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A prospect theory Nash bargaining solution and its stochastic stability☆

Ryoji Sawa[1]

Faculty of Engineering, Information and Systems, University of Tsukuba, Tennoudai 1-1-1, Tsukuba, Ibaraki 305–8573, Japan

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ABSTRACT

We consider the long-run outcomes of bargaining games when players obey prospect theory. We extend the evolutionary bargaining model of Young (1993) to a two-stage Nash demand game. Two players simultaneously choose whether to exercise an outside option in the first stage and play the Nash demand game in the second stage, which will be reached only if neither player exercises the outside option. We address the influence on the stochastically stable division of reference-dependent preferences where the reference point is the value of the outside option. We show that the division consistently differs from the Nash bargaining solution under expected utility theory. Inspired by this, we propose a *prospect theory Nash bargaining solution*, which coincides with the stochastically stable division.

1. Introduction

Bargaining is prevalent and a critical element in many situations, for example, in business, politics, and diplomacy. Bargaining situations have some degree of uncertainty in the outcomes, and bargainers' perceptions of the situation often play an important role. For example, bargainers may have their best alternative to a negotiation agreement (Fisher and Ury, 1981; Raiffa, 1982), which is the minimum acceptance level they would require to say yes. The distance from such an acceptance level can be the basis for bargainers' evaluations of the outcomes. It is a heuristic principle people naturally use to reduce the complexity in a bargaining process. Such heuristics, however, may influence our decisions. For example, reference-dependent valuations lead to decision biases in risky situations (see Kahneman and Tversky, 1979, for example).

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E-mail address: rsawa@sk.tsukuba.ac.jp

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One of such biases is loss aversion, the tendency for people to prefer avoiding losses to acquiring gains. This study asks how such cognitive biases affect the outcome of a bargaining process in the long run.[2]

The setting we consider is a two-player, two-stage game called the *Nash demand with outside options game*. In stage 1, each player $i \in \{1, 2\}$ chooses whether to exercise an outside option that guarantees her money r_i. If neither exercises the option, they enter stage 2 and play the Nash demand game (Nash, 1950) with a pie worth v. We assume that the opportunity cost of participating in the Nash demand game serves as the reference point for each player in stage 2. The opportunity cost of entering into stage 2 is to give up the outside option. The players think of an outcome in stage 2 as a gain only if it exceeds what they would have received by not going to stage 2. This can be viewed as the status quo bias—a bias that favors the retention of the status quo (Samuelson and Zeckhauser, 1988). Player i knows she can secure r_i by not participating in the Nash demand game, and then r_i would naturally become her reference point in stage 2.[3]

We employ stochastic stability analysis, pioneered by Foster and Young (1990), Kandori et al. (1993) and Young (1993a), to characterize long-run equilibria with behavioral players. In each period, two agents are drawn from large populations to play the game. Each agent samples demands made by opponent agents in past periods. She chooses a best response to the sampled demands, but her choice is subject to stochastic errors. The selection result is obtained by checking the robustness of equilibria against stochastic errors.

Our first main result, Theorem 1, shows that players achieve an efficient outcome in all stochastically stable conventions, and their shares of the pie depend on their outside options. The outside option value will be a perceived loss in the case of unsuccessful bargaining. When the option value is large, a player tends to drive a hard bargain and obtains a larger share in a stochastically stable convention relative to that under expected utility theory; when the option value is small, a player is reluctant to drive such a hard bargain.

Inspired by the stochastic stability analysis, we define the *prospect theory Nash bargaining solution*. It has a similar form to the ordinary Nash bargaining solution, i.e., it is a product of the players' utilities. However, each utility is offset by the player's perceived potential loss at the bargaining stage, which contrasts the two solutions. Our second main result, Theorem 2, shows that the stochastically stable convention converges to our solution under certain conditions. The prospect theory Nash bargaining solution has the following property. Given the opponent's reference point r_2, there exists a threshold level $r^* \geq r_2$ such that (i) Player 1 will end up with more than half of the pie if $r_1 > r^*$ and (ii) Player 1 will end up with less than half if $r_2 < r_1 < r^*$.

One of our contributions is to combine stochastic stability analysis with behavioral economics. Matcing the two fields has led to the solution concept above. As Bergin and Lipman (1996) point out, the selection result of stochastic stability analysis is sensitive to the specification of stochastic shocks, which represent people's seemingly irrational decisions. It highlights the importance of modeling bounded rationality. This study is along a line of research combining the two fields. Agents in this study are boundedly rational in three respects: (i) they are myopic, (ii) they make mistakes in their strategy choices, and (iii) they obey prospect theory. The first two are common in stochastic stability analysis, whereas the last one is the key feature of this study.

Related literature: The most closely related studies are Young (1993b) and Tröger (2002). Young (1993b) shows that the stochastically stable outcomes of the Nash demand game (without any outside option) coincide with the Nash bargaining solution.[4] The division would be a fifty-fifty split if players were to obey expected utility theory and have similar preferences. Our stochastically stable outcome with $r_1 = r_2 = 0$ coincides with it. Tröger (2002) extends Young (1993b) to the two-stage game in which one player chooses the investment level in stage 1 and the players play the Nash demand game in stage 2, where the size of the pie depends on the investment level. Our setting with $r_2 = 0$ is similar to the setting in Tröger (2002). We use his result as a benchmark under expected utility theory. We find two properties of prospect theory influential on bargaining outcomes. *Diminishing sensitivity* directs a player to typically drive a hard bargain when her share is close to her option value. It contributes to an increase in one's share. *Loss aversion* places psychological pressure on a player to avoid getting nothing in exchange for her option. This pressure deteriorates their attitudes toward driving a hard bargain and leads to a smaller share. Our analysis shows when one effect dominates the other and which effect dominates.

Other related studies are Sáez-Martí and Weibull (1999), Ellingsen and Robles (2002), Young (1998), Naidu et al. (2010), Hwang and Newton (2017), and Hwang et al. (2018). Sáez-Martí and Weibull (1999) employ a similar model to Young (1993b) except that a population of agents are clever in the sense that they know the opponent population's preferences and choose the best reply to the opponent population's best reply. Ellingsen and Robles (2002) are similar to Tröger (2002) except that they employ the evolutionary model of Kandori et al. (1993). Young (1998), Naidu et al. (2010), Hwang and Newton (2017), and Hwang et al. (2018) consider two-player coordination games with zero payoffs off-diagonal, called *the contract game*. This class of games is different from the Nash demand game, but the main feature is common—an

[2] We focus on the properties of reference-dependence, loss aversion, and diminishing sensitivity of prospect theory, and abstract away from the non-linear transformation of probabilities.

[3] A similar inpterpretation of the reference point is used in the finance literature (see Barberis et al., 2001; Barberis and Huang, 2008). For example, Barberis and Huang (2008) use the wealth level that "he would have experienced by investing at the risk-free rate" as the reference point for evaluating risky assets. This is an opportunity cost of investing in risky assets.

[4] More precisely, Young (1993b) assumes that the sample sizes of the demands may differ across two populations and shows that the outcomes coincide with the asymmetric Nash bargaining solution, which maximizes the weighted product of the two players' utilities. The sample sizes appear as the weights in the solution.

efficient outcome is achieved when players choose the corresponding strategies. Among them, Hwang et al. (2018) propose a novel solution concept, *the Logit bargaining solution*, that is built upon a stochastic stability analysis under the logit choice rule. See Section 6.4 of Newton (2018) for the relationship between bargaining solutions and the stochastic stability analysis.

