat{Q}_\lambda,\hat{Q}_\lambda^\dagger)$ instead of the set of the particle-hole pairs $(a_\mu^\dagger b_i^\dagger, b_i a_\mu)$,

$$\hat{Q}_\lambda^\dagger=\sum_{\mu i}\{\psi_\lambda(\mu i)a_\mu^\dagger b_i^\dagger-\varphi_\lambda(\mu i)b_i a_\mu\}\ ,$$

$$\langle\phi_0|[\hat{Q}_\lambda,\hat{Q}_{\lambda'}^\dagger]|\phi_0\rangle=\delta_{\lambda\lambda'}\ ,\qquad \langle\phi_0|[\hat{Q}_\lambda,\hat{Q}_{\lambda'}]|\phi_0\rangle=0\ ,$$

$$\langle\phi_0|[\hat{Q}_\lambda,[\hat{H},\hat{Q}_{\lambda'}^\dagger]]|\phi_0\rangle=\omega_\lambda\delta_{\lambda\lambda'}\ ;\qquad \omega_\lambda>0\ . \tag{A.5}$$

In this case, Eq.(A.1) is written as

$$\langle\phi_0|\,[i\hat{Q}^\dagger_\lambda,\ \{e^{-i\hat{G}}\hat{H}e^{i\hat{G}}-\frac{\partial\,[H]}{\partial\eta^*}\hat{X}_{\mathrm{coll}}^\dagger-\frac{\partial\,[H]}{\partial\eta}\hat{X}_{\mathrm{coll}}\}\,]\,|\phi_0\rangle = 0$$

and h.c. , (A.6)

which can be decomposed into

$$\langle\phi_0|\,[i\hat{Q}^\dagger_\lambda,\ \{e^{-i\hat{G}}\hat{H}e^{i\hat{G}}-\frac{\partial\,[H]}{\partial\eta^*}\hat{X}_{\mathrm{coll}}^\dagger-\frac{\partial\,[H]}{\partial\eta}\hat{X}_{\mathrm{coll}}\}\,]\,|\phi_0\rangle = 0$$

and h.c. , $\hat{Q}^\dagger_\lambda \neq \hat{Q}^\dagger_{\lambda_0}$, (A.7a)

$$\langle\phi_0|\,[i\hat{X}_{\mathrm{coll}}^\dagger,\ \{e^{-i\hat{G}}\hat{H}e^{i\hat{G}}-\frac{\partial\,[H]}{\partial\eta^*}\hat{X}_{\mathrm{coll}}^\dagger-\frac{\partial\,[H]}{\partial\eta}\hat{X}_{\mathrm{coll}}\}\,]\,|\phi_0\rangle = 0$$

and h.c. , (A.7b)

where $\hat{Q}^\dagger_{\lambda_0}$ is the conventional RPA-phonon creation operator with the lowest eigenvalue ω_{λ_0} . Since Eq.(A.7b) with Eq.(A.2) leads to the canonical equations of collective motion (2.21), Eq.(A.1) is finally reduced to Eq.(A.7a).

With the use of the notations

$$\hat{G}(\eta,\eta^*) = \sum_\lambda \{g^{(\lambda)}(\eta,\eta^*)\hat{Q}_\lambda+g^{(\lambda)*}(\eta,\eta^*)\hat{Q}^\dagger_\lambda\} \quad , \tag{A.8}$$

$$\begin{aligned}\hat{X}_{\mathrm{coll}}^\dagger &= e^{-i\hat{G}}\frac{\partial}{\partial\eta}e^{i\hat{G}} \\ &= i\frac{\partial\hat{G}}{\partial\eta}+\frac{1}{2!}[i\frac{\partial\hat{G}}{\partial\eta},i\hat{G}]+\frac{1}{3!}[[i\frac{\partial\hat{G}}{\partial\eta},i\hat{G}],i\hat{G}]+\cdots \\ &= i\frac{\partial\hat{G}}{\partial\eta}+\sum_{n\geq2}\frac{1}{n!}[\cdots[i\frac{\partial\hat{G}}{\partial\eta},\overbrace{i\hat{G}],\cdots],i\hat{G}}^{n-1}]\quad ,\end{aligned} \tag{A.9}$$

$$\begin{aligned}e^{-i\hat{G}}\hat{H}e^{i\hat{G}} &= \hat{H}+[\hat{H},i\hat{G}]+\frac{1}{2!}[[\hat{H},i\hat{G}],i\hat{G}]+\cdots \\ &= \hat{H}+\sum_{n\geq1}\frac{1}{n!}[\cdots[\hat{H},\overbrace{i\hat{G}],\cdots],i\hat{G}}^{n}]\quad ,\end{aligned} \tag{A.10}$$

Eq. (A.7a) is written as

$$\{\omega_\lambda g^{(\lambda)} + \frac{\partial g^{(\lambda)}}{\partial \eta}\frac{\partial [H]}{\partial \eta^*} - \frac{\partial g^{(\lambda)}}{\partial \eta^*}\frac{\partial [H]}{\partial \eta}\}$$

$$-\sum_{n\geqq 2}\frac{1}{n!}<\phi_0|\,[i\hat{Q}^\dagger_\lambda, \{[\cdots[\hat{H}, i\hat{G}], \cdots], i\hat{G}]\}]\,|\phi_0>$$

$$+\sum_{m\geqq 1}\frac{1}{(2m+1)!}<\phi_0|\,[i\hat{Q}^\dagger_\lambda, \{[\cdots[(i\frac{\partial\hat{G}}{\partial\eta}\frac{\partial[H]}{\partial\eta^*}$$

$$-i\frac{\partial\hat{G}}{\partial\eta^*}\frac{\partial[H]}{\partial\eta}), \overbrace{i\hat{G}], \cdots], i\hat{G}]}^{2m}\}]\,|\phi_0> = 0\ , \quad \lambda \neq \lambda_0\ , \qquad \text{(A.11)}$$

where we have used the fact

$$\sum_{m\geqq 1}\frac{1}{(2m)!}<\phi_0|\,[i\hat{Q}^\dagger_\lambda, \{[\cdots[(i\frac{\partial\hat{G}}{\partial\eta}\frac{\partial[H]}{\partial\eta^*} - i\frac{\partial\hat{G}}{\partial\eta^*}\frac{\partial[H]}{\partial\eta}), \overbrace{i\hat{G}], \cdots], i\hat{G}]}^{2m-1}\}]\,|\phi_0>$$

$$= 0\ . \qquad \text{(A.12)}$$

In the same way, Eq. (A.2) is written as

$$\eta^* - \sum_\lambda\{g^{(\lambda)}\frac{\partial g^{(\lambda)*}}{\partial\eta} - \frac{\partial g^{(\lambda)}}{\partial\eta}g^{(\lambda)*}\}$$

$$-2\sum_{m\geqq 2}\frac{1}{(2m)!}<\phi_0|\,[\cdots[i\frac{\partial\hat{G}}{\partial\eta}, \overbrace{i\hat{G}], \cdots], i\hat{G}]}^{2m-1}\,|\phi_0> = 0\ . \qquad \text{(A.13)}$$

