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Generalized cohomological field theories in the higher order formalism. (English)

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Commun. Math. Phys. 399, No. 3, 1439-1500 (2023)

Using ideas coming from the BCOV theory [*M. Bershadsky* et al., Commun. Math. Phys. 165, No. 2, 311–427 (1994; Zbl 0815.53082)], *S. Barannikov* and *M. Kontsevich* [Int. Math. Res. Not. 1998, No. 4, 201–215 (1998; Zbl 0914.58004)] gave a method to construct, out of a differential graded Batalin-Vilkovsky (dg BV) algebra abiding by certain conditions, a formal Dubrovin-Frobenius manifold. They applied it in the case of the dg BV algebra of polyvector fields on a Calabi-Yau manifold X, in which case their results produce a structure of a formal Dubrovin-Frobenius manifold on the Dolbeault cohomology of X. There is a mirror partner of the latter result for the de Rham cohomology of a symplectic manifold abiding by the hard Lefschetz theorem [*S. A. Merkulov*, Int. Math. Res. Not. 1998, No. 14, 727–733 (1998; Zbl 0931.58002)]. In principle, one can drop some pieces of structure of a Dubrovin-Frobenius manifold, like the inner product and the unit, where the underlying algebraic statement follows from the explicit description of the homotopy quotient of the operad BV by the Batalin-Vilkovsky operator Δ, established by *G. C. Drummond-Cole* and *B. Vallette* [Sel. Math., New Ser. 19, No. 1, 1–47 (2013; Zbl 1264.18010)]. Miraculously, the homotopy quotient of BV by Δ is to be represented by a non-differential operad, meaning that an unexpected formality theorem holds. The operad in question is the operad of hypercommutative algebras HyperCom.

Algebraically, a BV algebra may be defined as a commutative algebra rigged out in a unary operation Δ of homological degree one which squares to zero and is a differential operator of order at most two with respect to the commutative product. About two decades ago Losev asked what would change in the result of homotopical trivialization if one considers as an input an analog of a BV algebra, where the BV operator is a differential operator of an arbitrary finite order, not necessarily of order two. In the past decade or so, a number of results emerged that strongly suggest that the case of an operator of order different from two may be both tractable and interesting. Most of these results are related to the Givental group action on the space of hypercommutative algebra structure on a given vector space V, or, more generally, the space

$$\mathrm{Hom}\,(\mathrm{HyperCom}, \mathcal{O})$$

for an arbitrary target operad $\mathcal{O}$.

This paper shows that one may be able to use the Givental group action to compute the homotopy quotient by an operator Δ by higher order, answering the question of Losev. Methods of this paper are entirely algebraic, being an elementary algebraic approach to computations with psi-classes and Givental symmetries, which completely bypasses the intersection theory on moduli spaces of curves. The geometry behind the generalized cohomological field theories and the higher order formalism is a topic of the authors' ongoing project in a separate paper.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18Nxx Higher categories and homotopical algebra
81Txx Quantum field theory; related classical field theories
53Dxx Symplectic geometry, contact geometry

Full Text: DOI arXiv

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paper as accurately as possible without claiming the completeness or perfect precision of the matching.