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**A(nother) model for the framed little disks operad.** (English) Zbl 07681905  
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The framed little  $n$ -disks operad  $D_n^{fr}$  is the operad of embeddings of  $n$ -dimensional unit disks in a unit disk, whose real homotopy type was computed for  $n = 2$  in [*J. Giansiracusa* and *P. Salvatore*, Contemp. Math. 519, 115–121 (2010; [Zbl 1209.18008](#)); *P. Boavida de Brito* and *M. Weiss*, Homology Homotopy Appl. 15, No. 2, 361–383 (2013; [Zbl 1291.18025](#))] and for  $n \geq 3$  in [*A. Khoroshkin* and *T. Willwacher*, “Real models for the framed little  $n$ -disks operads”, Preprint, [arXiv:1705.08108](#)]. A real model for the framed little disks operad is a Hopf cooperad which is quasi-isomorphic to the homotopy Hopf cooperad of differential forms  $\Omega(D_n^{fr})$ .

This paper is motivated by the realization that the framed little disks operad over  $\mathbb{R}$  has a large group of homotopy automorphisms, which are not visible as automorphisms in the existing Hopf cooperad models. Let  $GC_n$  be the Kontsevich graph complex, which is a dg Lie algebra. It carries a grading by loop order, so that one can extend it to a Lie algebra

$$\mathbb{R}L \times GC_n$$

where  $L$  denotes the generator of the loop order grading. The dg Lie algebra

$$\mathfrak{a}'_n := \text{Der}((H(BSO(n))) \times ((H(BSO(n))) \widehat{\otimes} (\mathbb{R}L \times GC_n)))$$

with

$$\mathfrak{a}_n := (\mathfrak{a}'_n)^m$$

as the twist contains a Maurer-Cartan element

$$m \in H(BSO(n)) \widehat{\otimes} GC_n$$

describing the homotopy type of the action of  $SO(n)$  on the unframed little disks operad [*A. Khoroshkin* and *T. Willwacher*, “Real models for the framed little  $n$ -disks operads”, Preprint, [arXiv:1705.08108](#), Theorems 1.1 and 1.3].

The principal objective in this paper is to introduce a dg Hopf cooperad  $LG_n$  built from suitably decorated Feynman diagrams, which is a model for the framed little disks operad and carries an action of the dg Lie algebra  $\mathfrak{a}_n$  compatible with the Hopf cooperad structure. The main result is a description of a model for the framed little disks operad in which all of the infinitesimal homotopy automorphisms can be realized as infinitesimal Hopf cooperad automorphisms.

Reviewer: [Hirokazu Nishimura \(Tsukuba\)](#)

**MSC:**

- [18G35](#) Chain complexes (category-theoretic aspects), dg categories
- [18G85](#) Graph complexes and graph homology
- [18M75](#) Topological and simplicial operads
- [53C15](#) General geometric structures on manifolds (almost complex, almost product structures, etc.)
- [53D55](#) Deformation quantization, star products

**Keywords:**

framed little disks operad; graph complex; real algebraic model; symmetry dg Lie algebra

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