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Nilpotent types and fracture squares in homotopy type theory. (English) Zbl 07283042

Math. Struct. Comput. Sci. 30, No. 5, 511-544 (2020)

Nilpotent spaces play a significant role in the homotopy theory of spaces. Many results holding for simply connected spaces are to be generalized to nilpotent spaces, which have a rich theory of localizations away from sets of numbers, including fracture squares that reconstruct a space out of some of its localizations.

The principal objective in this paper is to develop the basic theory of nilpotent spaces in Homotopy Type Theory [*The Univalent Foundations Program*, Homotopy type theory. Univalent foundations of mathematics. Princeton, NJ: Institute for Advanced Study; Raleigh, NC: Lulu Press (2013; Zbl 1298.03002)], which give constructive proofs holding in any ∞-topos.

The synopsis of the paper goes as follows.

§2 establishes the equivalence between the two characterizing properties of nilpotency (Theorem 2.60), for which the relationship between unpointed Eilenberg-MacLane spaces and doubly pointed Eilenberg-MacLane spaces are studied. It is shown that the type of unpointed n-dimensional Eilenberg-MacLane spaces is equivalent to the type of doubly pointed $(n+1)$-dimensional Eilenberg-MacLane spaces (Theorem 2.25).

§3 establishes, following the suggestion of Shulman [https://homotopytypetheory.org/2014/06/30/fibrations-with-em-fiber/], that cohomology isomorphisms between nilpotent types induce isomorphisms in all homotopy groups.

§4 investigates the localization of a nilpotent type and its effect on homotopy groups, showing that the localization of a nilpotent type localizes its homotopy groups in the expected way (Theorem 4.19).

§5 constrcuts a fracture square for simply connected types without assuming Whitehead's principle (Theorem 5.4). This section as well as the previous one builds on [*E. Rijke* et al., "Modalities in homotopy type theory", Preprint, arXiv:1706.07526].

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MSC:

18N45 Categories of fibrations, relations to K-theory, relations to type theory
03B70 Logic in computer science
55P60 Localization and completion in homotopy theory
18E35 Localization of categories, calculus of fractions

Keywords:

homotopy type theory; nilpotent space; fracture square; Eilenberg-MacLane space; principal fibration

Full Text: DOI arXiv

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