Thus, Eqs. (A.11) and (A.13), with

$$[H] = <\phi_0|e^{-i\hat{G}}\hat{H}e^{i\hat{G}}|\phi_0> - <\phi_0|\hat{H}|\phi_0>$$

$$= \sum_{n\geqq 1}\frac{1}{n!}<\phi_0|\,[\cdots[\hat{H}, \overbrace{i\hat{G}], \cdots], i\hat{G}]}^{n}\,|\phi_0>\ , \qquad \text{(A.14)}$$

become the basic equations to specify the coefficients $g^{(\lambda)}(\eta,\eta^*)$ of $\hat{G}(\eta,\eta^*)$ in Eq. (A.8).

We are now in a position to determine the coefficients $g^{(\lambda)}(\eta,\eta^*)$ as well as the collective Hamiltonian $[H]$, which are appropriate for our specified collective motion. To do this, we make the following expansion of $g^{(\lambda)}(\eta,\eta^*)$ with respect to (η,η^*),

$$g^{(\lambda)}(\eta,\eta^*) = g^{(\lambda)}(1)+g^{(\lambda)}(2)+\cdots = \sum_{n\geqq 1} g^{(\lambda)}(n),$$

$$g^{(\lambda)}(n) \equiv \sum_{\substack{rs \\ (r+s=n)}} g_{rs}^{(\lambda)}(\eta^*)^r(\eta)^s \ . \tag{A.15}$$

Since the basic equations (A.11) and (A.13) with the (η,η^*)-expansion (A.15) are supposed to be valid for continuous ranges of η and η^*, we can equate the coefficients of each power of (η,η^*) in these equations to zero. Thus, by starting with the coefficients with the lowest power of (η,η^*) and by proceeding to the higher (η,η^*)-coefficients step by step, we can determine the unknown quantities $g_{rs}^{(\lambda)}$ of $g^{(\lambda)}(\eta,\eta^*)$ in Eq.(A.15) as well as the collective Hamiltonian $[H]$ self-consistently.

The important task in this expansion method is the choice of the lowest order term $g^{(\lambda)}(1)$ to satisfy our specified condition on the collective motion. Since the term with the lowest power of (η,η^*) in Eq.(A.13) leads us to

$$\eta^* = \sum_\lambda \{g^{(\lambda)}(1)\frac{\partial g^{(\lambda)*}(1)}{\partial\eta} - \frac{\partial g^{(\lambda)}(1)}{\partial\eta}g^{(\lambda)*}(1)\} \ , \tag{A.16}$$

and the term with the lowest power of (η,η^*) in the collective Hamiltonian (A.14) is

$$[H]^{(0)} \equiv \frac{1}{2}\langle\phi_0|\,[\,[\hat{H},i\hat{G}(1)]\,,i\hat{G}(1)]\,|\phi_0\rangle$$

$$= \sum_\lambda \omega_\lambda g^{(\lambda)*}(1)g^{(\lambda)}(1) \tag{A.17}$$

with

$$\hat{G}(1) \equiv \sum_{\lambda} \{ g^{(\lambda)}(1)\hat{Q}_\lambda + g^{(\lambda)*}(1)\hat{Q}^\dagger_\lambda \} \quad , \qquad \text{(A.18)}$$

we can choose

$$g^{(\lambda)}(1) = i\eta^*\delta_{\lambda\lambda_0} \; , \qquad g^{(\lambda)*}(1) = -i\eta\delta_{\lambda\lambda_0} \; , \qquad \text{(A.19)}$$

so that Eq. (A.16) is satisfied and $[H]^{(0)}$ becomes the RPA-phonon Hamiltonian

$$[H]^{(0)} = \omega_{\lambda_0}\eta^*\eta \; . \qquad \text{(A.20)}$$

With the use of (A.19) and (A.20), Eqs. (A.11) and (A.13) can be written in the following forms, respectively;

$$\{\omega_\lambda + \omega_{\lambda_0}(\eta\frac{\partial}{\partial\eta} - \eta^*\frac{\partial}{\partial\eta^*})\} g^{(\lambda)} = \{\frac{\partial g^{(\lambda)}}{\partial\eta^*}\frac{\partial [H]^{(\text{int})}}{\partial\eta} - \frac{\partial g^{(\lambda)}}{\partial\eta}\frac{\partial [H]^{(\text{int})}}{\partial\eta^*}\}$$

$$+\sum_{n\geqq 2}\frac{1}{n!}<\phi_0 | [i\hat{Q}^\dagger_\lambda, \{ \overbrace{[\cdots[\hat{H}, i\hat{G}], \cdots], i\hat{G}]}^{n} \}] | \phi_0>$$

$$-\sum_{m\geqq 1}\frac{1}{(2m+1)!}<\phi_0 | [i\hat{Q}^\dagger_\lambda, \{ [\cdots [(i\frac{\partial\hat{G}}{\partial\eta}\frac{\partial [H]}{\partial\eta^*}$$

$$-i\frac{\partial\hat{G}}{\partial\eta^*}\frac{\partial [H]}{\partial\eta}), \overbrace{i\hat{G}], \cdots], i\hat{G}]}^{2m} \}] | \phi_0> \; , \quad \lambda \neq \lambda_0 \; , \qquad \text{(A.21)}$$

$$(1-\eta\frac{\partial}{\partial\eta})\{\Delta g^{(\lambda_0)}\} - \eta^*\frac{\partial}{\partial\eta}\{\Delta g^{(\lambda_0)*}\}$$

$$= i\{ \{\Delta g^{(\lambda_0)*}\}\frac{\partial}{\partial\eta}\{\Delta g^{(\lambda_0)}\} - \{\Delta g^{(\lambda_0)}\}\frac{\partial}{\partial\eta}\{\Delta g^{(\lambda_0)*}\} \}$$

$$+i\sum_{\lambda\neq\lambda_0}\{\frac{\partial g^{(\lambda)}}{\partial\eta}g^{(\lambda)*} - g^{(\lambda)}\frac{\partial g^{(\lambda)*}}{\partial\eta}\}$$

$$-2i\sum_{m\geqq 2}\frac{1}{(2m)!}<\phi_0 | [\cdots [i\frac{\partial\hat{G}}{\partial\eta}, \overbrace{i\hat{G}], \cdots], i\hat{G}]}^{2m-1} | \phi_0> \equiv u \; , \qquad \text{(A.22)}$$

with

$$\Delta g^{(\lambda_0)} \equiv \sum_{n\geqq 2} g^{(\lambda_0)}(n) \ . \tag{A.23}$$

