数理物質科学研究科　博士論文の要約

Theoretical Study on Topological Phenomena in Honeycomb Structures with Kekulé Distortion

(ケクレ変形をもつ蜂の巣構造におけるトポロジカル現象の理論研究)

Huang Hong

Doctoral Program in
Materials Science and Engineering
Student ID: 201830112
Doctor of Philosophy in
Engineering

Abstract

Topology, which plays a central role in topological insulators, has attracted increasing interests in material sciences [1–3]. Surface or edge states of topological insulators that propagate robustly against disorder are protected by the topology in bulk band structures. Besides, in quantum spin Hall insulators (2D topological insulators) [4, 5], the propagation direction of edge state is fixed by spin. Such a spin-momentum locking effect can reduce the possibility of back-scattering. These properties can be exploited for achieving innovative applications such as non-dissipative transportation and spintronics. Although studies were first focused on electronic systems, the idea of topology has also been developed into bosonic systems and various wave phenomena [6–8].

Normally, a spin-orbit coupling (SOC) term is necessary for achieving the quantum spin Hall insulator. In 2015, Longhua Wu and Xiao Hu proposed a scheme that a topological state can be achieved in photonic crystals purely based on conventional dielectric materials [9]. Starting from a honeycomb lattice, a hexagonal unit cell containing six cylinders is chosen, where the C_{6v} symmetry is preserved. Changing lattice constant while fixing the distance between cylinders inside the unit cell that preserves a pseudo time-reversal symmetry originated from the C_{6v} symmetry and the time-reversal symmetry can open a bulk band gap at the Γ point. A nontrivial topology appears by reducing the lattice constant, accompanying a *p*-*d* band inversion. In resultant topological modes, a pseudospin-momentum locking phenomenon can be observed, where the angular momentum of the wave function of the out-of-plane electric field serves as the pseudospin. The existence of topological modes and pseudospin-momentum locking in this design have been confirmed experimentally [10, 11]. Only coupling textures that preserve the C_{6v} symmetry are sufficient in the consideration of this simple design, making it easy to apply to other materials.

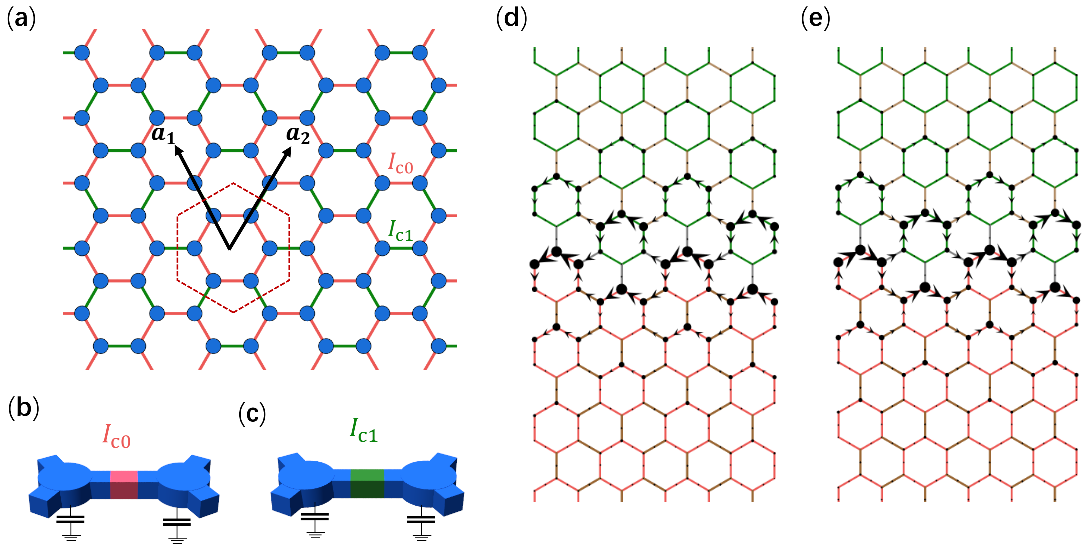

FIG. 1. (a) Josephson junction array (JJA) on honeycomb lattice. Each superconductor is shunted to a common ground by a capacitor with the common capacitance C and connected to each other by Josephson junctions. Superconductors are grouped into hexagonal unit cells indicated by the dashed line and Josephson critical current is I_{c0}/I_{c1} inside/between the unit cells. Lattice vectors are $\boldsymbol{a_1}$ and $\boldsymbol{a_2}$. (b) and (c) Schematics of Josephson junctions in (a), where superconducting parts are represented by blue, and insulating parts are presented by red/green for I_{c0}/I_{c1}. (d) and (e) Distributions of wavefunction amplitude (size of dot) and time-averaged energy flow (arrow between sites) for the topological Josephson plasmon mode at opposite momenta.

