Bridgeman, Jacob C.; **Lootens, Laurens**; **Verstraete, Frank**

Invertible bimodule categories and generalized Schur orthogonality. (English) Zbl 07732076

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Fusion categories are the categories of representations of weak Hopf algebras [*T. Hayashi*, "A canonical Tannaka duality for finite seimisimple tensor categories", Preprint, arXiv:math/9904073; *D. Nikshych* et al., Topology Appl. 127, No. 1–2, 91–123 (2003; Zbl 1021.16026); *D. Nikshych*, J. Algebra 275, No. 2, 639–667 (2004; Zbl 1066.16042); *V. Ostrik*, Transform. Groups 8, No. 2, 177–206 (2003; Zbl 1044.18004)], while in physics, they can be used to define a large class of 3-dimensional topological field theories [*N. Yu. Reshetikhin* and *V. G. Turaev*, Commun. Math. Phys. 127, No. 1, 1–26 (1990; Zbl 0768.57003); Invent. Math. 103, No. 3, 547–597 (1991; Zbl 0725.57007); *V. G. Turaev*, Quantum invariants of knots and 3-manifolds. 3rd edition. Berlin: Walter de Gruyter (2016; Zbl 1346.57002); *V. Turaev* and *A. Virelizier*, Monoidal categories and topological field theory. Basel: Birkhäuser/Springer (2017; Zbl 1423.18001); *V. G. Turaev* and *O. Y. Viro*, Topology 31, No. 4, 865–902 (1992; Zbl 0779.57009); *J. W. Barrett* and *B. W. Westbury*, Trans. Am. Math. Soc. 348, No. 10, 3997–4022 (1996; Zbl 0865.57013)], providing a classification of 2-dimensional rational conformal field theories [*J. Fuchs* et al., "TFT construction of RCFT correlators. I: Partition functions", Preprint, arXiv:hep-th/0204148; *I. Runkel* et al., "Categorification and correlation functions in conformal field theory", Preprint, arXiv:math/0602079; *J. Fröhlich* et al., "Duality and defects in rational conformal field theory", Preprint, arXiv:hep-th/0607247]. It is often convenient to specify a fusion category in terms of its *skeletal data*.

A pair of fusion categories $\mathcal{C}$ and $\mathcal{D}$ are called *Morita equivalent* if there exists an invertible bimodule category ${}_{\mathcal{C}}\mathcal{M}_{\mathcal{D}}$ between them [*M. Müger*, J. Pure Appl. Algebra 180, No. 1–2, 81–157 (2003; Zbl 1033.18002); J. Pure Appl. Algebra 180, No. 1–2, 159–219 (2003; Zbl 1033.18003)]. Morita equivalent categories give equivalent Turaev-Viro invariants of 3-manifolds [*M. Müger*, J. Pure Appl. Algebra 180, No. 1–2, 81–157 (2003; Zbl 1033.18002); *V. Turaev* and *A. Virelizier*, Int. J. Math. 31, No. 10, Article ID 2050076, 57 p. (2020; Zbl 1473.57032)], or physically, Levin-wen models in the same phase [*A. Kitaev* and *L. Kong*, Commun. Math. Phys. 313, No. 2, 351–373 (2012; Zbl 1250.81141); *M. A. Levin* and *X.-G. Wen*, Phys. Rev. B 71, No. 4, Article ID 045110, 21 p. (2005; doi:10.1103/PhysRevB.71.045110); *L. Lootens* et al., "Mapping between Morita equivalent string-net states with a constant depth quantum circuit", Preprint, arXiv:2112.12757].

This paper aims to answer affirmatively the question whether, given the data for a bimodule category, there is a simple way to check invertibility, making use of the annular algebra associated to ${}_{\mathcal{C}}\mathcal{M}$, whose representation category is equivalent to the dual $\mathcal{C}^*_{\mathcal{M}} = \mathrm{End}_{\mathcal{C}}(\mathcal{M})$.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18Dxx Categorical structures
81Txx Quantum field theory; related classical field theories
81Rxx Groups and algebras in quantum theory

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