The quantity $[H]^{(\mathrm{int})}$ in Eq. (A.21) is defined by

$$
\begin{aligned}
[H]^{(\mathrm{int})} &\equiv [H] - [H]^{(0)} \\
&= \omega_{\lambda_0}\{i\eta^* \Delta g^{(\lambda_0)*} - i\eta \Delta g^{(\lambda_0)}\} + \omega_{\lambda_0}\Delta g^{(\lambda_0)*}\Delta g^{(\lambda_0)} + \sum_{\lambda\neq\lambda_0}\omega_\lambda g^{(\lambda)*}g^{(\lambda)} \\
&\quad + \sum_{n\geqq 3}\frac{1}{n!}\langle\phi_0|\,\overbrace{[\cdots[\hat{H}, i\hat{G}], \cdots], i\hat{G}]}^{n}\,|\phi_0\rangle \\
&\equiv h(3)+h(4)+\cdots = \sum_{n\geqq 3} h(n) \ ,
\end{aligned}
$$

$$h(n) \equiv \sum_{\substack{rs \\ (r+s=n)}} h_{rs}(\eta^*)^r(\eta)^s \ . \tag{A.24}$$

Comparing the coefficients of (η, η^*) of both the sides of Eq. (A.22), we have

$$(s-1)g_{rs}^{(\lambda_0)} + (s+1)g_{s+1,r-1}^{(\lambda_0)*} = -u_{rs} \ ; \qquad r+s \geqq 2 \ ; \tag{A.25}$$

where the right-hand side of Eq. (A.22) is symbolically written as

$$u \equiv \sum_n u(n) \equiv \sum_n \sum_{\substack{rs \\ (r+s=n)}} u_{rs}(\eta^*)^r(\eta)^s \ . \tag{A.26}$$

From Eq. (A.25) and its complex conjugate equation, we have

$$g_{rs}^{(\lambda_0)} = -\frac{1}{2(r+s-1)}\{(2-r)u_{rs} + (s+1)u_{s+1,r-1}^*\} \ ; \qquad r+s \geqq 2 \ , \tag{A.27a}$$

which is formally expressed as

$$g^{(\lambda_0)}(\eta, \eta^*) = -\frac{1}{2}\left(\eta\frac{\partial}{\partial\eta} + \eta^*\frac{\partial}{\partial\eta^*} - 1\right)^{-1}\left\{\left(2-\eta^*\frac{\partial}{\partial\eta^*}\right)u + \eta^*\frac{\partial}{\partial\eta}u^*\right\} \ . \tag{A.27b}$$

In the same sense, Eq. (A.21) can be formally expressed as

$$g^{(\lambda\neq\lambda_0)}(\eta,\eta^*) = \{\omega_\lambda+\omega_{\lambda_0}(\eta\frac{\partial}{\partial\eta}-\eta^*\frac{\partial}{\partial\eta^*})\}^{-1}$$

$$\cdot\{\{\frac{\partial g^{(\lambda\neq\lambda_0)}}{\partial\eta^*}\frac{\partial[H]^{(\text{int})}}{\partial\eta}-\frac{\partial g^{(\lambda\neq\lambda_0)}}{\partial\eta}\frac{\partial[H]^{(\text{int})}}{\partial\eta^*}\}$$

$$+\sum_{n\geq2}\frac{1}{n!}<\phi_0|[i\hat{Q}^\dagger_{\lambda\neq\lambda_0},\{[\cdots\overbrace{[\hat{H},i\hat{G}],\cdots],i\hat{G}]}^{n}\}]|\phi_0>$$

$$-\sum_{m\geq1}\frac{1}{(2m+1)!}<\phi_0|[i\hat{Q}^\dagger_{\lambda\neq\lambda_0},\{[\cdots[(i\frac{\partial\hat{G}}{\partial\eta}\frac{\partial[H]}{\partial\eta^*}$$

$$-i\frac{\partial\hat{G}}{\partial\eta^*}\frac{\partial[H]}{\partial\eta}),\overbrace{i\hat{G}],\cdots],i\hat{G}]}^{2m}\}]|\phi_0>\} \quad . \tag{A.28}$$

The expressions (A.27b) and (A.28) of the basic equations are convenient for the (η,η^*)-expansion method. With these equations with the (η,η^*)-expansion, we can easily determine the higher order terms $g^{(\lambda)}(n)$ (i.e., $g^{(\lambda)}_{rs}$ with $r+s=n$) successively, starting with the lowest-order term $g^{(\lambda)}(1)$ given by Eq.(A.19). Thus, for instance, we obtain

$$g^{(\lambda_0)}(2) = 0 \quad , \tag{A.29a}$$

$$g^{(\lambda\neq\lambda_0)}(2) = \{\omega_\lambda+\omega_{\lambda_0}(\eta\frac{\partial}{\partial\eta}-\eta^*\frac{\partial}{\partial\eta^*})\}^{-1}$$

$$\cdot\frac{1}{2}<\phi_0|[i\hat{Q}^\dagger_\lambda,[[\hat{H},i\hat{G}(1)],i\hat{G}(1)]]|\phi_0> \quad , \tag{A.29b}$$

$$h(3) = \frac{1}{3!}<\phi_0|[[[\hat{H},i\hat{G}(1)],i\hat{G}(1)],i\hat{G}(1)]|\phi_0> \quad , \tag{A.30}$$

and

$$g^{(\lambda_0)}(3) = -\frac{1}{2}(\eta\frac{\partial}{\partial\eta}+\eta^*\frac{\partial}{\partial\eta^*}-1)^{-1}\{(2-\eta^*\frac{\partial}{\partial\eta^*})u(3)+\eta^*\frac{\partial}{\partial\eta}u^*(3)\} \quad , \tag{A.31a}$$

$$g^{(\lambda\neq\lambda_0)}(3) = \{\omega_\lambda+\omega_{\lambda_0}(\eta\frac{\partial}{\partial\eta}-\eta^*\frac{\partial}{\partial\eta^*})\}^{-1}$$

$$\cdot\{\{\frac{\partial g^{(\lambda)}(2)}{\partial\eta^*}\frac{\partial h(3)}{\partial\eta}-\frac{\partial g^{(\lambda)}(2)}{\partial\eta}\frac{\partial h(3)}{\partial\eta^*}$$

