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Natural deduction for quantum logic. (English) Zbl 07603237

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A natural deduction for quantum logic was proposed in [*Y. Delmas-Rigoutsos*, J. Philos. Log. 26, No. 1, 57–67 (1997; Zbl 0868.03029)]. Different from this, this paper proposes a natural deduction system corresponding to Nishimura's quantum sequent calculus ***GOM*** [*H. Nishimura*, J. Symb. Log. 45, 339–352 (1980; Zbl 0437.03034)]. While Nishimura's sequential system adopts conjunction ($\wedge$) and negation ($\neg$) as basic operations, this paper adopts the Sasaki hook [*U. Sasaki*, J. Sci. Hiroshima Univ., Ser. A 17, 293–302 (1954; Zbl 0055.25902)] as a kind of quasi-implication as well. Since the Sasaki hook fails to satisfy the deduction theorem, special care is required in dealing with assumptions.

Once a natural deduction system for quantum logic is obtained, the corresponding quantum λ-calculus is introduced via the Curry-Howard correspondence [*H. B. Curry*, Proc. Natl. Acad. Sci. USA 20, 584–590 (1934; JFM 60.0850.01); *H. B. Curry* and *R. Feys*, Combinatory logic. With two sections by William Craig. Amsterdam: North- Holland Publishing Company (1958; Zbl 0081.24104); `https://www.cs.cmu.edu/~crary/819-f09/Howard80.pdf`]. The proofs of the natural deduction system can be reversibly translated into the terms of the λ-calculus. The strong normalization property for the quantum λ-calculus is demonstrated. The proof of the strong normalization property follows [*J.-Y. Girard* et al., Proofs and types. Cambridge etc.: Univ. Press (1989; Zbl 0671.68002)]. Some λ-calculi based on intuitionistic linear logic were studied under the name of quantum λ-calculus [*P. Selinger* and *B. Valiron*, in: Semantic techniques in quantum computation. Cambridge: Cambridge University Press. 135–172 (2010; Zbl 1344.68052); *A. van Tonder*, SIAM J. Comput. 33, No. 5, 1109–1135 (2004; Zbl 1057.81016)].

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MSC:

03G12	Quantum logic
03B60	Other nonclassical logic
03F03	Proof theory in general (including proof-theoretic semantics)
68N18	Functional programming and lambda calculus
81P10	Logical foundations of quantum mechanics; quantum logic (quantum-theoretic aspects)

Keywords:

quantum logic; natural deduction; λ-calculus; curry-howard isomorphism; normalization

Full Text: DOI

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