In the thesis, we focus on the exploration of topological modes in typical wave systems such as Josephson plasma in superconductor Josephson junction arrays (JJAs), magnon transportation in magnetic materials and photonic crystal fibers (PCF). First, we start from topological Josephson plasmon modeson honeycomb lattice with superconductors shunted individually to a common ground by capacitors with capacitance C uniform in the system. Superconducting currents determined by dc and ac Josephson relation cause charging of superconductors, while capacitors linked to superconductors act as restoring forces, leading to Josephson plasmon modes, which are oscillations in charge and phase of the superconductors. For honeycomb lattice JJAs, Dirac cones are formed at the K and K' points of the Brillouin zone (BZ) in the frequency dispersion relation of Josephson plasmon modes [13]. Here we introduce a real-space pattern of Josephson critical current, where Josephson critical currents inside and between hexagonal unit cells are put same but different from each other, as shown in Fig. 1(b) and 1(c). When the critical current between hexagonal unit cells becomes larger than that inside the unit cell, a *p*-*d* band inversion takes place resulting in a nontrivial topology. As shown in Fig. 1(d) and 1(e), topological Josephson plasmon modes appear at the interface between topological and trivial domains (green/light brown lines for I_{c0}/I_{c1} in trivial domains and red/dark brown lines for I_{c0}/I_{c1} in topological domains), showing counterclockwise and clockwise circulations of Poynting vectors in unit cells correspond to up and down pseudospins respectively. The net energy flows for the up and down pseudospin point to opposite directions, manifesting the pseudospin-momentum locking phenomenon realized in JJAs.

Next, we consider topological magnons in magnetic materials. Magnons are quanta of spin-wave excitations in magnetic systems, which are described in terms of Holstein-Primakoff transformation to the Heisenberg spin model. To introduce topology into magnetic materials, one promising way is using the Dzyaloshinskii-Moriya (DM) interaction which plays the same

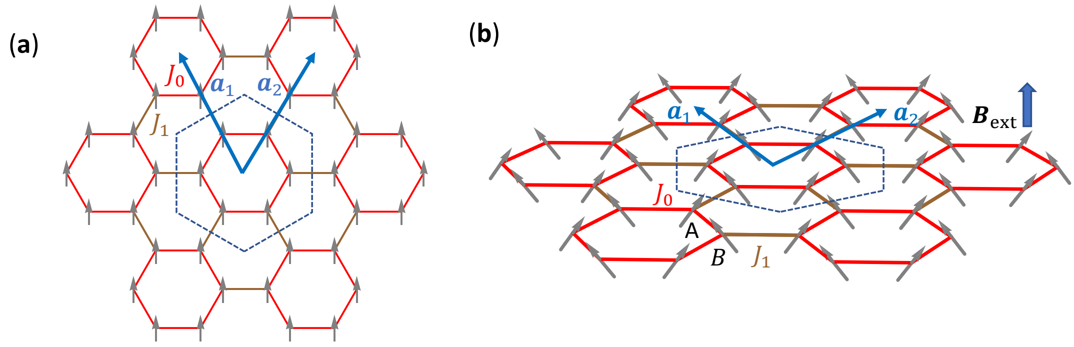

FIG. 2. (a) Ferromagnet on honeycomb lattice with nearest-neighbor exchange couplings. Hexagonal unit cells indicated by the dashed line are chosen with J_0/J_1 representing exchange couplings inside/between hexagonal unit cells. Unit vectors are represented by $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$. (b) Antiferromagnet on honeycomb lattice under an external magnetic field $B_{\rm ext}$ perpendicular to the lattice. Same unit cells are chosen as in (a) but spins of sublattice A and B are pointing in different directions.

role as SOC in eletronic systems [14, 15]. However, a sufficiently strong DM interaction is not easy to realize. Here, we propose topological magnon modes on honeycomb lattice by tuning nearest-neighbor (n.n.) exchange couplings, where no DM interaction is required [16]. Dirac-type linear magnon dispersions appear in the frequency magnon band structure of a ferromagnet on honeycomb lattice where only n.n. exchange couplings are considered. With a C_{6v}-symmetric texture in the strength of exchange couplings as shown in Fig. 2, where those between hexagonal unit cells are larger than those inside a unit cell, a frequency band gap opens, which yields a p-d band inversion and the nontrivial topology. Topological magnon modes robust to defects and disorder appear along the interface when topological and trivial structures are put side by side.