$$+\frac{1}{2}<\phi_0|[i\hat{Q}^\dagger_\lambda,\{[[\hat{H},i\hat{G}(2)],i\hat{G}(1)]+[[\hat{H},i\hat{G}(1)],i\hat{G}(2)]\}]|\phi_0>$$

$$+\frac{1}{3!}<\phi_0|[i\hat{Q}^\dagger_\lambda,\{[[[\hat{H},i\hat{G}(1)],i\hat{G}(1)],i\hat{G}(1)]\}]|\phi_0>$$

$$-\frac{1}{3!}<\phi_0|[i\hat{Q}^\dagger_\lambda,\{[[(i\frac{\partial\hat{G}(1)}{\partial\eta}\frac{\partial[H]^{(0)}}{\partial\eta^*}$$

$$-i\frac{\partial\hat{G}(1)}{\partial\eta^*}\frac{\partial[H]^{(0)}}{\partial\eta}),i\hat{G}(1)],i\hat{G}(1)]\}]|\phi_0>\}, \qquad (A.31b)$$

$$h(4) = \omega_{\lambda_0}i\{g^{(\lambda_0)*}(3)\eta^*-g^{(\lambda_0)}(3)\eta\}+\sum_{\lambda\neq\lambda_0}\omega_\lambda g^{(\lambda)*}(2)g^{(\lambda)}(2)$$

$$+\frac{1}{4!}<\phi_0|[[[[\hat{H},i\hat{G}(1)],i\hat{G}(1)],i\hat{G}(1)],i\hat{G}(1)]|\phi_0>$$

$$+\frac{1}{3!}<\phi_0|\{[[[\hat{H},i\hat{G}(2)],i\hat{G}(1)],i\hat{G}(1)]$$

$$+[[[\hat{H},i\hat{G}(1)],i\hat{G}(2)],i\hat{G}(1)]$$

$$+[[[\hat{H},i\hat{G}(1)],i\hat{G}(1)],i\hat{G}(2)]\}|\phi_0>, \qquad (A.32)$$

where

$$\hat{G}(n) \equiv \sum_\lambda\{g^{(\lambda)}(n)\hat{Q}_\lambda+g^{(\lambda)*}(n)\hat{Q}^\dagger_\lambda\}. \qquad (A.33)$$

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Figure Captions

Fig. 1. Diffeomorphic mapping $M^2 \rightarrow \Sigma^2$.

Fig. 2. Schematic illustration of TDHF trajectory starting near to Σ^2 in three different cases. SCC trajectory is indicated by solid line on Σ^2. TDHF trajectory is indicated by dashed line (behind Σ^2) and dotted-solid line (in front of Σ^2).

Fig. 3. Three-Level SU(3) Model.

Fig. 4. SCC trajectories with total-energy values E=8, 5 and 1. Shadow region indicates unstable region with $\omega^2 \leqq 0$.

Fig. 5. Contour map of non-local effects I.

Fig. 6. Poincaré-section map of TDHF trajectories with total energy (a) E=8, (b) E=5 and (c) E=1.

Fig. 7. TDHF trajectory represented by double-circles in three-dimensional coordinate space. (a) E=8, (b) E=5 and (c) E=1.

Fig. 8. Definition of double-circle coordinates.

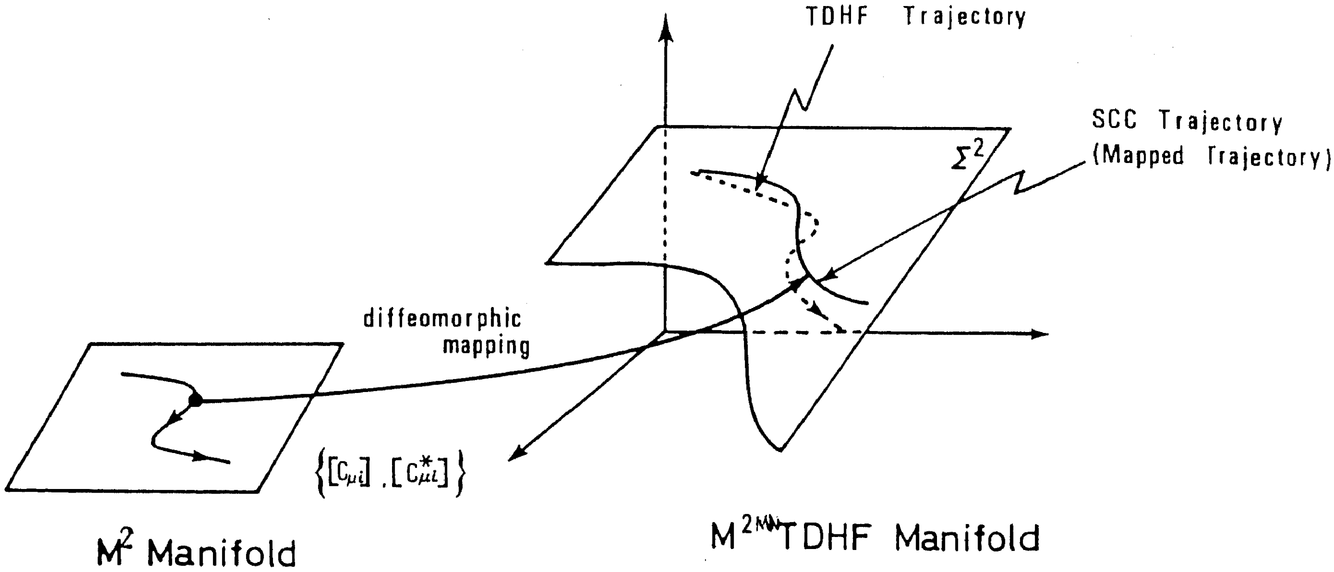
TDHF Trajectory
SCC Trajectory
(Mapped Trajectory)
Σ^2
diffeomorphic
mapping
$\{[C_{\mu i}], [C^*_{\mu i}]\}$
M^2 Manifold
M^{2MN} TDHF Manifold

