Chatterjee, Saikat; **Koushik, Praphulla**
Extension of topological groupoids and Hurewicz morphisms. (English) [Zbl 07739321]

This paper introduces the notion of Morita equivalence of topological groupoid extensions, investigating its relation with a gerbe over a topological stacck. The paper offers a topological version of the authors' previous work [Bull. Sci. Math. 163, Article ID 102886, 30 p. (2020; Zbl 1453.14003)] within the smooth setup. The most distinctive feature in the topological setup is the representability of a morphism of stacks obtained from a topoloigcal space (Lemma 2.10), which in turn shows the existence of a pair of atlases related by a map of local sections (Lemma 2.11).

A Serre fibration is a continuous map of topological spaces that has homotopy lifting property with respect to CW complexes, while a Hurewicz fibration enjoys homotopy lifting property with respect to all topological spaces. A Hurwicz (respectively, Serre) stack is a topological stack that is representable to a topological groupoid

$$[X_1 \rightrightarrows X_0]$$

whose source map is a Hurwicz (respectively, Serre) morphism of stacks. The notions of Serre and Hurewicz stacks were introduced in [*B. Noohi*, Adv. Math. 252, 612–640 (2014; Zbl 1310.14006)]. The main contributions of this paper are the introduction of a Hurwicz (respectively, Serre) gerbe and the observation on its relation with a Hurwicz (respectively, Serre) morphism of stacks.

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MSC:

18F20	Presheaves and sheaves, stacks, descent conditions (category-theoretic aspects)
22A22	Topological groupoids (including differentiable and Lie groupoids)
53C08	Differential geometric aspects of gerbes and differential characters

Keywords:

topological groupoid extensions; gerbes; Hurewicz morphisms

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