Zapata-Carratala, Carlos

Dimensioned algebra: mathematics with physical quantities. (English) Zbl 07629583

Matematica 1, No. 4, 849-885 (2022)

Dimensional analysis first became a part of the physical science vernacular during the 19th century with the work of *Fourier* [Theórie analytique de la chaleur. Nouv. éd. Breslau: Kohner (1883; JFM 15.0954.01); [Analytische Theorie der Wärme. Deutsche Ausgabe von B. Weinstein. Berlin: Julius Springer (1884; JFM 16.1075.01)] after partial precedents in the writings of Descartes and Euler on mathematics. Dimensional analysis has not become part of the mainstream scientific discourse until the early 20th century with the works of *E. Buckingham* ["On physically similar systems; illustrations of the use of dimensional equations", Phys. Rev. 4, No. 4, 345–376 (1914; doi:10.1103/PhysRev.4.345)] and *Lord Rayleigh* [Phil. Mag. (6) 24, 864–869 (1912; JFM 43.0334.04)].

It was not until the work of *J. Janyška* et al. [Acta Appl. Math. 110, No. 3, 1249–1276 (2010; Zbl 1208.15021); "Semi-vector spaces and units of measurement", Preprint, arXiv:0710.1313] on semi-vector spaces and positive spaces that we find all the standard features of dimensional analysis in a transparent and mathematically rigorous framework. This paper aims to define a theory of algebra informed by how physical quantities are used in practice. To this end, the author gives the definition of a generalized notion of set and binary operation, unfurling an analogue of the ordinary theory of commutative algebra and discussing the generalizations of objects such as abelian groups, rings, modules and algebras.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18B99 Special categories
70G55 Algebraic geometry methods for problems in mechanics
13P25 Applications of commutative algebra (e.g., to statistics, control theory, optimization, etc.)
00A79 Physics

Keywords:

commutative algebra; category theory; mathematical physics; metrology; observables

Full Text: DOI arXiv

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