The evolutionary approach has been adopted in other bargaining settings too. For example, double auction settings are studied by Binmore et al. (2003), Agastya (2004), Santamaria-Garcia (2009), and coalitional bargaining settings are studied by Agastya (1997, 1999), Arnold and Schwalbe (2002), Newton (2012), Rozen (2013), Nax, 2019, and Sawa (2019).[5] Similarly to the above mentioned studies, our focus is on the evolutionary approach. We are aware that there are other approaches to bargaining. As well-known, John Nash adopted two approaches to bargaining: the axiomatic approach (Nash, 1950), and the strategic approach (Nash, 1953), both of which support the Nash bargaining solution. An important next direction is to explore the outcomes of these approaches in the bargaining game with behavioral agents. If their outcomes coincide with the one derived from our evolutionary approach, it will strengthen the validity of our finding. The analysis via other approaches is an exciting question for future research.

Incorporating findings of behavioral economics into the literature on learning has burgeoned recently. We name a few developments in evolutionary models and other learning models.[6] In evolutionary models, there exist two strands of research.[7] Studies in the first strand take behavioral biases as given and examine the influence of such biases on the equilibrium selection results. Norman (2009), Wood (2015), Zusai (2018), Sawa and Wu (2018a,b), Bilancini and Boncinelli (2020), and this study fall within this category. Norman (2009) and Zusai (2018) consider switching costs of strategies, which can be interpreted as status quo bias. Wood (2015) considers behavioral biases in beliefs of the strategy distribution. Sawa and Wu (2018a,b) consider reference-dependent preferences similarly to this paper but they focus on normal form games. Bilancini and Boncinelli (2020) consider condition-dependent mistake models where mistake probabilities depend on the past experiences. Studies in the other strand examine which evolutionary models lead people to have observed behavioral biases, for example, Mohlin (2012), Heller (2014), and Heller and Mohlin (2019).[8] Heller (2014) offers an evolutionary model of people with heterogeneous types of risk aversion and shows that people with overconfidence tend to survive. Mohlin (2012) and Heller and Mohlin (2019) study the evolution of cognitive sophistication. In adaptive learning, there are models incorporating features akin to loss aversion.[9] For example, in “experience-weighted attraction,” unchosen strategies are reinforced based on the payoffs they would have earned. Funai (2019) characterizes conditions under which such learning processes converge to a quantal response equilibrium. In social learning, Eyster and Rabin (2010) propose a model of naïve inference in which each player plays a best response to her beliefs that her predecessors follow their own signals, neglecting the possibility that some predecessors might herd. Their model incorporates the psychology of limited attention to how other players extract information from observations.

Our results provide a further insight into bargaining power in the Nash demand game. Many studies have developed insights into bargaining power. For example, a higher reservation price strengthens the player’s bargaining power (Nash, 1953). Gallo (2014) shows that when the game is played in a network, the bargaining power is represented by the degree of the player in the network. Köbberling and Peters (2003) consider players obeying rank-dependent utility theory and show that utility risk aversion and probabilistic risk aversion have opposite effects on the player’ bargaining power (with regard to Kalai–Smorodinsky bargaining solution). We find mixed effects of prospect theory on bargaining power. Players with prospect theory are more sensitive to small compromises, which leads to stronger bargaining power. While players with prospect theory also tend to avoid the no-deal outcome due to loss aversion, which leads to weaker bargaining power. The latter effect is similar to what Driesen et al. (2012) found in the Rubinstein’s (1982) alternating offers bargaining game. Increased loss aversion may result in decreased bargaining power of the player. The mixture of the two effects determine the player’s bargaining power (see Section 4.3).

The paper is organized as follows. Section 2 describes utility functions obeying prospect theory. Section 3 introduces the game and the evolutionary dynamics. In Section 4, we define the Nash bargaining solution under prospect theory, and show that the stochastically stable division coincides with it. Section 5 offers a comparison of the stochastically stable divisions under prospect theory and under expected utility theory, a discussion on outside options, and a brief literature review on experimental studies. Section 6 concludes.

2. Preliminaries

Experimental studies have revealed that people’s preferences may violate the axioms of the expected utility theory. Tversky and Kahneman (1992), henceforth TK92, propose a functional form of utility that is consistent with such experimental results; a player’s subjective expected utility is calculated based on a utility function kinked at a reference point. For

[5] Binmore et al. (2003) call their double auction setting *the cushioned demand game*. In the same paper, they also study the contract game via the evolutionary approach.

[6] Sobel (2000) classifies learning models as individual learning models, social learning models, and game theoretical learning models. Game theoretical models include evolutionary models.

[7] Behavioral biases in evolutionary settings are also experimentally examined. See Lim and Neary (2016), Mäs and Nax (2016), Alós-Ferrer and Ritschel (2018), and Hwang et al. (2018) for example.

[8] This strand is called indirect evolutionary approach in Güth (1995).

[9] See “learning direction theory” in Selten and Stoecker (1986), “experience-weighted attraction” in Camerer and Ho (1999), “regret matching” in Hart and Mas-Colell (2000), “adaptive heuristics” in Hart (2005), and others.

monetary payoff M,

$$u(M;r)=\begin{cases}(M-r)^{\alpha} & \text{if } M-r\geq 0,\\ -\lambda(r-M)^{\beta} & \text{otherwise,}\end{cases} \tag{1}$$

where $0<\alpha<1$, $0<\beta<1$, $\lambda>1$, and $r\in\mathbb{R}$ denotes the reference point to which gains and losses are defined. Utility function (1) satisfies the two properties reported in Kahneman and Tversky (1979): the marginal utility of gains and losses decreases with their size (*diminishing sensitivity*), and people strongly prefer avoiding losses to obtaining gains (*loss aversion*). It is estimated that $\alpha=\beta=0.88$ and $\lambda=2.25$ in TK92.

Inspired by these studies, we impose the following assumption on utility functions.

Assumption 1 (prospect theory preference)**.** A player obeying prospect theory with a reference point r has a utility function of the following form.

$$u(M;r)=\begin{cases}u^{+}(M-r) & \text{if } M-r\geq 0,\\ -u^{-}(r-M;\lambda) & \text{otherwise,}\end{cases} \tag{2}$$

where $\lambda>1$ represents the degree of the player's loss aversion. For all $x>0$ and all $\lambda>1$, $u^{+}(x)$ and $u^{-}(x;\lambda)$ are twice differentiable and satisfy the following properties,

$$\lim_{x\to+0}u^{+}(x)=u^{+}(0)=\lim_{x\to-0}u^{-}(x;\lambda)=0,\quad \frac{\partial u^{-}(x;\lambda)}{\partial\lambda}>0,$$
$$u^{+}(x)<u^{-}(x;\lambda)\quad\text{(loss aversion)},$$
$$\frac{\mathrm{d}u^{+}(x)}{\mathrm{d}x}>0,\quad \frac{\partial(-u^{-}(x:\lambda))}{\partial x}>0,\quad \frac{\mathrm{d}^2u^{+}(x)}{\mathrm{d}x^2}\leq 0,\quad \frac{\partial^2u^{-}(x;\lambda)}{\partial x^2}\leq 0\quad\text{(diminishing sensitivity)}.$$

It ensures that the utility functions have diminishing sensitivity and loss aversion. The utility function in Eq. (1) with $\alpha=\beta$ and $\lambda>1$ satisfies the assumption. A player obeying prospect theory is assumed to have a utility function satisfying Assumption 1. A player obeying expected utility theory is assumed to have a utility function given by (2) with $r=0$. Since monetary outcomes are non-negative in our game, a player with $r=0$ has a concave utility of money.

