Macpherson, James
Extension of the 2-representation theory of finitary 2-categories to locally (graded) finitary 2-categories. (English) ⎍Zbl 07533810⎍

The study of the 2-representation theory of finitary and fiat 2-categories was pioneered by *V. Mazorchuk* and *V. Miemietz* [Compos. Math. 147, No. 5, 1519–1545 (2011; Zbl 1232.17015); Mosc. Math. J. 14, No. 3, 595–615 (2014; Zbl 1342.17003); Int. Math. Res. Not. 2016, No. 24, 7471–7498 (2016; Zbl 1404.18014); Math. Z. 282, No. 1–2, 411–434 (2016; Zbl 1375.18037)] and further exploited in various other works [*M. Mackaay* et al., Indiana Univ. Math. J. 68, No. 1, 1–33 (2019; Zbl 1472.18019); *A. Chan* and *V. Mazorchuk*, Math. Proc. Camb. Philos. Soc. 166, No. 2, 325–352 (2019; Zbl 1411.18008)], having important applications in quotients of 2-Kac-Moody algebras [*V. Mazorchuk* and *V. Miemietz*, Trans. Am. Math. Soc. 368, No. 11, 7623–7644 (2016; Zbl 1365.18006)] and Soergel bimodules [*M. Mackaay* and *V. Mazorchuk*, J. Pure Appl. Algebra 221, No. 3, 565–587 (2017; Zbl 1404.18013); *J. Zimmermann*, J. Pure Appl. Algebra 221, No. 3, 666–690 (2017; Zbl 1360.18008); *T. Kildetoft* et al., Trans. Am. Math. Soc. 371, No. 8, 5551–5590 (2019; Zbl 1409.18005)]. This paper extends the theory to certain 2-categories with infinitely many objects, called locally finitary 2-categories, generalizing the classical classification results of simple transitive 2-representations of weakly fiat 2-categories to this environment.

The synopsis of the paper goes as follows.

§2 gives the initial definitions for locally finitary 2-categories and their 2-representations, presenting some minor results which demonstrate that the cell structures of the 2-category is highly analogous in the generalization.

§3 generalizes various results in [*M. Mackaay* et al., Indiana Univ. Math. J. 68, No. 1, 1–33 (2019; Zbl 1472.18019)], leading up to Theorem 3.6 as well as Theorem 3.12, which classifies the simple transitive 2-representations for locally finitary 2-categories associated to certain infinite-dimensional algebras.

§4 considers generalizations of various results in [*V. Mazorchuk* and *V. Miemietz*, Compos. Math. 147, No. 5, 1519–1545 (2011; Zbl 1232.17015); Mosc. Math. J. 14, No. 3, 595–615 (2014; Zbl 1342.17003); Int. Math. Res. Not. 2016, No. 24, 7471–7498 (2016; Zbl 1404.18014); Math. Z. 282, No. 1–2, 411–434 (2016; Zbl 1375.18037); Trans. Am. Math. Soc. 368, No. 11, 7623–7644 (2016; Zbl 1365.18006)], eventually aiming to establish Theorem 4.33 classifying all simple transitive 2-representations of strongly regular locally weakly fiat 2-categories as being equivalent to cell 2-representations. The result is utilized to classify all simple transitive 2-representations of cyclotomic 2-Kac-Moody algebras.

§5 presents an application of the result by demonstrating that cyclotomic 2-Kac-Moody algebras of given weights are locally weakly fiat 2-categories, submitting to the aforementioned theorem.

§6 examines the further generalization to locally restricted G-finitary 2-categories for some countable abelian group G, constructing a degree zero 2-category associated to such a 2-category and using it to construct a degree zero coalgebra 1-morphism for a given graded transitive 2-representation of the original 2-category. The setup allows of proving Theorem 6.21 claiming that the previously constructed internal 2-representations associated to a transitive 2-representation is to be viewed as a degree zero construction in a canonical fashion. This is applied to the cyclotomic 2-Kac-Moody categories of given weights, showing that their cell 2-representations are all graded simple 2-representations, which leads to Theorem 6.25 claiming that any simple transitive 2-representation is in fact a graded 2-representation.

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MSC:

Cited in **2** Documents

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