Kachour, Camell

Algebraic models of cubical weak ∞-categories with connections. (English) Zbl 07528817

Categ. Gen. Algebr. Struct. Appl. 16, No. 1, 143-187 (2022)

The author defines the category of cubical categorical stretchings, which is the cubical analogue of the category of globular categorical stretchings in [*J. Penon*, Cah. Topologie Géom. Différ. Catégoriques 40, No. 1, 31–80 (1999; Zbl 0918.18006)]. The key ingredient is a cubical analogue of the globular contractions built there. A monad $\mathbb{W}$ on the category of cubical sets whose algebras are the models of cubical weak ∞-categories with connections is given. The monad is the cubical analogue of the monad $\mathbb{P}_C^0$ there, whose $\mathbb{P}_C^0$-algebras are the globular ∞-categories of Penon. Main proofs use sketch theory initiated by Charles Ehresmann and his students, especially [*L. Coppey* and *C. Lair*, Diagrammes 13, 112 p. (1985; Zbl 0594.18006)].

In [*K. Kachour*, Cah. Topol. Géom. Différ. Catég. 49, No. 1, 1–68 (2008; Zbl 1202.18004); Theory Appl. Categ. 30, 775–807 (2015; Zbl 1320.18004); *J. Penon*, Cah. Topologie Géom. Différ. Catégoriques 40, No. 1, 31–80 (1999; Zbl 0918.18006)] some computations were described for globular higher structures born with globular stretcchings. It was proved in [loc. cit.] that in dimension 2, the globular weak ∞-categories of Penon are bicategories. The author gives in §4.3 a precise definition of the dimension for algebras of models of cubical weak ∞-categories. Computations in dimension 2 lead to long computations and go beyond the scope of this paper, but the reader can verify that the models of dimension 2 are weak double categories in the sense of [*D. Verity*, Repr. Theory Appl. Categ. 2011, No. 20, 266 p. (2011; Zbl 1254.18001)]. The author exhibits an example of cubical coherence cell in dimension 2 in §4.1.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18B40 Groupoids, semigroupoids, semigroups, groups (viewed as categories)
18C15 Monads (= standard construction, triple or triad), algebras for monads, homology and derived functors for monads
18C20 Eilenberg-Moore and Kleisli constructions for monads
20L99 Groupoids (i.e. small categories in which all morphisms are isomorphisms)
55U35 Abstract and axiomatic homotopy theory in algebraic topology

Keywords:

cubical weak ∞-groupoids with connections; homology theory; homotopy theory; computer science

Full Text: DOI

References:

[1] Antolini, R.,Geometric realisations of cubical sets with connections, and classifying spaces of categories, Appl. Categ. Structures 10(5) (2002), 481-494. · Zbl 1021.18005

[2] Ashley, N.,SimplicialT-complexes and crossed complexes: a nonabelian version of a theorem of Dold and Kan, Dissertationes Math. (Rozprawy Mat.) 265 (1988), 1-61. With a preface by R. Brown. xviii, 396. · Zbl 1003.55500

[3] Batanin, M.,On the Penon method of weakening algebraic structures, J. Pure Appl. Algebra 172(1) (2002), 1-23. · Zbl 1003.18010

[4] Bourke, J.,Iterated algebraic injectivity and the faithfulness conjecture,https:// arxiv.org/pdf/1811.09532.pdf. · Zbl 1458.18012

[5] Al-Agl, F.A., Brown, R., and Steiner, R.,Multiple categories: The equivalence of a globular and a cubical approach, Adv. Math. 170 (2002), 71-118. · Zbl 1013.18003

[6] Brown, R., Higgins, Ph., and Sivera, R., "Nonabelian Algebraic Topology", Europ. Math. Soc., Tracts in Mathematics 15, 2011. · Zbl 1237.55001

[7] Cheng, E. and Makkai, M.,A note on Penon's definition of weakn-category, Cah. Topol. G´eom. Diff´er. Cat´eg. 50 (2009), 83-10.

[8] Coppey, L. and Lair, Ch.Le¸cons de th´eorie des esquisses, Universit´e Paris VII, (1985).

[9] Foltz, F.,Sur la cat´egorie des foncteurs domin´es, Cah. Topol. G´eom. Diff´er. Cat´eg. 11(2) (1969), 101-130.

[10] Grandis, M. and Mauri, L.,Cubical sets and their site, Theory Appl. Categ. 11 (2003), 185-201. · Zbl 1022.18009

[11] Hirschhorn, Ph., "Model Categories and Their Localizations", Math. Surveys Monogr. 99, 2003. · Zbl 1017.55001

[12] Kachour, K.,D´efinition alg´ebrique des cellules non-strictes, Cah. Topol. G´eom. Diff´er. Cat´eg. 1 (2008), 1-68. · Zbl 1202.18004

[13] Kachour, C.,Algebraic models of cubical weak higher structures, To appear in Categ. General Alg. Structures Appl. · Zbl 1355.18007

[14] Kachour, C.,Algebraic definition of weak(⊠, n)-categories, Theory Appl. Categ. 30(22) (2015), 775-807. · Zbl 1320.18004

[15] Kachour, C.,An algebraic approach to weakω-groupoids, Australian Category Seminar, 14 September 2011.http://web.science.mq.edu.au/groups/co cgi-bin/speaker-info.cgi?name=Camell+Kachour

[16] Kachour, C. and Weber, M., "Introduction to Higher Cubical Operads", First Part: The Cubical Operad of Cubical Weak⊠-Categories, Pr´epublication de l'IHES, http://preprints.ihes.fr/2017/M/M-17-19.pdf

[17] Kachour, C. and Weber, M., "Introduction to Higher Cubical Operads", Second Part: The Functor of Fundamental Cubical Weak⊠-Groupoids for Spaces, Pr´epublication de l'IHES,http://preprints.ihes.fr/2018/M/M-18-03.pdf

[18] Kachour, C.,Combinatorial approach to the categoryΘ0of cubical pasting diagrams, https://arxiv.org/abs/2102.09787

[19] Kachour, C., (⊠, n)-ensembles cubiques, talk in CLE, Paris, November 2016.https: //sites.google.com/site/logiquecategorique/Contenus/201611-kachour

[20] Lafont, Y., Metayer, F., and Worytkiewicz, K.,A folk model structure on omega-cat, Adv. Math. 224 (2010), 1183-1231. · Zbl 1236.18017

[21] Lair, Ch., "Condition syntaxique de triplabilit´e d'un foncteur alg´ebrique esquiss´e", Diagrammes 1, 1979.

[22] Maltsiniotis, G.,La cat´egorie cubique avec connections est une cat´egorie test stricte, (preprint) (2009), 1-16.

[23] Penon, J.,Approche polygraphique des⊠-cat´egories non-strictes, Cah. Topol. G´eom. Diff´er. Cat´eg. 40(1) (1999), 31-80.

[24] Penon, J.,⊠-cat´egorification de structures ´equationnelles, S´eminaires Itin´erants de Cat´egories (S.I.C) 'a Amiens, Septembre 2005.

[25] Tuy´eras, R., "Sketches in Higher Category Theory", Ph.D. Thesis, Cambridge University Press 95(1), 2017.

[26] Verity, D., "Enriched Categories, Internal Categories and Change of Base", Ph.D. Thesis,http://www.tac.mta.ca/tac/reprints/articles/20/tr20.pdf · Zbl 1254.18001