Mederos, B.; Pérez-Cabrera, I.; Takane, M.; Tapia Sánchez, G.; Zavala, B.

A method to construct all the paving matroids over a finite set. (English) Zbl 07557831

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Paving matroids were defined in [*J. Hartmanis*, Can. J. Math. 11, 97–106 (1959; Zbl 0089.37002)] through the concept of d-partitions in number theory. Paving matroids play a significant role in computer science via greedy algorithms and the matroid oracles [*C. Heunen* and *V. Patta*, Appl. Categ. Struct. 26, No. 2, 205–237 (2018; Zbl 1403.18002)]. This paper aims to give a characterization of paving matroids leading to a concrete construction of their hyperplances and to an algorithm for finding them. A counterexample to a characterization of paving matroids by *J. G. Oxley* [Matroid theory. 2nd ed. Oxford: Oxford University Press (2011; Zbl 1254.05002), 1.3.10] is given. A simple proof of Rota's basis conjecture [*G.-C. Rota*, Mitt. Dtsch. Math.-Ver. 6, No. 2, 45–52 (1998; Zbl 1288.00005)] is given for the case of sparse-paving matroids and for the case of paving matroids of rank r on a set of cardinality $n \leq 2r$.

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MSC:

18-XX Category theory; homological algebra

Keywords:

simple matroid; paving matroid; sparse-paving matroid; lattice; hyperplanes of a matroid; circuits of a matroid; Rota's basis conjecture

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