At last, we discuss using the concept of topology in constructing optical fibers. Light is confined inside the core by the total internal reflection in conventional optical fibers. In topological photonic crystals, topological modes have good confinement at the interface, so that it is possible to construct PCFs that benefit from topology. In the topological PCF as shown in Fig. 3, the core is formed by a topological photonic crystal surrounded by a trivial photonic crystal as the cladding, where each hexagonal unit cell contains six identical triangular holes, and in topological and trivial photonic crystals the distance between triangular holes and the center of unit cells is finite tuned. We clarify that even with nonzero momentum along the fiber axis, p-d band inversion still exists which yields the nontrivial photonic topology [17]. Topological interface modes are localized at the interface of the core and cladding, carrying orbital angular momenta depending on the pseudospin. Another mechanism to provide confined modes in topological PCFs is to exploit band-inversion-induced reflection [18]. Below the bulk band gap, d and p modes are dominant around the Γ point in the topological and trivial photonic crystal respectively, inducing a reflection at the interface. There appear several confined bulk modes below the lower band edge for the topological PCF, which also carry orbital angular momenta depending on the pseudospin. We find that the mode close the lower band edge is very well confined in the core tipological photonic crystal, which may be useful for constructing stable and strong fiber laser.

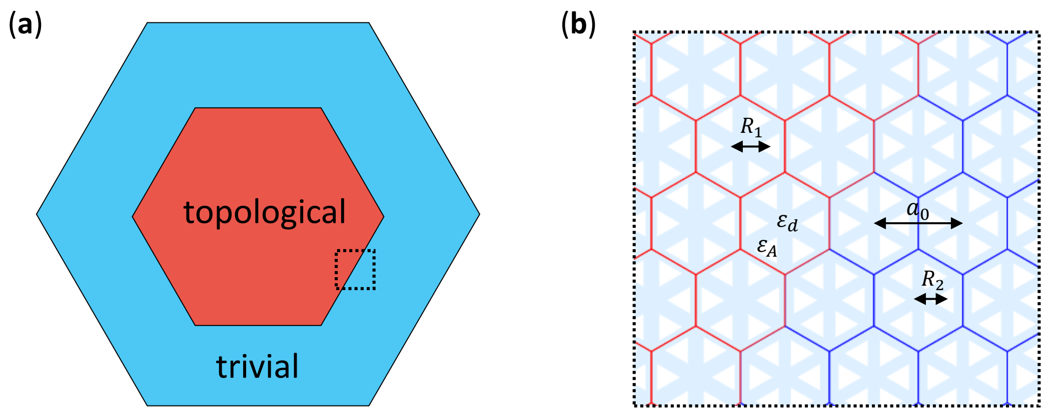

FIG. 3. (a) Cross-section of photonic crystal fiber (PCF). A topological photonic crystal forms the core surrounded by a trivial photonic crystal as the cladding. (b) Zoom-in picture of the part denoted by the dashed line in (a). Each unit cell contains six identical triangular holes, which are away from the center of the unit cell by R_1/R_2 in the topological/trivial photonic crystal. a_0 is the lattice distance in both topological and trivial photonic crystals and $\varepsilon_A/\varepsilon_d$ is the dielectric constant of air/dielectric material.

[1] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. **82**, 3045 (2010).
[2] X.-L. Qi and S.-C. Zhang, Rev. Mod. Phys. **83**, 1057 (2011).
[3] H. Weng, R. Yu, X. Hu, X. Dai, and Z. Fang, Adv. Phys. **64**, 227 (2015).
[4] C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005).
[5] B. A. Bernevig, T. L. Hughes, and S.-C. Zhang, Science **314**, 1757 (2006).
[6] F. D. M. Haldane and S. Raghu, Phys. Rev. Lett. **100**, 013904 (2008).
[7] Z. Yang, F. Gao, X. Shi, X. Lin, Z. Gao, Y. Chong, and B. Zhang, Phys. Rev. Lett. **114**, 114301 (2015).
[8] S. D. Huber, Nat. Phys. **12**, 621 (2016).
[9] L.-H. Wu and X. Hu, Phys. Rev. Lett. **114**, 223901 (2015).
[10] Y. Li, Y. Sun, W. Zhu, Z. Guo, J. Jiang, T. Kariyado, H. Chen, and X. Hu, Nat. Commun. **9**, 4598 (2018).
[11] Y. Yang, Y.-F. Xu, T. Xu, H.-X. Wang, J.-H. Jiang, X. Hu, and Z.-H. Hang, Phys. Rev. Lett. **120**, 217401 (2018).
[12] H. Huang, T. Kariyado, and X. Hu, Opt. Mater. Express **11**, 448 (2021).
[13] S. Banerjee, J. Fransson, A. M. Black-Schaffer, H. Ågren, and A. V. Balatsky, Phys. Rev. B **93**, 134502 (2016).
[14] L. Zhang, J. Ren, J.-S. Wang, and B. Li, Phys. Rev. B **87**, 144101 (2013).
[15] S. A. Owerre, J. Phys. Condens. Matter **28**, 386001 (2016).
[16] H. Huang, T. Kariyado, and X. Hu, Sci. Rep. **12**, 1 (2022).
[17] H. Huang, T. Kariyado, and X. Hu, in preparation.
[18] Z.-K. Shao, H.-Z. Chen, S. Wang, X.-R. Mao, Z.-Q. Yang, S.-L. Wang, X.-X. Wang, X. Hu, and R.-M. Ma, Nature Nanotechnology **15**, 67 (2020).