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On a quotient S -set induced by countably infinite decreasing chains. (English) Zbl 07757173
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Alexandroff topological spaces have been introduced in 1937 by *P. Alexandroff* [Rec. Math. Moscou, n. Ser. 2, 501–519 (1937; [Zbl 0018.09105](#))] to construct a homology theory for discrete spaces. In [*S. J. Andima* and *W. J. Thron*, *Pac. J. Math.* 75, 297–318 (1978; [Zbl 0384.54018](#))], the correspondence between the topological, order-theoretic and category-theoretic views has been used to interpret various results of each of the above three natures in alternative ways.

This paper makes use of monoid actions $S \times X \rightarrow X$ in order to define the quotient of an S -set induced by a suitable S -congruence which provides an example of Alexandroff topological space abiding by several non-trivial properties.

The synopsis of the paper goes as follows.

- §2 introduces the main notions and notations concerning set systems and set operations, together with some basic results on them.
- §3 shows that the maximal members of the set system $\mathcal{A}_{\sigma, X}$ is a covering for X , establishing other properties of essentially attractive and essentially Noetherian Alexandroff spaces. The relation between monoid actions and Alexandroff spaces is investigated.
- §4 investigates the main properties of two specific kinds of chains definable on an Alexandroff space, namely, a descending chain $\mathcal{D}_{\sigma, X}$ and an ascending one $\mathcal{E}_{\sigma, X}$.
- §5 analyzes the main properties of a quotient S -set induced by a subset of points of a monoid S forming an infinite decreasing chain.

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MSC:

- [18A22](#) Special properties of functors (faithful, full, etc.)
- [20M15](#) Mappings of semigroups
- [20M30](#) Representation of semigroups; actions of semigroups on sets
- [20M50](#) Connections of semigroups with homological algebra and category theory
- [54A05](#) Topological spaces and generalizations (closure spaces, etc.)
- [54A25](#) Cardinality properties (cardinal functions and inequalities, discrete subsets)
- [06A15](#) Galois correspondences, closure operators (in relation to ordered sets)

Keywords:

[Alexandroff spaces](#); [closure operators](#); [monoids](#); [monoid actions](#); [categories](#)

Full Text: [DOI](#)

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