Magnot, Jean-Pierre
On diffeological principal bundles of non-formal pseudo-differential operators over formal ones. (English) Zbl 07740305

The correspondence between formal and non-formal pseudo-differential operators plays a significant role in analysis. This paper aims to give a geometric description of this correspondence. The author describes the structure of diffeological bundle of non-formal classical pseudo-differential operators over formal ones, and its structure group. To this end, the author gives results on diffeological principal bundles with no local trivialization including an Ambrose-Singer theorem, using the smoothing connections [*J.-P. Magnot*, Int. J. Geom. Methods Mod. Phys. 10, No. 9, Article ID 1350043, 31 p. (2013; Zbl 1278.58001)]].

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MSC:

53C05	Connections (general theory)
57R55	Differentiable structures in differential topology
58B05	Homotopy and topological questions for infinite-dimensional manifolds
58B10	Differentiability questions for infinite-dimensional manifolds
58J40	Pseudodifferential and Fourier integral operators on manifolds

Keywords:

diffeological bundles; pseudo-differential operators; Ambrose-Singer theorem

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