Remark 1. We mean by loss aversion the preference of avoiding losses to commensurate gains, or $u^{+}(x)<u^{-}(x;\lambda)$ for $x>0$. We think that most utility functions satisfying Assumption 1 are kinked functions, i.e. $\lim_{x\to+0}\frac{\mathrm{d}u^{+}(x)}{\mathrm{d}x}\neq\lim_{x\to-0}\frac{\partial-u^{-}(x;\lambda)}{\partial x}$. An exception is

$$u(M;r)=\begin{cases}a\log\log(M-r+e) & \text{if } M-r\geq 0,\\ -a\log\log(r-M+e)-\lambda\log((r-M)^2+1) & \text{otherwise,}\end{cases}$$

where e denotes the natural base. For sufficiently large $a>0$, the function satisfies Assumption 1 with $\lim_{x\to+0}\frac{\mathrm{d}u^{+}(x)}{\mathrm{d}x}=\lim_{x\to-0}\frac{\partial-u^{-}(x;\lambda)}{\partial x}=\frac{a}{e}$.

The players' subjective expected utility is computed by taking the probability weighted value of $u(M;r)$. TK92 suggest weighting the values of gains and losses with a non-linear transformation of probabilities. For simplicity, we abstract from this feature and compute the subjective expected utilities as follows:[10]

$$EU(s)=\sum_{M\in\mathscr{M}}P(M|s)u(M;r), \tag{3}$$

where s denotes the player's choice, $\mathscr{M}$ denotes the set of monetary outcomes, and $P(M|s)$ denotes the probability of outcome M conditional on choice s.

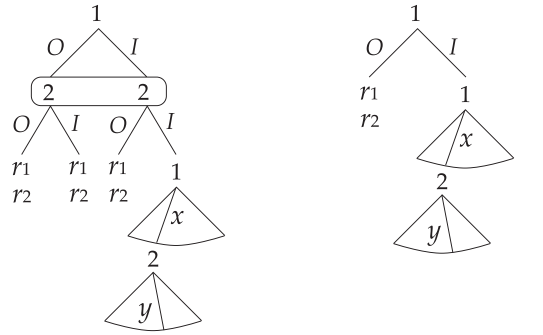
3. Model and evolution

3.1. Model

The *Nash demand with outside options game* is a two-stage game played by two players. Each player $i\in\{1,2\}$ has an outside option that earns her the monetary outcome $r_i\geq 0$. In the first stage, each player $i\in\{1,2\}$ simultaneously chooses $e_i\in\{I,O\}$, that is, whether to enter the subgame (I) or to exercise the outside option (O). If at least one player chooses O, each player i receives $r_i\geq 0$, and the game ends. In the second stage, which will be reached only if $e_1=e_2=I$, the players bargain over a *pie* $v>0$ by simultaneously announcing demands $x,y>0$. We assume that $r_1+r_2<v$. The players' monetary outcomes are given by

$$(M_1,M_2)=\begin{cases}(x,y) & \text{if } e_1=e_2=I \text{ and } x+y\leq v,\\ (0,0) & \text{if } e_1=e_2=I \text{ and } x+y>v,\\ (r_1,r_2) & \text{otherwise.}\end{cases} \tag{4}$$

[10] It is not uncommon to use Eq. (3). See Barberis et al. (2001), for example.



(a) Game I: Both players can opt out.

(b) Game II: Only Player 1 can opt out.

Fig. 1. Nash demand with outside options game.

We say that the pie is split if $e_1 = e_2 = I$ and $x + y \le v$. Let $s_i = (e_i, x)$ denote an arbitrary strategy of Player i, and let $M_i(s)$ denote the monetary outcome for Player i when the strategy profile is $s = (s_1, s_2)$. We assume that the value of the player's outside option serves as her reference point. Player i's (subjective) utility is given by Eq. (2), that is, $u_i(M_i(s); r_i)$.

The game is depicted in Fig. 1(a). This game has many equilibria. Suppose that the strategy space for each player i is given by $\{I, O\} \times [0, v]$. Then, the set of pure-strategy subgame perfect equilibria is $SPE = SPE_1 \cup SPE_2 \cup SPE_3$, where

$$SPE_1 = \{(e_1, x_1), (e_2, x_2) : e_1 = e_2 = I, x_1 + x_2 = v, x_1 \ge r_1, x_2 \ge r_2\},$$
$$SPE_2 = \{(e_1, x_1), (e_2, x_2) : e_1 = O, e_2 \in \{I, O\}, x_1 + x_2 = v, x_1 \le r_1\},$$
$$SPE_3 = \{(e_1, x_1), (e_2, x_2) : e_1 \in \{I, O\}, e_2 = O, x_1 + x_2 = v, x_2 \le r_2\}.$$

We select equilibria by means of the stochastic stability analysis, and examine the dependence of selection results on the preferences and the option values.

We discretize the strategy space to perform the stochastic stability analysis. We assume a base unit $\delta > 0$ such that v/δ is an integer. The strategy space in the subgame is given by[11]

$$X = \{\delta, 2\delta, \ldots, v - \delta\}.$$

For the sake of simplicity, we assume that $r_i \notin X$ for all $i \in \{1, 2\}$. Let $S = \{I, O\} \times X$, which is the strategy space of the entire game for each player. Define

$$X^{<r_2}_{>r_1} = \{x \in X : r_1 < x < v - r_2\}, \qquad x_{\max} = \max\{X^{<r_2}_{>r_1}\}, \qquad x_{\min} = \min\{X^{<r_2}_{>r_1}\}.$$

Note that $\{s_1 = (I, x), s_2 = (I, v - x)\}$ is a subgame perfect equilibrium for all $x \in X^{<r_2}_{>r_1}$.

A key difference from Young (1993b) is the introduction of outside options together with the assumption that they work as reference points in utility function (2). If Player i can secure r_i by choosing the option, then she would assess the outcome based on r_i. We employ this interpretation and assume that players evaluate outcomes relative to r_i. For example, in the experiments of Kahneman et al. (1990), subjects are given coffee mugs, and it is observed that the entitlement to mugs instantaneously serves as their reference point. There exist other factors that could serve as a reference point, e.g., expected monetary payoffs from the bargaining game. We leave such analysis to future work.

3.2. Stochastic evolutionary dynamics

The evolutionary process follows Young (1993b). Players 1 and 2 are represented by finite populations A and B of agents, respectively. In each period $t = 1, 2, \ldots,$ a pair of agents $(a_t, b_t) \in A \times B$ is randomly chosen to play the Nash demand with outside options game. These agents rely on histories of length m of the Nash demand subgame. The history in period t is defined as

$$\omega_t = \left\{(x_t^1, y_t^1), (x_t^2, y_t^2), \ldots, (x_t^m, y_t^m)\right\},$$

where (x_t^i, y_t^i) denotes the announced demands in the ith most recent play of the Nash demand subgame. When at least one agent chooses the outside option in period t, the history remains unchanged, i.e., $\omega_{t+1} = \omega_t$. The history space is given by $\Omega = \times_{i=1}^m (X \times X)$. At the beginning of period t, the process is in history ω_t. For $t = 1$, $\omega_1 \in \Omega$. The choice of ω_1 does not affect the results. In each period t, agent a_t draws a sample from ω_t, denoted by $K_1(\omega_t)$, which consists of k demands made by Player 2. Similarly to a_t, agent b_t draws a sample, denoted by $K_2(\omega_t)$, which consists of k demands of Player 1. We

[11] Our results will still hold even if we replace $X = \{\delta, \ldots, v - \delta\}$ with $X = \{0, \delta, \ldots, v - \delta, v\}$.

assume that $m \geq 2k$. That is, there is an upper bound on the accuracy of the agents' samples. This guarantees a convergence result of the dynamic process (see Lemma 1).

