Clingman, Tslil; Moser, Lyne

2-limits and 2-terminal objects are too different. (English) Zbl 07629355

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It is well known in ordinary category theory that limits are equivalent to terminal objects in the slice category of cones. The principal objective in this paper is to show by counter-examples that the 2-categorical analogues of the theorem relating 2-limits and 2-terminal objects in various choices of slice 2-categories of 2-cones fail, even if relaxing the 2-cones to pseudo- or lax-natural transformations or considering bi-type limits and bi-terminal objects.

The synopsis of the paper goes as follows.

§2 introduces the notions of 2-limits and strict-slices of 2-cones, establishing that a 2-limit is always 2-terminal in the strict-slice of 2-cones while providing a counter-example against the converse.

§3 turns to the larger 2-categories of pseudo- and lax-slicees of 2-cones, demonstrating by counter-examples that 2-limits are in fact unrelated to 2-terminal objects in these.

§4 introduces pseudo- and lax-limits, investigating their relationships with 2-terminal objects in the different slices.

§5 addresses the case of bi-type limits, showing that these are particularly always bi-terminal in the pseudo-slice of appropriate cones and adapting the results obtained for the 2-type cases to the bi-type cases.

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MSC:

18N10 2-categories, bicategories, double categories
18A30 Limits and colimits (products, sums, directed limits, pushouts, fiber products, equalizers, kernels, ends and coends, etc.)
18A25 Functor categories, comma categories
18A05 Definitions and generalizations in theory of categories

Keywords:

2-dimensional limits; 2-dimensional terminal objects; slice 2-categories

Full Text: DOI arXiv

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