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Eilenberg-Moore and Kleisli type categories for bimonads on arbitrary categories. (English)

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In category theory, the problem of developing a suitable Hopf algebra theory on an arbitrary category in place of classical Hopf algebras over vector spaces has been considered in various places, using various approaches. The first natural generalization was to the level of braided monoidal categories and it was achieved by replacing the classical flip map of vector spaces with the braiding of the base category. Many important results from the classical theory of Hopf algebras over vector spaces have been transferred to this context.

Another way of extending the Hopf algebra theory to a more general context is by using (co)monads, which in turn requires the existence of a monoidal structure on the base category. The notion generalizing bialgebras in this setting is called *bimonads*, while the notion generalizing Hopf algebras in the setting is called *Hopf monads* [*A. Bruguières* and *A. Virelizier*, Adv. Math. 215, No. 2, 679–733 (2007; Zbl 1168.18002); *I. Moerdijk*, J. Pure Appl. Algebra 168, No. 2–3, 189–208 (2002; Zbl 0996.18005)]. Bimonads are monads for which the associated endofunctor and natural transformations are required to be comonoidal. Hopf monoids are defined by assuming that the base category is autonomous.

A somewhat different approach aiming to generalize bialgebras/Hopf algebras to arbitrary categories without assuming the existence of a monoidal structure was considered in [*R. Wisbauer*, Appl. Categ. Struct. 16, No. 1–2, 255–295 (2008; Zbl 1154.18001); *B. Mesablishvili* and *R. Wisbauer*, J. K-Theory 7, No. 2, 349–388 (2011; Zbl 1239.18002)]. A bimonad is an endofunctor with both a monad and a comonad structure which are linked together by a so-called mixed distributive law which in some sense plays the role of the flip map. A Hopf bimonad is a bimonad which admits a natural transformation on the underlying endofunctor satisfying a compatibility similar to the classical antipode condition.The principal objective in this paper is to show that there is a rich theory behind the bimonads/Hopf monads in [loc. cit.]. The synopsis of the paper goes as follows.

- §1 gives preliminaries.
- §2 is concerned with the Eilenberg-Moore category associated to a bimonad H, which is the category of triples consisting of H-algebra and H-coalgebra structures on the same object which fuffill a compatibility condition generalizing the one defining the classical Hopf modules. Limits and colimits in the Eilenberg-Moore category of a bimonad are explicitly constructed.
- §3 considers, for a given bimonad H, two Kleisli type categories, proving that the Kleisli adjunctions from the (co)monad theory still hold in the present setting. The notions of free and cofree bimodules are introduced as the images of the left adjoint and the right adjoint of the forgetful functors from the Eilenberg-Moore category of the bimonad H to the Eilenberg-Moore category of the underlying comonad and monad respectively. As in the classical case of (co)monads, it is shown that the two Kleisli categories are equivalent with the categories of free and respectively cofree bimodules. Characterizations of monomorphisms and epimorphisms in the Eilenberg-Moore category of a bimonad is also obtained.

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MSC:

18C20 Eilenberg-Moore and Kleisli constructions for monads
16T05 Hopf algebras and their applications
18A35 Categories admitting limits (complete categories), functors preserving limits, completions
18A20 Epimorphisms, monomorphisms, special classes of morphisms, null morphisms

Keywords:

(co)monad; bimonad/Hopf monad; Eilenberg-Moore category; Kleisli category; (co)free bimodule; Hopf

module

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