Each agent calculates a best response to the frequency distribution of the demands in $K_i(\omega_t)$, assuming that the subgame will be played:

$$BR(K_i(\omega_t)) = \underset{s_i \in \{I,O\} \times \tilde{X}(K_i(\omega_t))}{\text{argmax}} \frac{1}{k} \sum_{y \in K_i(\omega_t)} u_i(M_i(s_i, (I, y)); r_i) \quad \text{for } i \in \{1, 2\}, \tag{5}$$

$$\text{where} \quad \tilde{X}(K_i(\omega_t)) = \underset{x \in X}{\text{argmax}} \frac{1}{k} \sum_{y \in K_i(\omega_t)} u_i(M_i((I, x), (I, y)); r_i).$$

$BR(K_i(\omega_t))$ is the set of best responses that choose an optimal action in each subgame, i.e., strategies induced by backward induction. If an agent has multiple best responses, she chooses each of them with positive probability.[12]

We assume that matching, sampling, and selecting a best response depend on ω_t but are otherwise independent of the period t. Then, the probability of ω_{t+1} depends only on ω_t, and the dynamic is a Markov chain with state space Ω.[13] We call this Markov chain the *unperturbed dynamic*. Agents always choose one of best responses to their sample in the unperturbed dynamic. Let P^0 denote the transition matrix of the unperturbed dynamic.

We allow agents to sometimes make mistakes or experiment with strategies that are not best responses in the perturbed version of the dynamics. Let $\varepsilon > 0$ be the probability that any given agent experiments in any given period. If an agent experiments, she chooses a strategy at random from strategy set S. Let P^ε denote the transition matrix of the Markov chain with mistake rate $\varepsilon > 0$. We call a Markov chain with P^ε for $\varepsilon > 0$ a *perturbed dynamic*.

The agents' choice in period t determines the state ω_{t+1}. If at least one agent exercises the outside option, then the history remains unchanged, i.e., $\omega_{t+1} = \omega_t$. Otherwise, the agents play the Nash demand game, and the history is updated by adding their demands (x_t, y_t) and removing the oldest record, i.e., $\omega_{t+1} = \{(x_t, y_t), (x_t^1, y_t^1), \dots, (x_t^{m-1}, y_t^{m-1})\}$. A transition from state ω_t to state ω_{t+1} is denoted by (ω_t, ω_{t+1}). Let $P^\varepsilon_{\omega,\omega'}$ denote the transition probability of (ω, ω').

We set some definitions for the stochastic stability analysis. A state ω is *coordinated* if $\omega = \{(x, v-x), \dots, (x, v-x)\}$ for some $x \in X$. Let $Q(x)$ denote the state that is coordinated on $(x, v-x)$. A state ω is in a *convention* if ω is coordinated on $(x, v-x)$ for some $x \in X^{<r_2}_{>r_1}$ or if $e = O$ is the best response for at least one player against any of her sample from ω. A convention with $e_1 = e_2 = I$ corresponds to a subgame-perfect equilibrium, whereas a convention with $e_i = O$ for some $i \in \{1, 2\}$ corresponds to a self-confirming equilibrium (Fudenberg and Levine, 1993). A convention is an absorbing state of the unperturbed dynamic. The next lemma shows that the unperturbed dynamic converges to some convention under certain conditions.

Lemma 1. *If $k \leq m/2$, the process induced by P^0 converges to a convention almost surely.*

Lemma 1 is our version of Theorem 1 in Young (1993b). We omit the proof since it is similar to that of Young (1993b).

4. Prospect theory Nash bargaining solution and stochastic stability results

4.1. Definition

We define a Nash bargaining solution under prospect theory similarly to the ordinary Nash bargaining solution. We place no restrictions on the outside options other than $r_1, r_2 \geq 0$ and $r_1 + r_2 < v$. Note that the condition that $r_1 + r_2 < v$ implies that it is efficient to play the Nash demand subgame in our two-stage game.

Definition 1 (Prospect theory Nash bargaining solution)**.** Suppose that the two players' utility functions, u_1 and u_2, have the form of Eq. (2). The *prospect theory Nash bargaining solution* is the unique division $(x^*, v-x^*)$, where x^* is given by

$$x^* = \underset{x \in [r_1, v-r_2]}{\text{argmax}} (u_1(x; r_1) - u_1(0; r_1))(u_2(v-x; r_2) - u_2(0; r_2)). \tag{6}$$

When $r_1 = r_2 = 0$, this expression reduces to the ordinary Nash bargaining solution.

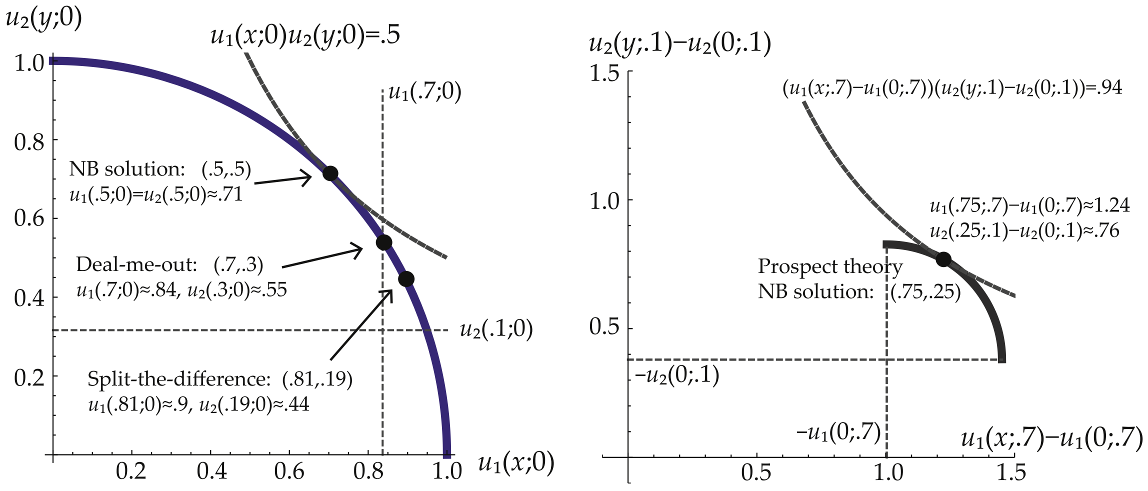
The solution is a product of the players' utilities, as in the ordinary Nash bargaining solution, except that the utility of player $i \in \{1, 2\}$ is offset by $u_i(0; r_i)$. We interpret $-u_i(0; r_i)$ as the players' fear of receiving nothing in the bargaining game. Eq. (6) implies that when a player's concern of the no-deal outcome is larger, i.e., $u_i(0; r_i)$ is smaller, she will likely obtain a smaller share in the prospect theory Nash bargaining solution.

Binmore et al. (1989) suggest two Nash bargaining solutions with the presence of an outside option, *split-the-difference* and *deal-me-out*. Suppose the outside option is worth (r_1, r_2), and let $Y = \{(x, y) : x \geq r_1, y \geq r_2, x + y \leq v\}$. Split-the-difference is the Nash bargaining solution (x, y) that maximizes the product of $u_1(x; 0) - u_1(r_1; 0)$ and $u_2(y; 0) - u_2(r_2; 0)$ on the set Y.[14] This solution will assign each player her option value plus what is given by a Nash bargaining solution over

[12] We assume no sticking behavior; agents do not stick with the 'current' strategy. When a pair of agents is drawn from the population, they do not have any initial strategy in hand. They draw a random sample from the history, compute best responses to it, and choose one of them.

[13] We may write history ω as state ω and use the terms interchangeably.

[14] The definition here follows Eq. (3) of Sutton (1986).