Figure 1

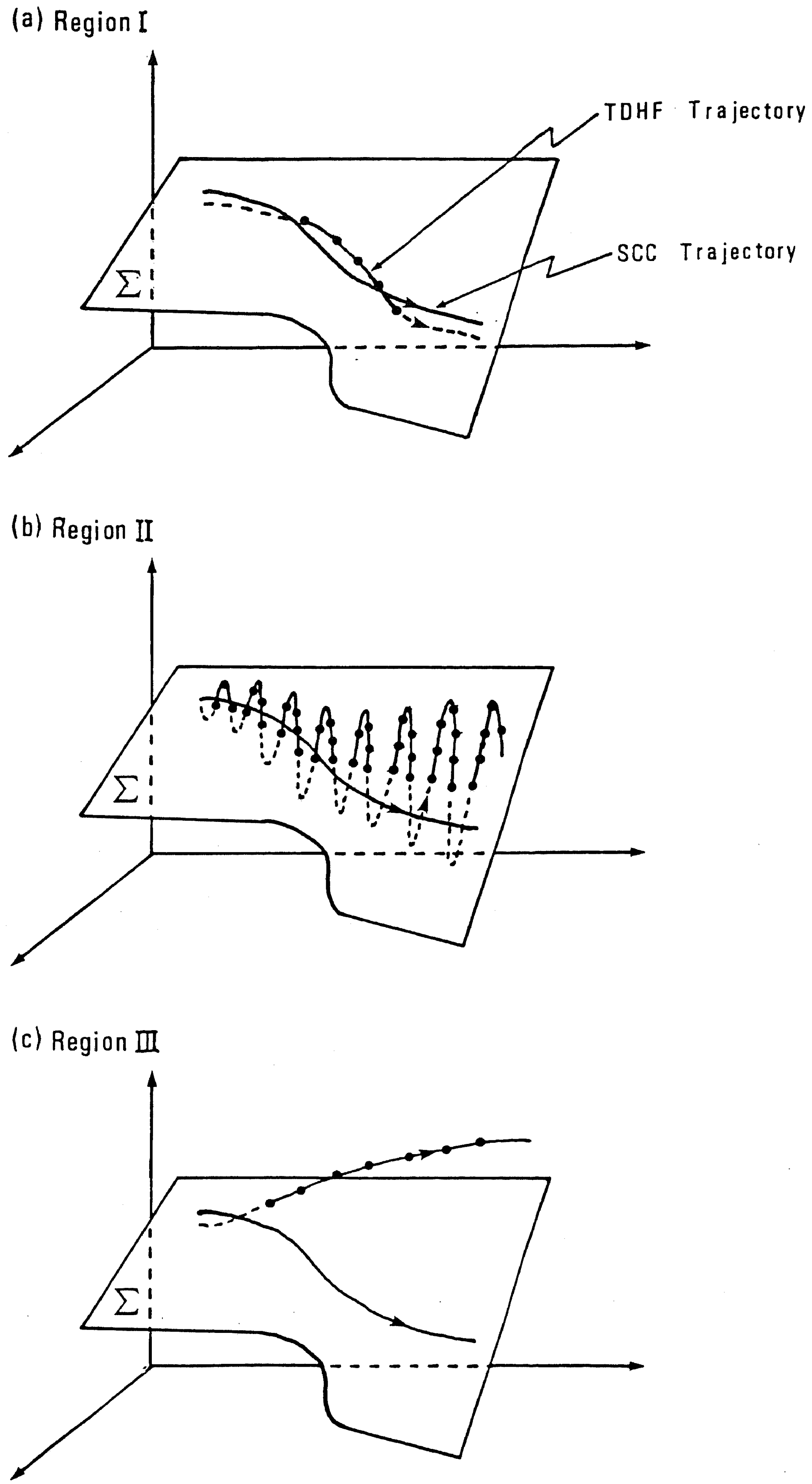
(a) Region I
TDHF Trajectory
SCC Trajectory
Σ
(b) Region II
Σ
(c) Region III
Σ

