Abrusci, V. Michele; **Casadio, Claudia**

Lambek's syntactic calculus and noncommutative variants of linear logic: laws and proof-nets. (English) Zbl 07440902

Casadio, Claudia (ed.) et al., Joachim Lambek: the interplay of mathematics, logic, and linguistics. Cham: Springer. Outst. Contrib. Log. 20, 1-37 (2021)

The Calculus of Syntactic Types [*J. Lambek*, Am. Math. Mon. 65, 154–170 (1958; Zbl 0080.00702); in: Categorial grammar, 153–172 (1988; Zbl 0692.03019)], aka the Lambek Calculus (LC), represents a milestone in the field of mathematical linguistics. After Girard's introduction of linear logic [*J.-Y. Girard*, Theor. Comput. Sci. 50, 1–102 (1987; Zbl 0625.03037)], it was shown [*V. M. Abrusci*, Z. Math. Logik Grundlagen Math. 36, No. 1, 11–15 (1990; Zbl 0719.03005); Z. Math. Logik Grundlagen Math. 36, No. 4, 297–318 (1990; Zbl 0810.03005)] that LC is no longer than the noncommutative fragment of multiplicative intuitionistic linear logic (MILL), specifically being a noncommutative fragment of linear logic without additive operations. Classical systems connected to LC such as Noncommutative Multiplicative Linear Logic (NMLL) and Cyclic Multiplicativ Linear Logic (CyMLL) have been investigated [*M. Pentus*, Lect. Notes Comput. Sci. 1234, 306–311 (1997; Zbl 0888.03036); *P. Di Gianantonio*, Lect. Notes Comput. Sci. 3210, 130–144 (2004; Zbl 1095.03070); *V. M. Abrusci* and *R. Maieli*, Lect. Notes Comput. Sci. 9160, 53–68 (2015; Zbl 1365.03039); Lect. Notes Comput. Sci. 9804, 43–59 (2016; Zbl 1478.03081); Math. Struct. Comput. Sci. 29, No. 6, 733–762 (2019; Zbl 1456.03092); *M. Pentus*, Lect. Notes Comput. Sci. 1234, 306–311 (1997; Zbl 0888.03036)].

This article is devoted to the relationship between LC and CyMLL, proposing a geometric representation of a well-known laws of LC (Residuation, Monotomicity, Application, Expansion, Type-raising, Composition, Geach and Switching laws) by means of cyclic-multiplicative proof-nets (CyM-PN), whose definitions and detailed representations for each of the mentioned laws offer a better understanding of their internal structure, allowing for their classification into three main families of graphs, namely, cyclic multiplicative proof-nets with one, two and three axiom links. Beholden to this geometric representation, some new laws of LC are obtained, on the basis of which some possible linguistic applications and examples are presented.

For the entire collection see [Zbl 1470.03008].

Reviewer: Hirokazu Nishimura (Tsukuba)

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References:

[1] Moortgat, M.: Symmetric Categorial Grammar: Residuation and Galois Connections. Linguistic analysis, 36(1-4), (2010), 143-166.

[2] Moortgat, M., Moot, R.: Proof-nets for the Lambek-Grishin Calculus. In E. Grenfestette, C. Heunen, M. Sadrzadeh (eds.),Compositional Methods in Physics and Linguistics, Oxford University Press (2013), 283-320. · Zbl 1350.03023

[3] Morrill, G.: A Categorial Type Logic. In C. Casadio, B. Coeke, M. Moortgat and P. Scott (eds.)Categories and Types in Logic, Language and Physics: Essays Dedicated to Jim Lambek on the Occasion of His 90th Birthday. Springer LNCS Volume 8222, FoLLI Publications in Logic, Language and Information (2014), 331-352. · Zbl 1285.03017

[4] Yetter, D. N.: Quantales and (Noncommutative) Linear Logic.The Journal of Symbolic Logic, 55(1), (1990), 41-64 · Zbl 0701.03026