

Iragi, Minani; Šlapal, Josef

Transitive quasi-uniform structures depending on a parameter. (English) Zbl 07697391
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It is well known that transitive quasi-uniform spaces play an important role, all but as general as that of quasi-uniform spaces, in the study of topological properties. The most attractive aspect of quasi-uniform spaces is that they can all be obtained by considering interior preserving open covers of their associated topological spaces, which is often called the Fletcher construction [*P. Fletcher* and *W. F. Lindgren*, Quasi-uniform spaces. New York Basel: Marcel Dekker, Inc. (1982; [Zbl 0501.54018](#))].

This paper constructs categorical quasi-uniform structures determined by a parameter, which leads to the description of subcategories defined by quasi-uniform structures and to a categorical treatment of all transitive quasi-uniformities compatible with a topology. The latter are thought of as a category \mathcal{C} rigged out in an $(\mathcal{E}, \mathcal{M})$ -factorization structure. A number of Galois connections related to quasi-uniform structures on \mathcal{C} are established, leading to the description of subcategories of \mathcal{C} determined by quasi-uniform structures.

The synopsis of the paper goes as follows.

§2 deals with preliminaries concerning

1. category theory [*J. Adámek* et al., Abstract and concrete categories. The joy of cats. New York etc.: John Wiley & Sons, Inc. (1990; [Zbl 0695.18001](#)); Repr. Theory Appl. Categ. 2006, No. 17, 1–507 (2006; [Zbl 1113.18001](#))], and
2. categorical closure and interior operators [*S. J. R. Vorster*, Quaest. Math. 23, No. 4, 405–416 (2000; [Zbl 0974.18003](#)); *D. Dikranjan* and *E. Giuli*, Topology Appl. 27, 129–143 (1987; [Zbl 0634.54008](#))] as well as
3. categorical topogenous, quasi-uniform and syntopogenous structures [*D. Holgate* and *M. Iragi*, Topology Appl. 263, 16–25 (2019; [Zbl 1420.18003](#)); *D. Holgate* et al., Appl. Categ. Struct. 24, No. 5, 447–455 (2016; [Zbl 1359.54003](#)); https://etd.uwc.ac.za/bitstream/handle/11394/6717/Iragi_m_msc_ns_2016.pdf?sequence=1&isAllowed=y].

§3 gives the necessary background on categorical quasi-uniform structures.

§4 is devoted to quasi-uniform structures determined by a fixed subclass of \mathcal{M} . It is shown (Theorem 4.1) that, for any fixed subclass \mathcal{N} of \mathcal{M} closed under pullbacks, there is a transitive quasi-uniform structure on \mathcal{C} determined by \mathcal{N} , which leads to a Galois connection between quasi-uniform structure on \mathcal{C} and a conglomerate of all subclasses of \mathcal{M} closed under pullbacks.

§5 addresses a description of the subcategories determined by quasi-uniform structures, which is achieved by two Galois connections:

1. one between quasi-uniform structures and idempotent closure operators, and
2. the other between quasi-uniform structures and the conglomerate of all full subcategories of \mathcal{C} .

§6 gives a list of examples.

Reviewer: [Hirokazu Nishimura \(Tsukuba\)](#)

MSC:

- [18A05](#) Definitions and generalizations in theory of categories
- [18F60](#) Categories of topological spaces and continuous mappings
- [54A15](#) Syntopogeneous structures
- [54B30](#) Categorical methods in general topology

Keywords:

closure operator; quasi-uniform structure; syntopogenous structure; Galois connection; interior operator

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