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The Hodge Chern character of holomorphic connections as a map of simplicial presheaves. (English) Zbl 07585320

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This paper defines a map of simplicial presheaves, the Chern character, assigning to every sequence of composable non-connection-preserving monomorphisms of vector bundles with holomorphic connections an appropriate sequence of holomorphic forms. The authors apply this Chern character map to Čech nerve of a good cover of a complex manifold, assembling the data by passing to the totalization to get a map of simplicial sets (Corollary 3.3). On the 0-simplicies, the totalized map gives a combinatorial formula for the Chern character of a bundle in the Čech-Hodge complex in terms of the transition functions [*N. R. O'Brian* et al., Am. J. Math. 103, 225–252 (1981; Zbl 0473.14008)]. Over the 1-simp;icies, the authors obtain a formula for the Hodge-Chern-Simons invariant of bundle isomorphisms in the Čech complex with respect to the domain and range connections and in terms of the transition functions of the bundle. The authors also this Chern character to complex Lie groupoids to obtain invariants of bundles on them in terms of the simplicial data.

There is also an infinity homotopy coherent version of all of this, where vector bundles are replaced by derived families whose clutching functions fit together only up to an infinite system of coherent homotopies. This relates to [*D. Toledo* and *Y. L. L. Tong*, Math. Ann. 237, 41–77 (1978; Zbl 0391.32008); *D. Toledo* and *Y. L. L. Tong*, Ann. Math. (2) 108, 519–538 (1978; Zbl 0413.32006)]. The homotopy coherent story will be addressed in a forthcoming paper, being to be employed to get invariants of the derived automorphisms of coherent sheaves on complex manifolds. Another foreseeable direction is to develop a commutative diagram of spaces, which, after applying π_0, results in the classical Grothendieck-Riemann-Roch (GRR) commutative square. This will extend the differential geometric discussion of GRR established in [*N. R. O'Brian* et al., Bull. Am. Math. Soc., New Ser. 5, 182–184 (1981; Zbl 0495.14010); *N. R. O'Brian* et al., Math. Ann. 271, 493–526 (1985; Zbl 0539.14005)] to the entire K-theory spectrum.

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19L10 Riemann-Roch theorems, Chern characters
58J28 Eta-invariants, Chern-Simons invariants

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