Figure 2

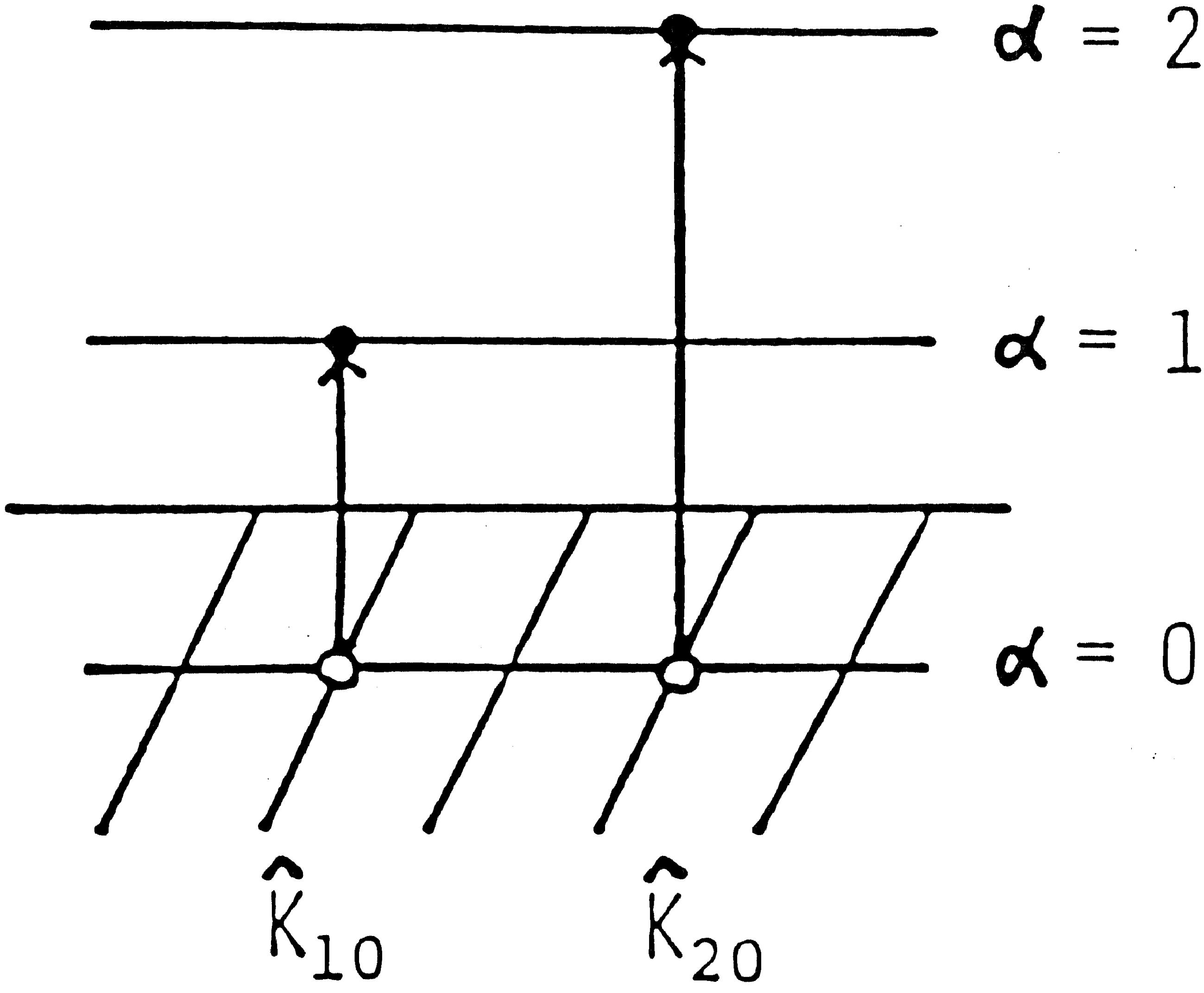

Figure 3

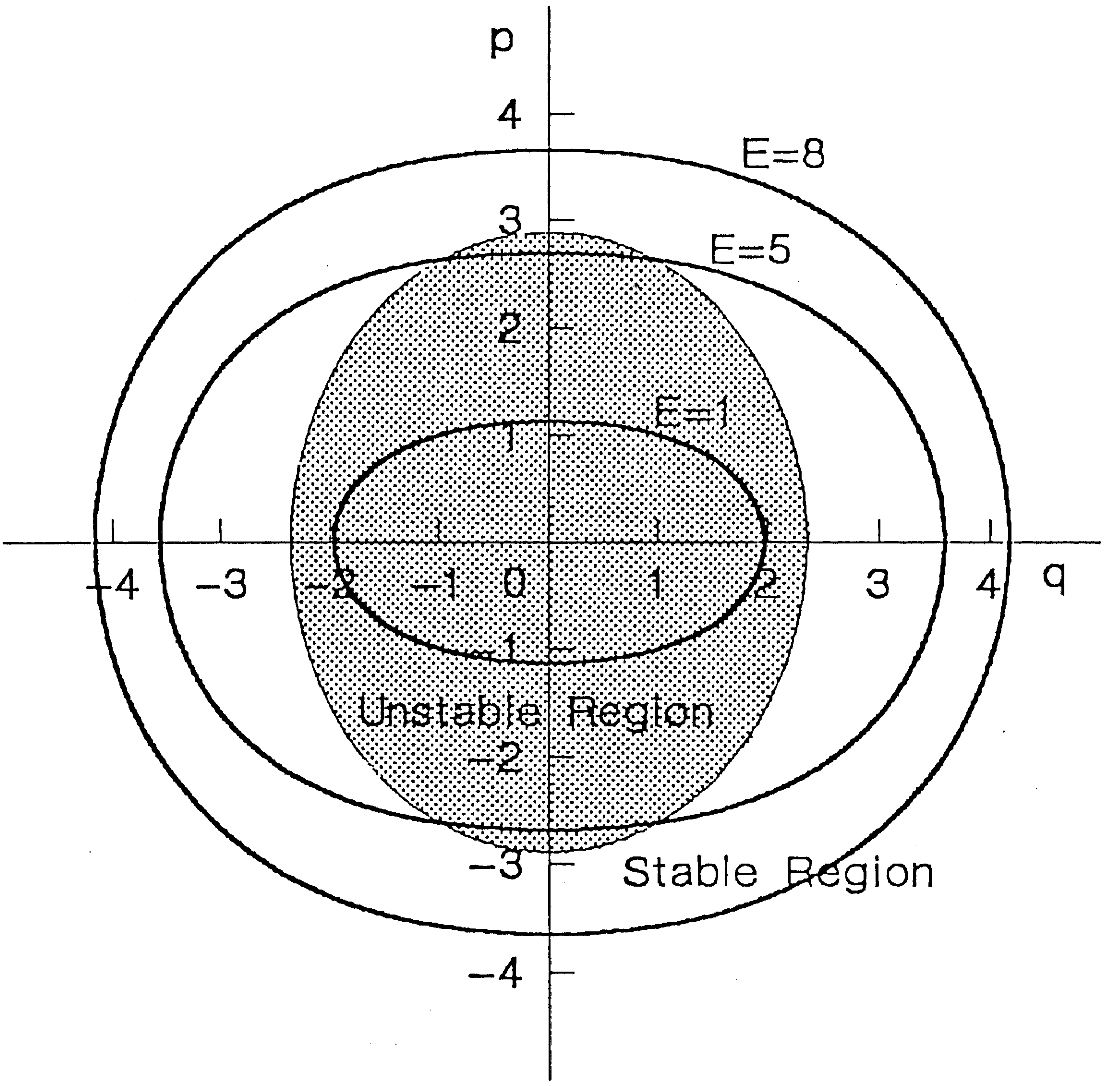
p
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
1
2
3
4
q
E=8
E=5
E=1
Unstable Region
Stable Region

