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Actads. (English) Zbl 07577054

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Having been coined by *J. P. May* [The geometry of iterated loop spaces. Berlin-Heidelberg-New York: Springer-Verlag (1972; Zbl 0244.55009)], operads are a central concept of modern algebraic topology. *J. C. Baez* and *J. Dolan* [Adv. Math. 135, No. 2, 145–206 (1998; Zbl 0909.18006)] introduced the +-construction to pass from monoids to plain operads and, via iteration, to opetopes. Opetopes do not explain how symmetric operads arise from monoids. While the +-construction can be applied to symmetric operads, there is an additional possibility of introducing permutations on each higher level by permuting variables without regard to the structures on the previous levels. This paper aims to explore the interplay of these permutations, the resulting structure being called *actads*.

The actads are indexed so that monoids are 0-actads and operads are 1-actads. A 3-base is visualized as a tree drawn in the way that, in the place of each node of a tree, one puts a triangle whose vertex is the node, and whose base contains the successor nodes, where each triangle can be recursively subdivided into a tree in a similar fashion.

In n-actads with $n > 1$, the permutations on the different levels give rise to a certain special kind of n-fold category. The concept of iterated algebras over an n-actad and various types of iterated units are investigated. Some examples of algebras over 2-actads are given, being shown to be used to construct certain new interesting homotopy types of operads. A connection between actads and ordinal notation is also discussed.

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18N99	Higher categories and homotopical algebra
55P47	Infinite loop spaces
03F15	Recursive ordinals and ordinal notations

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References:

[1] Adamek, J.; Rosicky, J., Locally Presentable and Accessible Categories (1994), Cambridge: Cambridge University Press, Cambridge · Zbl 0795.18007 · doi:10.1017/CBO9780511600579

[2] Adams, J. F., Stable Homotopy and Generalised Homology (1974), Chicago: The University of Chicago Press, Chicago · Zbl 0309.55016

[3] Baez, J. C.; Dolan, J., Higher-dimensional algebra III. n-categories and the algebra of opetopes, Adv Math, 135, 145-206 (1998) · Zbl 0909.18006 · doi:10.1006/aima.1997.1695

[4] Bénabou, J., Introduction to Bicategories, Reports of the Midwest Category Seminar, 1-77 (1967), Berlin-Heidelberg: Springer-Verlag, Berlin-Heidelberg · Zbl 1375.18001 · doi:10.1007/BFb0074299

[5] Berger, C.; Moerdijk, I., On the derived category of an algebra over an operad, Georgian Math J, 16, 13-28 (2009) · Zbl 1171.18005 · doi:10.1515/GMJ.2009.13

[6] Birkhoff, G.; Lipson, J. D., Heterogeneous algebras, J Combin Theory, 8, 115-133 (1970) · Zbl 0211.02003 · doi:10.1016/S0021-9800(70)80014-X

[7] Borceux, F., Handbook of Categorical Algebra 2: Categories and Structures (1994), Cambridge: Cambridge University Press, Cambridge · Zbl 0843.18001 · doi:10.1017/CBO9780511525865

[8] Cheng, E., The category of opetopes and the category of opetopic sets, Theory Appl Categ, 11, 353-374 (2003) · Zbl 1036.18004

[9] Cheng, E., Weak n-categories: Opetopic and multitopic foundations, J Pure Appl Algebra, 186, 109-137 (2004) · Zbl 1036.18005

· doi:10.1016/S0022-4049(03)00139-7

[10] Cheng, E., Weak n-categories: Comparing opetopic foundations, J Pure Appl Algebra, 186, 219-231 (2004) · Zbl 1042.18004 · doi:10.1016/S0022-4049(03)00140-3

[11] Cohen, F. R.; Lada, T. J.; May, J. P., The Homology of Iterated Loop Spaces (1976), Berlin-Heidelberg: Springer-Verlag, Berlin-Heidelberg · Zbl 0334.55009 · doi:10.1007/BFb0080464

[12] Curien P-L, Thanh C H, Mimram S. Syntactic approaches to opetopes. arXiv:1903.05848, 2019 · Zbl 07567918

[13] Elmendorf, A. D.; Mandell, M. A., Rings, modules, and algebras in infinite loop space theory, Adv Math, 205, 163-228 (2006) · Zbl 1117.19001 · doi:10.1016/j.aim.2005.07.007

[14] Finster E. Implementing the opetopes: Programming with higher dimensional trees. http://ericfinster.github.io/files/impl-ops.pdf, 2018

[15] Fiore, M.; Saville, P., List objects with algebraic structure, 1-18 (2017), Wadern: Schloss Dagstuhl—Leibniz-Zentrum für Informatik, Wadern · Zbl 1441.68016

[16] Gambino, N.; Hyland, M., Wellfounded trees and dependent polynomial functors, Types for Proofs and Programs, 210-225 (2004), Berlin-Heidelberg: Springer-Verlag, Berlin-Heidelberg · Zbl 1100.03055 · doi:10.1007/978-3-540-24849-1$_1$4

