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The category $\mathcal{S}h_{j^I}\mathbf{MSet}$ of sheaves in \mathbf{MSet} . (English. English summary)

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Given a monoid M , this paper is concerned with the subcategory $\mathcal{S}h_{j^I}\mathbf{BM}$ of the topos \mathbf{BM} of M -sets determined by the weak Lawvere-Tierney topology j^I associated to the ideal closure operator determined by a right ideal I of M . It is shown that $\mathcal{S}h_{j^I}\mathbf{BM}$ is actually a subtopos of \mathbf{BM} and that the Lawvere-Tierney topology associated to this subtopos is the idempotent hull of j^I . The authors generalize the results in [M. M. Ebrahimi and M. Mahmoudi, *Bull. Iranian Math. Soc.* **21** (1995), no. 1, 25–33; MR1431587] to the category of I -separated objects (in Section 3). They investigate injectivity of the initial Boolean algebra in $\mathcal{S}h_{j^I}\mathbf{BM}$ with respect to a special class of monomorphisms, giving an equivalent statement for the topos to be De Morgan after [Z. Khanjanzadeh and A. Madanshekaf, *Comm. Algebra* **46** (2018), no. 5, 1868–1888; MR3799178] (in Section 4). *Hirokazu Nishimura*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.