Figure 4

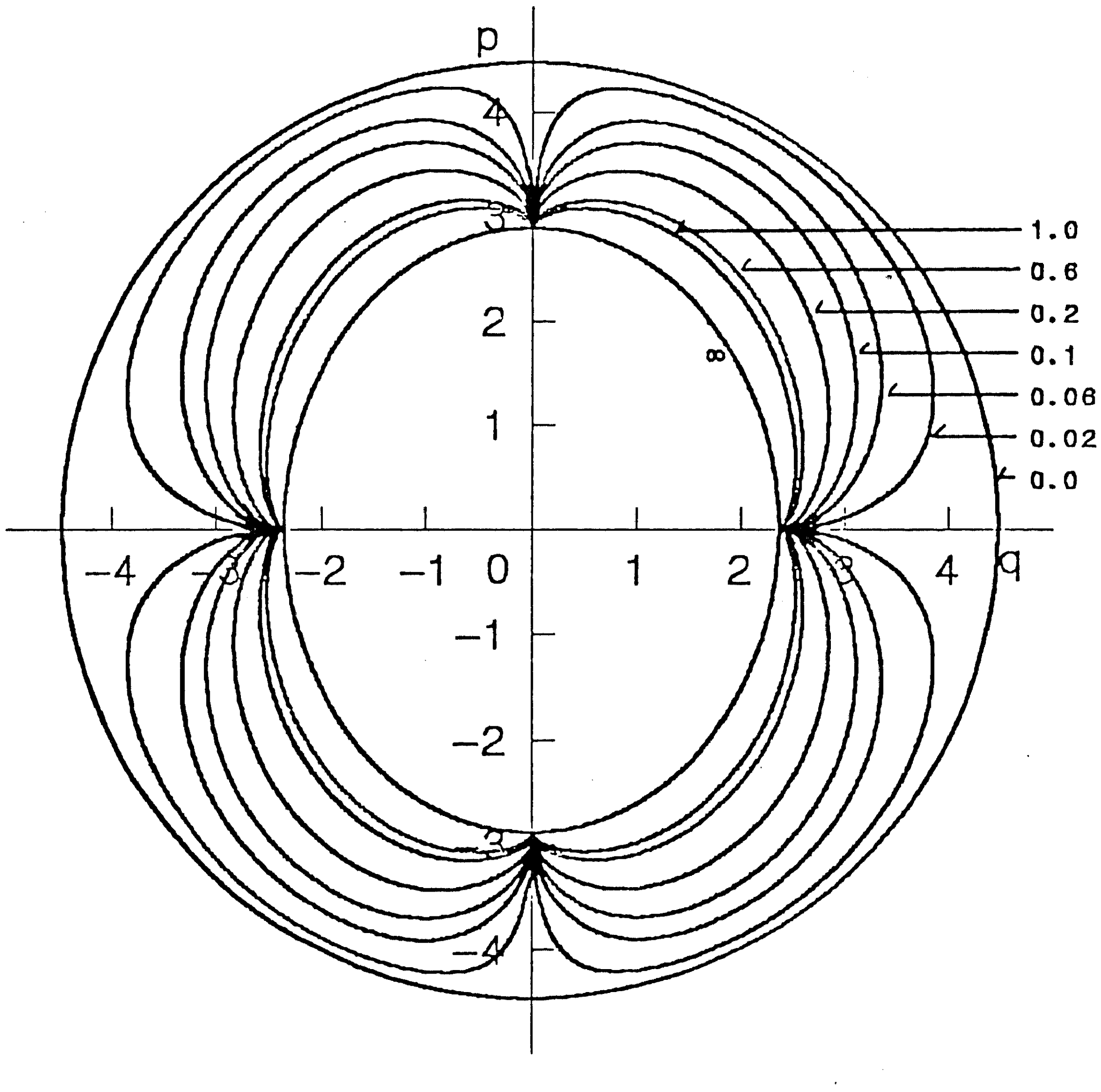

Figure 5

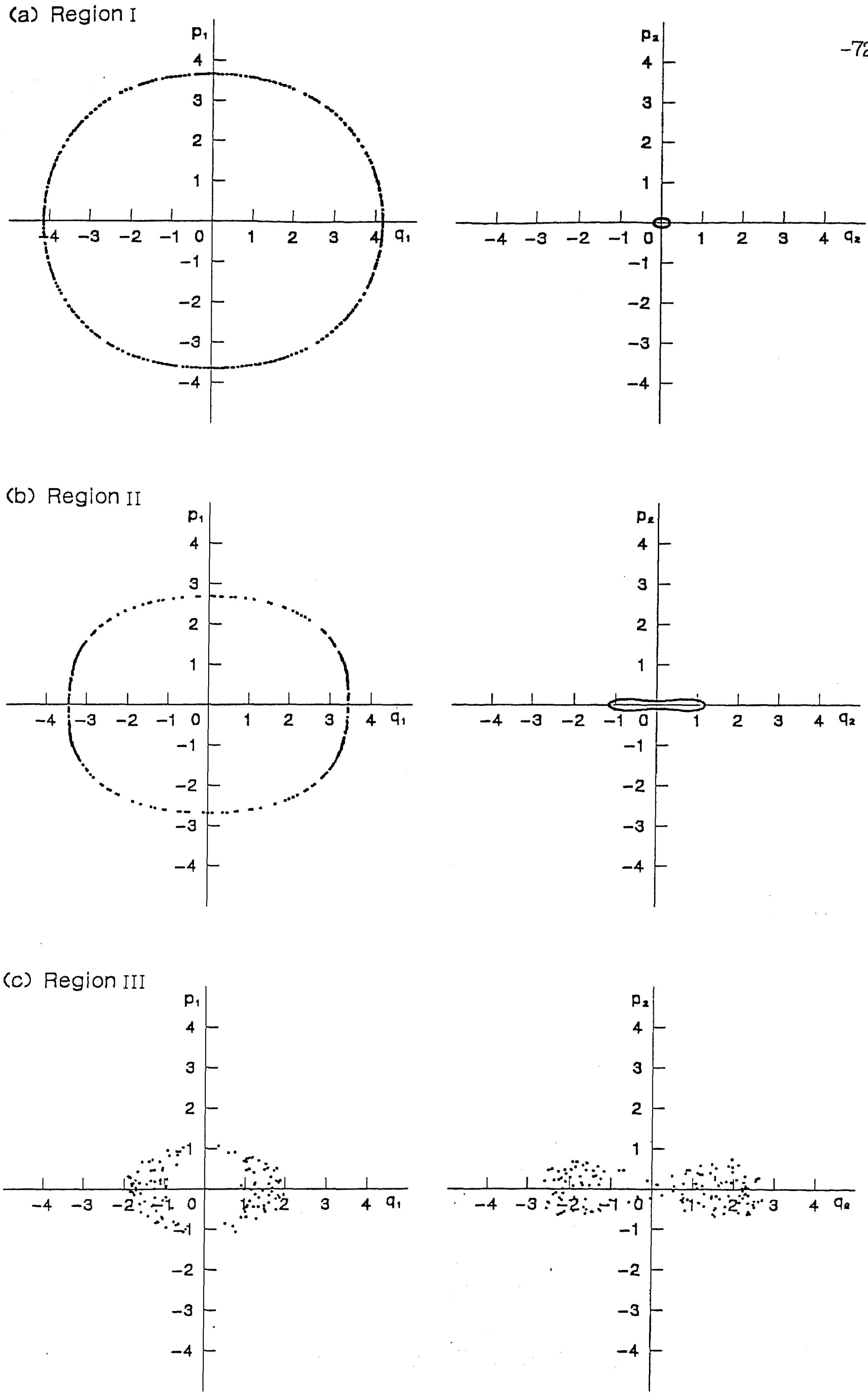