(a) Nash bargaining solutions (b) Prospect theory Nash bargaining solution

Fig. 2. Comparison of the four bargaining solutions.

the remaining pie, which is worth $v - r_1 - r_2$. The remaining pie is split by the solution maximizing $v_1(x)v_2(v - r_1 - r_2 - x)$ over $x \in [0, v - r_1 - r_2]$, where $v_i(z) = u_i(z; 0) - u_i(r_i; 0)$ for $i \in \{1, 2\}$. Deal-me-out is the Nash bargaining solution maximizing the product of $u_1(x; 0)$ and $u_2(y; 0)$ on the set Y. In deal-me-out, the outside options are used only as constraints on the range of divisions in the bargaining game. The next example illustrates the difference among the two solutions and the prospect theory Nash bargaining solution.

Example 1. Suppose a bargaining game with $v = 1$ and the utility function given by Eq. (1) with $\alpha = \beta = 0.5$ and $\lambda = 1.2$. Fig. 2(a) shows the possibility frontier of $(u_1(x; 0), u_2(y; 0))$ with $x + y \le v$. The thickest line is the set of $(u_1(x; 0), u_2(y; 0))$ with $x + y = v$. Any point below the line is inefficient. The ordinary Nash bargaining solution will split the pie equally. The fifty-fifty division maximizes the product of the players' utilities, which is $u_1(0.5; 0)u_2(0.5; 0) = 0.5$. Fig. 2(a) also sketches two other solutions when there is an outside option worth $(0.7, 0.1)$. The split-the-difference division is $(0.81, 0.19)$, whereas the deal-me-out division is $(0.7, 0.3)$.

Fig. 2(b) shows the prospect theory Nash bargaining solution with $r_1 = 0.7$ and $r_2 = 0.1$. The utility possibility frontier is elevated by $-u_1(0; 0.7)$ and $-u_2(0; 0.1)$ in the figure, as suggested in Eq. (6). It shows that players obeying prospect theory would end up with a division around $(0.75, 0.25)$.

4.2. Stochastic stability analysis and main results

The process induced by P^ε for $\varepsilon \in (0, 1)$ is an aperiodic, irreducible Markov chain, and thus it has the unique stationary distribution $\pi_\varepsilon \in \mathbb{R}_+^\Omega$. Let $\pi_\varepsilon(\omega)$ denote the weight π_ε places on state ω. Let $\pi_0 = \lim_{\varepsilon \to 0} \pi_\varepsilon$. We define a stochastically stable state as below.

Definition 2. A state ω is *(generically) stochastically stable* if $\pi_0(\omega) > 0$ for all sufficiently large k and $m \ge 2k$.

The definition is consistent with that of *generic stability* in Young (1993b). If the mistake rate ε is sufficiently small, then the process is in one of the stochastically stable states in the long run with probability arbitrarily close to one. Thus, the limiting stationary distribution π_0 provides a sound prediction of the long-run outcomes.

To tip the process from one convention to another requires a succession of mistakes. We say that the number of mistakes in the sequence of transitions that leads from state s to s' is the *cost* of the transition from s to s'. The number of mistakes for the process to move from a convention to another is called the *escaping cost* of the convention. Let

$$x_\delta^* \in X_\delta^* \equiv \underset{x \in X_{>r_1}^{<r_2}}{\operatorname{argmax}}\, c_\delta^*(x),$$

where

$$c_\delta^*(x) = \min\left\{\frac{u_1(x; r_1) - u_1(x - \delta; r_1)}{u_1(x; r_1) - u_1(0; r_1)}, \frac{u_2(v - x; r_2) - u_2(v - x - \delta; r_2)}{u_2(v - x; r_2) - u_2(0; r_2)}\right\}. \tag{7}$$

The next theorem characterizes the stochastically stable division. The proofs of it and the subsequent results are relegated to Appendix A.2.

Theorem 1. *For sufficiently small* δ, $Q(x)$ *is stochastically stable if and only if* $x \in X^*_\delta$.

We discuss the intuition behind Theorem 1. The two terms in Eq. (7) represent the escaping costs from the convention $Q(x)$ to $Q(x-\delta)$ and to $Q(x+\delta)$, respectively. To see this, suppose that the process is in a convention in period t, that is $\omega_t = \{(x, v-x), \ldots, (x, v-x)\}$ for some $x \in X^{<r_2}_{>r_1}$. Consider the case that the process moves toward $Q(x-\delta)$. A best response of Player 1 will become $x-\delta$ if Player 2 makes i successive mistakes of demanding $v-x+\delta$ such that

$$\underbrace{u_1(x-\delta; r_1)}_{\substack{\text{utility when}\\ \text{compromising}}} \geq \underbrace{\frac{i}{k} u_1(0; r_1)}_{\substack{\text{perceived loss}\\ \text{of claiming } x}} + \underbrace{\frac{k-i}{k} u_1(x; r_1)}_{\substack{\text{perceived gain}\\ \text{of claiming } x}}. \tag{8}$$

The left-hand side (LHS) denotes the probability-weighted utility of demanding $x-\delta$, and the right-hand side (RHS) denotes that of demanding x. The first term of the RHS reflects the perceived loss from unsuccessful bargaining, which results in $u_1(0; r_1) \leq 0$. This term indicates that a player under prospect theory cares more about the no-deal outcome than a player under expected utility theory. Rearranging the inequality, we obtain the first term of the RHS of Eq. (7):

$$i > k \frac{u_1(x; r_1) - u_1(x-\delta; r_1)}{u_1(x; r_1) - u_1(0; r_1)}.$$

Similarly to that, the second term of the RHS of Eq. (7) corresponds to the minimum cost of the transition from $Q(x)$ to $Q(x+\delta)$. The process can escape from $Q(x)$ to other conventions, e.g., $Q(x-2\delta)$. However, those moves require a larger number of mistakes, i.e., those are more unlikely moves (Lemmas A.1–A.3 in the Appendix). We can roughly see this by replacing $u_1(x-\delta; r_1)$ with $u_1(x-2\delta; r_1)$ in Eq. (8).

For a convention in which some player chooses the outside option, the minimum escaping cost from it is at most two (Lemmas A.4–A.5). This is because if both players make a mistake, then the process moves either into another convention in which some player chooses the option or into some state in which both players choose to enter the subgame. For the latter case, the process can move to $Q(x)$ for some $x \in X^{<r_2}_{>r_1}$ without any further mistake. $Q(x^*_\delta)$ is the convention that is the most robust against mistakes since x^*_δ maximizes the minimum escaping cost.

The next theorem offers a coincidence result. It shows that the stochastically stable convention under prospect theory is arbitrarily close to our version of the Nash bargaining solution as δ approaches zero.

Theorem 2. *Let* x^* *be the prospect theory Nash bargaining solution. Then,* $\lim_{\delta \to 0} x^*_\delta = x^*$.

Theorem 3 of Young (1993b) shows that the stochastically stable division maximizes $u_1(x)u_2(v-x)$ when $u_1(\cdot)$ and $u_2(\cdot)$ obey expected utility theory, k is common for both players, and δ approaches zero. Definition 1 and Theorem 2 reveal that the effects of prospect theory appear as constant terms in our Nash bargaining solution, i.e., $u_1(0; r_1)$ and $u_2(0; r_2)$. Since these effects are independent from the bargaining result, Theorem 2 coincides with Theorem 3 of Young (1993b) when the players' utility functions are replaced by $u_1(x; r_1) - u_1(0; r_1)$ and $u_2(v-x; r_2) - u_2(0; r_2)$. The next example illustrates an implication of Theorem 2. The effect of the reference points on the division will be further discussed in the next section.