[17] Ginzburg, V.; Kapranov, M., Koszul duality for operads, Duke Math J, 76, 203-272 (1994) · Zbl 0855.18006 · doi:10.1215/S0012-7094-94-07608-4

[18] Hermida, C.; Makkai, M.; Power, J., On weak higher-dimensional categories I: Part 1, J Pure Appl Algebra, 154, 221-246 (2000) · Zbl 0971.18005 · doi:10.1016/S0022-4049(99)00179-6

[19] Hermida, C.; Makkai, M.; Power, J., On weak higher-dimensional categories I: Part 2, J Pure Appl Algebra, 157, 247-277 (2001) · Zbl 0985.18006 · doi:10.1016/S0022-4049(00)00129-8

[20] Hermida, C.; Makkai, M.; Power, J., On weak higher-dimensional categories I: Part 3, J Pure Appl Algebra, 166, 83-104 (2002) · Zbl 0992.18005 · doi:10.1016/S0022-4049(01)00014-7

[21] Hinich, V. A.; Schechtman, V. V., On homotopy limit of homotopy algebras, K-Theory, Arithmetic and Geometry, 240-264 (1987), Berlin-Heidelberg: Springer-Verlag, Berlin-Heidelberg · Zbl 0631.55011 · doi:10.1007/BFb0078370

[22] Kock, J.; Joyal, A.; Batanin, M., Polynomial functors and opetopes, Adv Math, 224, 2690-2737 (2010) · Zbl 1221.18001 · doi:10.1016/j.aim.2010.02.012

[23] Leinster, T., Higher Operads, Higher Categories (2004), Cambridge: Cambridge University Press, Cambridge · Zbl 1160.18001 · doi:10.1017/CBO9780511525896

[24] Mandell, M. A., E_⊠ algebras and p-adic homotopy theory, Topology, 40, 43-94 (2001) · Zbl 0974.55004 · doi:10.1016/S0040-9383(99)00053-1

[25] May, J. P., A general algebraic approach to Steenrod operations, The Steenrod Algebra and Its Applications: A Conference to Celebrate N.E. Steenrod's Sixtieth Birthday, 153-231 (1970), Berlin-Heidelberg: Springer-Verlag, Berlin-Heidelberg· doi:10.1007/BFb0058524

[26] May, J. P., The Geometry of Iterated Loop Spaces (1972), Berlin-Heidelberg: Springer-Verlag, Berlin-Heidelberg · Zbl 0244.55009 · doi:10.1007/BFb0067491

[27] May, J. P., E_⊠-spaces, group completions and permutative categories, 61-94 (1974), Cambridge: Cambridge University Press, Cambridge · Zbl 0281.55003

[28] May, J. P., The spectra associated to permutative categories, Topology, 17, 225-228 (1978) · Zbl 0417.55011 · doi:10.1016/0040-9383(78)90027-7

[29] May, J. P., Multiplicative infinite loop space theory, J Pure Appl Algebra, 26, 1-69 (1982) · Zbl 0532.55013 · doi:10.1016/0022-4049(82)90029-9

[30] Palm, T., Dendrotopic sets for weak infinity-categories (2003), Toronto: York University, Toronto

[31] Palm, T., Dendrotopic sets, Galois Theory, Hopf Algebras, and Semiabelian Categories, 411-461 (2004), Providence: Amer Math Soc, Providence · Zbl 1075.18004

[32] Priddy, S. B., Koszul resolutions, Trans Amer Math Soc, 152, 39-60 (1970) · Zbl 0261.18016 · doi:10.1090/S0002-9947-1970-0265437-8

[33] Steenrod, N. E., A convenient category of topological spaces, Michigan Math J, 14, 133-152 (1967) · Zbl 0145.43002 · doi:10.1307/mmj/1028999711

[34] Szawiel, S.; Zawadowski, M., The web monoid and opetopic sets, J Pure Appl Algebra, 217, 1105-1140 (2013) · Zbl 1283.18005 · doi:10.1016/j.jpaa.2012.09.030

[35] Szawiel, S.; Zawadowski, M., Theories of analytic monads, Math Structures Comput Sci, 24, e240604 (2014) · Zbl 1342.18006 · doi:10.1017/S0960129513000868

[36] Szawiel S, Zawadowski M. Polynomial and analytic functors and monads, revisited. arXiv:1506.04317, 2015 · Zbl 1316.18007

[37] Veblen, O., Continuous increasing functions of finite and transfinite ordinals, Trans Amer Math Soc, 9, 280-292 (1908) · Zbl 39.0102.01 · doi:10.1090/S0002-9947-1908-1500814-9

[38] Zawadowski M. On positive face structures and positive-to-one computads. arXiv:0708.2658, 2007

[39] Zawadowski M. On ordered face structures and many-to-one computads. arXiv:0708.2659, 2007

[40] Zawadowski, M., Lax monoidal fibrations, Models, Logics, and Higher-Dimensional Categories, 341-426 (2011), Providence: Amer Math Soc, Providence · Zbl 1243.18005 · doi:10.1090/crmp/053/17