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Additive Grothendieck pretopologies and presentations of tensor categories. (English)

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The notion of a Grothendieck topology on a small category allows of defining a category of sheaves and the development of sheaf cohomology [*M. Artin* (ed.) et al., Séminaire de géométrie algébrique du Bois-Marie 1963–1964. Théorie des topos et cohomologie étale des schémas. (SGA 4). Un séminaire dirigé par M. Artin, A. Grothendieck, J. L. Verdier. Avec la collaboration de N. Bourbaki, P. Deligne, B. Saint-Donat. Tome 1: Théorie des topos. Exposés I à IV. 2e éd. Berlin-Heidelberg-New York: Springer-Verlag (1972; Zbl 0234.00007); *G. Tamme*, Introduction to étale cohomology. Translated by Manfred Kolster. Berlin: Springer-Verlag (1994; Zbl 0815.14012)]. The notion of Grothendieck topologies extends canonically to enriched categories [*F. Borceux* and *C. Quinteiro*, Rapp., Sémin. Math., Louvain, Nouv. Sér. 245–260, 171–193 (1996; Zbl 0883.18007); Cah. Topologie Géom. Différ. Catégoriques 37, No. 2, 145–162 (1996; Zbl 0883.18006)]. Besides, by the Gabriel-Popesco theorem [*N. Popesco* and *P. Gabriel*, C. R. Acad. Sci., Paris 258, 4188–4190 (1964; Zbl 0126.03304)], a category is Grothendieck abelian iff it can be realized as the category of additive sheaves on a preadditive site. However, the notion of a Grothendieck pretopology does not extend naively to the additive setting.

This paper defines a notion on preadditive categories playing a role similar to the notion of a Grothendieck pretopology on an unenriched category. Each such additive pretopology defines an additive Grothendieck topology, sufficing for defining the sheaf cohomology.

The synopsis of the paper goes as follows.

- §1 gives prelimiinaries on Grothendieck topologies and categories.
- §2 introduces additive pretopologies, establishing the connection with additive topologies, demonstrating the characterization of sheaves, and proving that the notion of a pretopology provides a universal property for the sheaf category.
- §3 studies subcanonical and noetherian topologies, as inspired by the corresponding unenriched notions in [*G. Tamme*, Introduction to étale cohomology. Translated by Manfred Kolster. Berlin: Springer-Verlag (1994; Zbl 0815.14012), I.§1.3 and I.§3.10]. An additive topology is subcanonical if the representable presheaves are sheaves, while it is noetherian if the sheafifications of the representable presheaves are compact. These notions are difficult to characterize directly from the topology, but they are easily described on the level of pretopologies. The notion of monoidal additive Grothendieck topologies is also introduced. The tensor product of two Grothendieck categories is addressed after [*W. Lowen* et al., Int. Math. Res. Not. 2018, No. 21, 6698–6736 (2018; Zbl 1408.18024)].
- §4 investigates the presentations of tensor categories.
- Appendix A looks back at the non-enriched Grothendieck pretopologies. The author generalizes the notion via some natural steps to arrive at its non-linear version he put forward as an additive Grothendieck pretopology.

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MSC:

18E10 Abelian categories, Grothendieck categories
18E35 Localization of categories, calculus of fractions
18F10 Grothendieck topologies and Grothendieck topoi
18D15 Closed categories (closed monoidal and Cartesian closed categories, etc.)

Keywords:

additive Grothendieck topology; Grothendieck category; Noetherian and subcanonical topologies; tensor category

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