Example 2 (Parameters estimated in TK92)**.** Suppose that the utility functions are given by Eq. (1) with $\alpha = \beta = 0.88$ and $\lambda = 2.25$. They are the estimates in TK92. Assume that Player 2 does not have an outside option, or $r_2 = 0$. We can show that when $\alpha = \beta$,

$$x^* < \frac{v}{2} \qquad \forall r_1 \in \left(0, \frac{v}{2} \frac{\lambda^\zeta}{1+\lambda^\zeta}\right),$$

where $\zeta = \frac{1}{1-\alpha}$. This together with $\alpha = \beta = 0.88$ and $\lambda = 2.25$ implies that Player 1's share is less than $\frac{v}{2}$ if $r_1 \in (0, 0.4994v)$. For almost all outside options that are less than $\frac{v}{2}$, Player 1 will get less than what Player 2 with no outside option will get. It may be surprising that a higher reference point may not always lead to a stronger bargaining position. We interpret this implication as follows. When people are strongly loss averse, e.g. $\lambda = 2.25$, a higher reference point leads to a strong impact of failure (the no-deal outcome) in the Nash demand subgame. Due to this, people tend to avoid failure. In contrast to this, the impact of failure for people with no outside option is cushioned to some extent because their opportunity cost of entering the subgame is zero. As a consequence, people with a higher reference point may be a softer negotiator in bargaining than people with no reference point are. See also Observation 1 in Section 4.3.

4.3. The effects of the reference point

We offer a comparative statics analysis for the stochastically stable division in response to the Player 1's reference point. Prospect theory has mixed effects on bargaining outcomes. Firstly, it makes the player's resistance to concession stronger when their share is closer to the reference point. This is because the sensitivity to outcomes is increasing as their share is closer to their reference point. Secondly, it generally makes the player's resistance to concession weaker. This is due to loss aversion, which places a higher disutility on the no-deal outcome. Furthermore, this disutility becomes higher when the reference point is higher. Loss averse players tend to compromise more easily in order to avoid the no-deal outcome.

The first effect dominates when the bargaining outcome is close to the player's reference point. The player with prospect theory obtains a larger share (than one with expected utility theory obtains). When the bargaining outcome is further from the player's reference point, the second effect dominates and the player obtains a smaller share.

The next proposition shows the influence of prospect theory on the long-run outcomes more precisely. There exists a threshold level r^* (depending on r_2), which determines the bargaining power of Player 1 relative to Player 2. Player 1's bargaining power is stronger than Player 2's if $r_1 > r^*$; it is weaker than Player 2's if $r_1 < r^*$.

Proposition 1. *Suppose that players' utility functions have the same form. Without loss of generality, let $r_1 > r_2$. If $\lim_{x\to 0} \frac{du^+(x)}{dx} = \infty$ and $\frac{d^3u^+(x)}{dx^3} > 0$, then there exists $r_2 \le r^*(r_2) < v/2$ such that $\mathrm{sgn}(r_1 - r^*(r_2)) = \mathrm{sgn}(x^*_\delta - \frac{v}{2})$ for sufficiently small δ.*

The third derivative condition is interpreted as the convexity of the marginal diminishing sensitivity. The condition implies that the sensitivity diminishes more slowly as x is farther from the reference point. Utility functions of TK92 satisfy it. The next observation is for utility functions of TK92.

Observation 1. Suppose that players' utility functions follow Eq. (1) with $\alpha = \beta$ and that Player 2 has no outside option, or $r_2 = 0$. Then, $r^*(r_2) = \frac{v}{2}\frac{\lambda\zeta}{1+\lambda\zeta}$ where $\zeta = \frac{1}{1-\alpha}$.

$r^*(0) \approx 0.4994v$ for $\lambda = 2.25$ and $\alpha = \beta = 0.88$ — the estimated parameters in TK92. Most options that are less than $\frac{v}{2}$ would deteriorate Player 1's share. Observation 1 implies that for the utility functions of TK92, $r^*(0)$ is increasing in λ, and decreasing in α. That is, if Player 1 is more loss averse (larger λ), Player 1's bargaining power is weaker than Player 2's in a larger range of r_1. If Player 1 has stronger diminishing sensitivity (larger α), Player 1's bargaining power is stronger than Player 2's in a larger range of r_1.

Proposition 1 implies that $x^*_\delta < \frac{v}{2}$ for all $r_1 < r^*(r_2)$, that is, Player 1 with $r_1 < r^*(r_2)$ obtains a smaller share than Player 2 even though Player 1 has a seemingly stronger bargaining position than Player 2 ($r_1 > r_2$). We briefly sketch the intuition of Proposition 1. Let

$$C(x, x-\delta) = k \cdot \frac{u_1(x; r_1) - u_1(x-\delta; r_1)}{u_1(x; r_1) - u_1(0; r_1)}, \tag{9}$$

$$C(x, x+\delta) = k \cdot \frac{u_2(v-x; r_2) - u_2(v-x-\delta; r_2)}{u_2(v-x; r_2) - u_2(0; r_2)}. \tag{10}$$

$C(x, x-\delta)$ is the minimum number of mistakes required for the transition from $Q(x)$ to $Q(x-\delta)$. $C(x, x+\delta)$ is that for the transition from $Q(x)$ to $Q(x+\delta)$. $C(x, x-\delta)$ and $C(x, x+\delta)$ can be interpreted as Player 1's resistance to concession and Player 2's, respectively. A stochastically stable convention maximizes the minimum of $C(x, x-\delta)$ and $C(x, x+\delta)$. We focus on Player 1's resistance to concession. For sufficiently small δ, $C(x, x-\delta)$ is approximated as

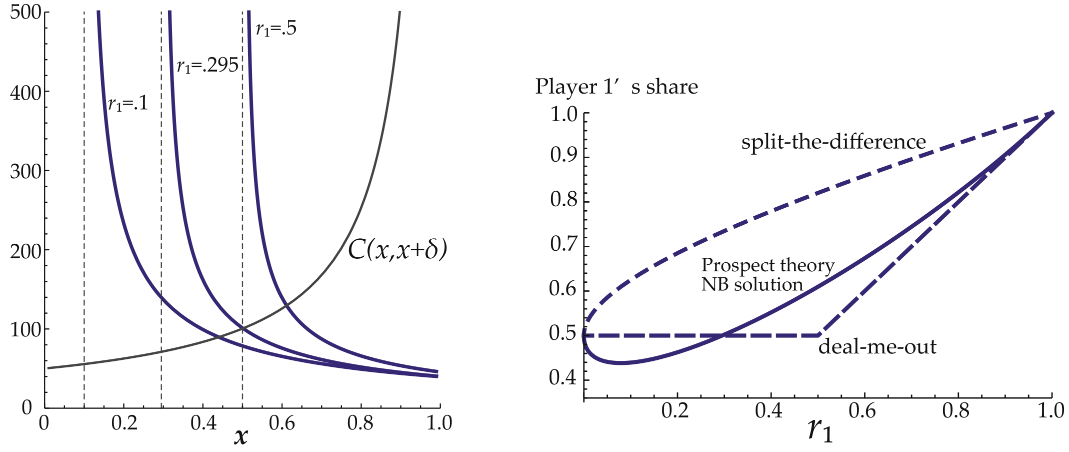
$$C(x, x-\delta) \approx k\delta \frac{u_1'(x; r_1)}{u_1(x; r_1) - u_1(0; r_1)}. \tag{11}$$

Due to diminishing sensitivity, $u_1'(x; r_1)$ becomes larger when x is closer to r_1. That is, Player 1 becomes more reluctant to compromise as her share is closer to r_1. When $r_1 > r^*(r_2)$, the effect of diminishing sensitivity dominates in $C(x, x-\delta)$. The stochastically stable division is in favor of Player 1—she tends to obtain a larger share.