Figure 6

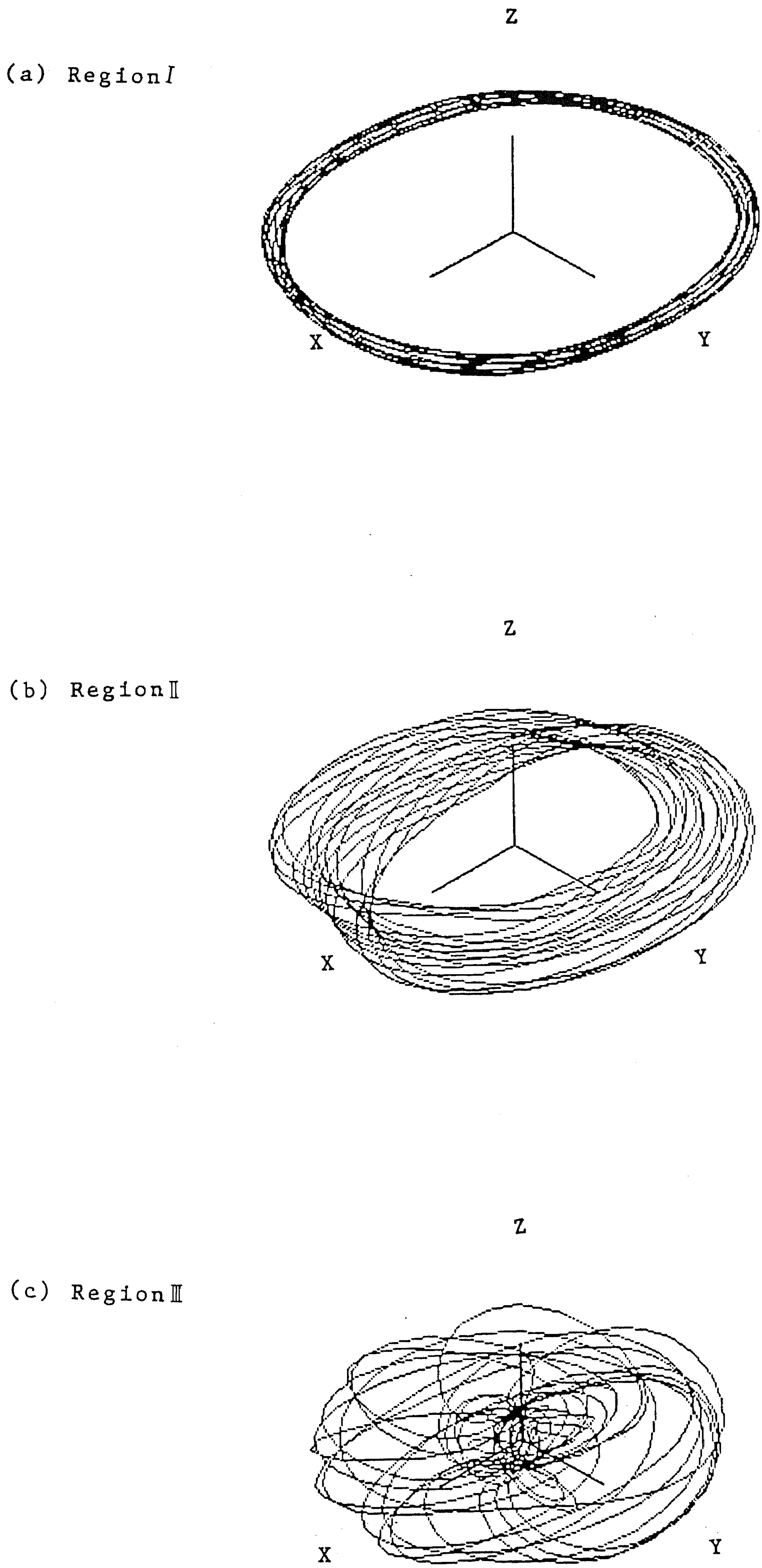

Figure 7

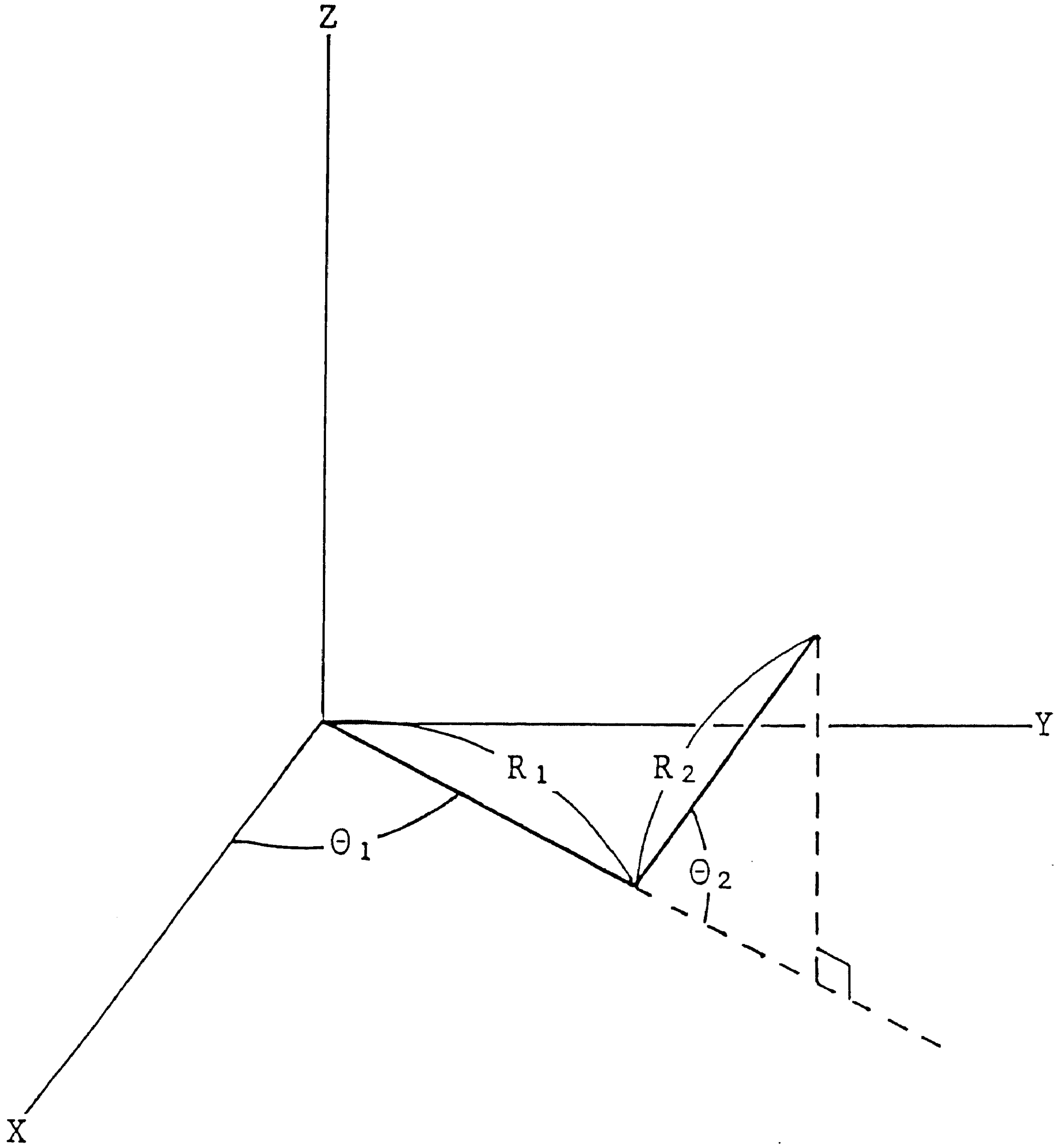

Figure 8