Iragi, Minani; **Holgate, David**
Quasi-uniform structures and functors. (English) Zbl 07692247
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The introduction of categorical closure operators by *D. Dikranjan* and *E. Giuli* [Topology Appl. 27, 129–143 (1987; Zbl 0634.54008)] was the point of departure for study of topological structures on categories, which eventually motivated the introduction of categorical interior [*S. J. R. Vorster*, Quaest. Math. 23, No. 4, 405–416 (2000; Zbl 0974.18003)] and neighborhood operators [*D. Holgate* and *J. Šlapal*, Topology Appl. 158, No. 17, 2356–2365 (2011; Zbl 1232.54018)]. While the categorical interior operators were shown to be pleasantly related to neighborhood operators, a nice relationship between closure and neighborhood operators had been lacking until the *categorical topogenous structures* were introduced [*D. Holgate* et al., Appl. Categ. Struct. 24, No. 5, 447–455 (2016; Zbl 1359.54003); `https://etd.uwc.ac.za/xmlui/handle/11394/7081`]. The conglomerate of categorical topogenous structures is order isomorphic to the conglomerate of all neighborhood operators containing both the conglomerate of all interior operators as reflective subcategories.

Categorical syntopogenous structures are a natural generalization of categorical topogenous structures, providing a convenient setting to investigate a quasi-uniform structure on a category [*D. Holgate* and *M. Iragi*, Topology Appl. 263, 16–25 (2019; Zbl 1420.18003)]. The use of syntopogenous structures allows of description of a quasi-uniformity as a family of categorical closure operators.

This paper studies a number of categorical quasi-uniform structures induced by functors. The authors depart from a category $\mathcal{C}$ with a proper $(\mathcal{E},\mathcal{M})$-factorization system, defining the continuity of a $\mathcal{C}$-morphism with respecct to two syntopogenous structures on $\mathcal{C}$ and using it to describe the quasi-uniformities induced by pointed and copointed endofunctors of $\mathcal{C}$.

Thinking of categories supplied with quasi-uniformities as large spaces, the continuity of $\mathcal{C}$-morphisms is generalized to functors. It is shown that for an $\mathcal{M}$-fibration or a functor that has a right adjoint, one can obtain a concrete construction of the coarsest quasi-uniformity for which the functor is continuous.

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MSC:

18A05	Definitions and generalizations in theory of categories	Cited in **1** Document
18F60	Categories of topological spaces and continuous mappings	
54A15	Syntopogeneous structures	
54B30	Categorical methods in general topology	

Keywords:

closure operator; syntopogenous structure; quasi-uniform structure; (co)pointed endofunctor and adjoint functor

Full Text: arXiv Link

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