The second term in the denominator, $-u_1(0; r_1)$, is Player 1's disutility from the no-deal outcome. We interpret it as Player 1's psychological pressure against the no-deal outcome. By choosing "In", Player 1 has to give up r_1 and enters the game in which she may end up receiving nothing. The fear of the no-deal outcome is represented by $-u_1(0; r_1)$. When $r_1 < r^*(r_2)$, this effect dominates, and the division is less in favor of Player 1. Note that $-u_1(0; r_1)$ is increasing in not only λ but also the option value r_1. Due to this, an increase in the option value may reduce one's resistance to concession and does not always bring a larger share to them.

Example 3. Suppose that the players' utility functions are given by Eq. (1). The parameters are $\alpha = \beta = 1/2$, $\lambda = 1.2$, $v = 1$, $\delta = 0.01$, and $k = 10,000$. Fig. 3(a) shows $C(x, x-\delta)$ and $C(x, x+\delta)$ for several values of r_1 and $r_2 = 0$. The intersections between $C(x, x-\delta)$ and $C(x, x+\delta)$ correspond to the stochastically stable divisions. Each cost curve becomes steeper as x approaches the reference point due to diminishing sensitivity. For $r_1 = 0.5$, Player 1 receives more than 0.5 (≈ 0.6) in the stochastically stable division. For $r_1 = 0.1$, Player 1 receives less than 0.5 (≈ 0.42). This is due to loss aversion; $C(x, x-\delta)$ for $r_1 = 0.1$ steeply decreases as x departs from r_1. The stochastically stable division is around $(0.5, 0.5)$ for $r_1 = 0.295$ since $r^*(r_2) \approx 0.295$ for $r_2 = 0$.

Fig. 3(b) compares our result with the split-the-difference and deal-me-out solutions, defined in Section 4.1. It shows Player 1's shares over r_1 as predicted by our solution, split-the-difference, and deal-me-out. Player 1's share in split-the-difference is the largest among the three solutions for all $r_1 \in (0, 1)$, whereas our solution and deal-me-out have relatively similar predictions. In contrast to the other two solutions, Player 1's share in our solution is not monotone over r_1. The share predicted by our solution is below $\frac{v}{2}$ and the smallest among the three solutions for $r_1 \in (0, 0.295)$. It becomes slightly larger than that of deal-me-out for $r_1 > 0.295$. This non-monotonicity property of our solution is consistent for $\lambda > 1$.



(a) $C(x, x-\delta)$ over x with three different r_1 (b) Player 1's share over r_1 for three solutions

Fig. 3. $C(x, x-\delta)$ over x and Player 1's share over r_1.

5. Discussion

5.1. Comparison to expected utility theory

We compare the effects of expected utility theory and prospect theory. For the sake of simplicity, we analyze the game depicted in Fig. 1(b), i.e. no outside option for Player 2. We call the game *the Nash demand with an outside option game*. Since "In" is always optimal for Player 2, we omit her first-stage decision and restrict the strategy space for Player 2 to X. Player 2 draws a sample $K_2(\omega_t)$ and calculates a best reply as follows:

$$BR(K_2(\omega_t)) = \arg\max_{y\in X} \frac{1}{k} \sum_{x\in K_2(\omega_t)} u_2(M_2((I,x),(I,y));0).$$

Let $X_{>r_1} = \{x \in X : x > r_1\}$. We slightly adjust the definition of convention: ω is in a *convention* if ω is coordinated on $(x, v-x)$ for some $x \in X_{>r_1}$ or if $e_1 = O$ is Player 1's best response against any sample from ω. Recall that $Q(x)$ for $x \in X$ denotes the state that is coordinated on $(x, v-x)$, i.e. $Q(x) = \{(x, v-x), \ldots, (x, v-x)\}$.

Tröger (2002) studies the stochastic stability of two-stage Nash demand game in which one player chooses the investment level in the first stage. We use his result as a benchmark result under expected utility theory. The following corollary is implied by Tröger (2002).

Corollary 1. *Suppose a Nash demand with an outside option game. Assume that players' utility functions are linear in monetary payoffs. The unique stochastically stable state is* $Q(x_\delta^b)$ *where* $x_\delta^b = \max\{x_{\min}, v/2\}$, $x_{\min} = \min\{X_{>r_1}\}$.

We examine whether Player 1 obeying prospect theory obtains a larger share relative to the benchmark division $(x_\delta^b, v - x_\delta^b)$. Note that the benchmark division approaches the deal-me-out solution as δ approaches zero.

Remark 2. The stochastically stable division under expected utility theory approaches the deal-me-out solution even in the game of Fig. 1(a) if both players have a common concave utility function, $u(\cdot)$. To see this, without loss of generality, assume that $0 \le r_2 \le r_1$ with $r_1 + r_2 < v$. Then, the stochastically stable division approaches the solution of the maximization problem below as δ approaches zero.

$$x^* = \operatorname*{argmax}_{x\in[r_1, v-r_2]} u(x)u(v-x).$$

Note that the solution is $(\frac{v}{2}, \frac{v}{2})$ if $r_1 \le \frac{v}{2}$ and $(r_1, v-r_1)$ if $r_1 > \frac{v}{2}$.

As for prospect theory, the following corollary is implied by Theorem 1.

Corollary 2. *Suppose a Nash demand with an outside option game. For sufficiently small* δ, $Q(x_\delta^*)$ *is stochastically stable if and only if* x_δ^* *maximizes* $c_\delta^*(x)$ *on* $X_{>r_1}$, *where*

$$c_\delta^*(x) = \min\left\{ \frac{u_1(x;r_1) - u_1(x-\delta;r_1)}{u_1(x;r_1) - u_1(0;r_1)},\ 1 - \frac{u_2(v-x-\delta;0)}{u_2(v-x;0)} \right\}. \tag{12}$$

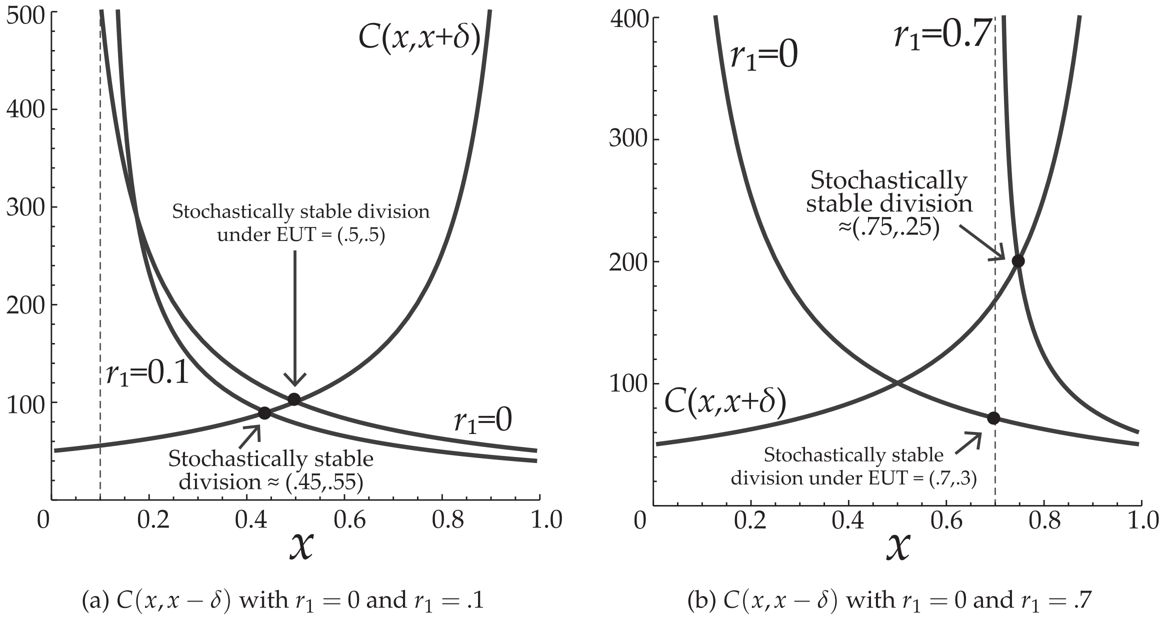


Fig. 4. $C(x, x-\delta)$ and $C(x, x+\delta)$ for $r_1 = 0.1$ and $r_1 = 0.7$.

