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Graded K-theory, filtered K-theory and the classification of graph algebras. (English)

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One of the beauties of the theory of Leavitt path algebras is that one can obtain a substantial amount of information about the structure of the algebra from the geometry of its associated graph. The theory of Leavitt path algebras is intrinsically related, via graphs, to the theory of symbolic dynamics and -algebras, where the major classification programs have been a domain of intense research in the last five decades. Since [*M. Rørdam*, Encycl. Math. Sci. 126, 145 p. (2002; Zbl 1016.46037); *G. Restorff*, J. Reine Angew. Math. 598, 185–210 (2006; Zbl 1111.46034)], filtered K-theory has been investigated and developed in [*S. Eilers* et al., Bull. Malays. Math. Sci. Soc. (2) 33, No. 2, 233–241 (2010; Zbl 1206.46051); Duke Math. J. 170, No. 11, 2421–2517 (2021; Zbl 1476.46064)], where it was shown that the sublattice of gauge invariant prime ideals and their quotient K-groups can be used as an invariant. It was shown in [*S. Eilers* et al., Can. J. Math. 70, No. 2, 294–353 (2018; Zbl 1396.46047)] that filtered K-theory is a complete invariant for unital graph C^*-algebras. The paper [*S. Eilers* et al., MATRIX Book Ser. 1, 229–249 (2018; Zbl 1448.19007)] introduced the filtered K-theory in the genuinely algebraic setting, showing that if two Leavitt path algebras with coefficients in complex numbers $\mathbb{C}$ have isomorphic filtered algebraic K-theory, then the associated graph C^*-algebras have isomorphic filtered K-theory.

This paper is devoted to graded K-theory as a capable invariant for the classification of graph algebras. This approach was initiated in [*R. Hazrat*, Math. Ann. 355, No. 1, 273–325 (2013; Zbl 1262.16047)] and further studied in [*P. Ara* and *E. Pardo*, J. K-Theory 14, No. 2, 203–245 (2014; Zbl 1311.16003); *R. Hazrat*, J. Algebra 384, 242–266 (2013; Zbl 1294.16005); J. Algebra 375, 33–40 (2013; Zbl 1284.16004)]. This paper aims to show that in the setting of graph algebras, graded K-theory determines a large portion of filtered K-theory, demonstrating that for two Leavitt path algebras over a field, if their graded Grothendieck groups are isomorphic, then certain precisely defined quotients of their filtered K-theories are also isomorphic.

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MSC:

16D70 Structure and classification for modules, bimodules and ideals (except as in 16Gxx), direct sum decomposition and cancellation in associative algebras)
16S88 Leavitt path algebras
18F30 Grothendieck groups (category-theoretic aspects)

Keywords:

Leavitt path algebra; graph C^*-algebra; graded K-theory; filtered K-theory; graded prime ideal; graded Grothendieck group

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