Coulembier, Kevin; **Etingof, Pavel**; **Ostrik, Victor**; **Pauwels, Bregje**
Monoidal abelian envelopes with a quotient property. (English) Zbl 07640140
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A fully faithful embedding of a pseudo-tensor category into a tensor category with an appropriate universal property is known as an abelina envelope of the pseudo-tensor category. The question of which pseudo-tensor categories do admit an abelian envelope has attracted a lot of attention recently [*D. Benson* and *P. Etingof*, Adv. Math. 351, 967–999 (2019; Zbl 1430.18013); *D. Benson* et al., "New incompressible symmetric tensor categories in positive characteristic", Preprint, arXiv:2003.10499; *J. Comes* and *V. Ostrik*, Algebra Number Theory 8, No. 2, 473–496 (2014; Zbl 1305.18019); *K. Coulembier* et al., Algebra Number Theory 16, No. 9, 2099–2117 (2022; Zbl 07640151); *K. Coulembier*, "Additive Grothendieck pretopologies and presentations of tensor categories", Preprint, arXiv:2011.02137; *P. Deligne*, in: Algebraic groups and homogeneous spaces. Proceedings of the international colloquium, Mumbai, India, January 6–14, 2004. New Delhi: Narosa Publishing House/Published for the Tata Institute of Fundamental Research. 209–273 (2007; Zbl 1165.20300); *I. Entova-Aizenbud* et al., Int. Math. Res. Not. 2020, No. 15, 4602–4666 (2020; Zbl 1477.18034)], leading to various interesting applications. It is observed empirically that every object in the tensor category is a quotient of an object in the pseudo-tensor category. On the other hand, it was observed in [*D. Benson* et al., "New incompressible symmetric tensor categories in positive characteristic", Preprint, arXiv:2003.10499] that when a pseudo-tensor category $\boldsymbol{D}$ can be embedded in a tensor category $\boldsymbol{T}$ such that every object in $\boldsymbol{T}$ is a quotient of an object in $\boldsymbol{D}$, $\boldsymbol{T}$ must be the abelian envelope of $\boldsymbol{D}$.

This paper aims to further investigate abelian envelopes, in particular in relation to the above quotient property. The first main result establishes an intrinsic criterion on a pseudo-tensor category determining whether there exists an abelian envelope with the quotient property. The other main results establish new ways to interpret known tensor categories as abelian envelopes. It is also investigated whether the Deligne product and extension of scalars are always tensor categories. It is finally demonstrated that if the extension of scalars is always a tensor category, then so is Deligne's product of tensor categories.

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MSC:

18-XX Category theory; homological algebra
81-XX Quantum theory

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