Both stochastically stable divisions suggest efficient outcomes; the players engage in the Nash demand game and split the pie. The two divisions generally differ when $r_1 > 0$. Player 1's share is non-decreasing over r_1 under expected utility theory, whereas, under prospect theory, Player 1 does not always benefit from the outside option. In fact, the outside option sometimes harms Player 1. The next example illustrates this.

Example 4. Recall that the stochastically stable division is around the intersection of $C(x, x-\delta)$ and $C(x, x+\delta)$ defined in Eqs. (9) and (10). Fig. 4 shows $C(x, x-\delta)$ for $r_1 \in \{0, 0.1, 0.7\}$ and $C(x, x+\delta)$ for $r_2 = 0$. The other parameters are the same as in Example 3.

Players obeying expected utility theory demand $(0.5, 0.5)$ for $r_1 = 0.1$ and $(0.7, 0.3)$ for $r_1 = 0.7$. Under prospect theory, the stochastically stable division is around $(0.45, 0.55)$ for $r_1 = 0.1$ and around $(0.75, 0.25)$ for $r_1 = 0.7$. Player 1 ends up with a smaller portion relative to the benchmark when $r_1 = 0.1$. While, Player 1 obtains a larger portion relative to the benchmark when $r_1 = 0.7$.

As x approaches her reference point r_1, Player 1 is more reluctant to compromise, i.e., $C(x, x-\delta)$ is higher. As discussed in Section 4.3, this is due to diminishing sensitivity; the marginal utility of gains or losses decreases with their size. Due to the greater marginal utility around r_1, when Player 1's share is closer to r_1, she has a stronger desire to protect her share, that is, she is less prone to making a compromise. For $r_1 = 0.7$, this property helps the intersection move to the right.

For $r_1 = 0.1$, Player 1 does worse than the benchmark. The property of loss aversion shifts $C(x, x-\delta)$ downward. This makes Player 1 compromise to $x-\delta$ more easily than she would with $r_1 = 0$. We interpret this as follows. Player 1 has an alternative ($e = O$) for a certain return, so choosing $e = I$ puts psychological pressure on her due to the possibility of ending up with nothing. It deteriorates her ability to drive a hard bargain.

The next result, Proposition 2, examines the influence of the degree of loss aversion on the division. Facing an opponent with expected utility theory, a player will yield a smaller share if she is more loss averse. Furthermore, if a player is extremely loss averse, i.e., in the limit of large λ, her share falls to the one equivalent to her option value.

Proposition 2. *Suppose a Nash demand with an outside option game. Let* $x^*_\lambda = \lim_{\delta\to 0} x^*_\delta$ *for a given* λ. *Then,* $\frac{\partial x^*_\lambda}{\partial \lambda} < 0$. *Furthermore,* $\lim_{\lambda\to\infty} x^*_\lambda = r_1$ *if* $\lim_{\lambda\to\infty} \frac{\partial u^-(0;\lambda)}{\partial \lambda} > 0$.

Proof. Let $U(x, \lambda) = (u_1(x; r_1) - u_1(0; r_1))u_2(v - x; 0)$. Theorem 2 shows that x^*_λ maximizes $U(x, \lambda)$. This implies that $\frac{\partial U(x,\lambda)}{\partial x} = 0$, and $\frac{\partial^2 U(x,\lambda)}{\partial x^2} < 0$ for $x = x^*_\lambda$. Differentiating the first equality above with respect to λ and rearranging it, we obtain

$$\frac{\partial x^*_\lambda}{\partial \lambda} = -\frac{\frac{\partial^2 U(x,\lambda)}{\partial \lambda \partial x}}{\frac{\partial^2 U(x,\lambda)}{\partial x^2}} < 0.$$

The inequality comes from that, for $x = x^*_\lambda$,

$$\frac{\partial^2 U(x,\lambda)}{\partial\lambda\partial x} = -\frac{\partial u^-(0;\lambda)}{\partial\lambda} u_2'(v - x^*_\lambda; 0) < 0.$$

For the second claim, observe that, for all $x > r_1$, if λ is sufficiently large, we have that $-u_1(0;r_1)u_2(v-r_1;0) > (u_1(x;r_1) - u_1(0;r_1))u_2(v-x;0)$. □

5.2. Young's (1993b) model with reference-dependent preferences

Our model adds two changes to the Nash demand game: (I) outside options r_i, and (II) the assumption of reference-dependent utility. Integrating the change (I) alone results in that the stochastically stable state is the deal-me-out solution, as discussed in Section 5.1. In this section, we consider a setting with the change (II) alone, i.e. Young's (1993b) bargaining model with reference-dependent utility. Formally, we call as the *Young's bargaining model* the model in which the strategy space is restricted to $\{I\} \times X$ for both players. We generalize the Nash bargaining solution as follows.

Definition 3. Suppose that $u_1(\cdot)$ and $u_2(\cdot)$ have the form of Eq. (2). The *generalized prospect theory Nash bargaining solution* is a division $(x^*_g, v - x^*_g)$ where x^*_g satisfies that

$$x^*_g \in \operatorname*{argmax}_{x\in[0,v]} (u_1(x;r_1) - u_1(0;r_1))(u_2(v-x;r_2) - u_2(0;r_2)). \tag{13}$$

Note that the Nash product is maximized over $[0, v]$ instead of $[r_1, v - r_2]$. The generalized prospect theory Nash bargaining solution is not necessarily unique due to that $u_i(\cdot; r_i)$ may not be concave over $[0, r_i)$ for $i \in \{1, 2\}$. For what follows, we assume that x^*_g is uniquely determined. Define

$$C^-(x) = \frac{u_1'(x;r_1)}{u_1(x;r_1) - u_1(0;r_1)}. \qquad \forall x \in (0,r_1) \cup (r_1, v],$$

$$C^+(x) = \frac{u_2'(v-x;r_2)}{u_2(v-x;r_2) - u_2(0;r_2)}. \qquad \forall x \in [0, v-r_2) \cup (v-r_2, v),$$

where $u_i'(y;r_i) = \frac{\partial u_i(y;r_i)}{\partial y}$ for $i \in \{1,2\}$. As discussed in Section 4.3, $k\delta C^-(x)$ approximates the number of required mistakes for the transition from $Q(x)$ to $Q(x-\delta)$ for sufficiently large k and sufficiently small δ. Similarly to that, $k\delta C^+(x)$ approximates that number for the transition from $Q(x)$ to $Q(x+\delta)$.

The next proposition characterizes a sufficient condition under which the generalized prospect theory Nash bargaining solution corresponds to the stochastically stable state in the Young's bargaining model.

Proposition 3. *Suppose the Young's bargaining model with the players' preferences satisfying Assumption 1 and $r_1 + r_2 < v$. For every $\delta > 0$, let $\tilde{x}^*_\delta$ denote the state for which $Q(\tilde{x}^*_\delta)$ is stochastically stable. If $C^-(x) > C^+(x)$ for all $x \in (0, r_1)$ and $C^+(x) > C^-(x)$ for all $x \in (v - r_2, v)$, then $\lim_{\delta\to 0} \tilde{x}^*_\delta = x^*_g$.*

We omit the proof and briefly explain the intuition. Recall that under Assumption 1, Player i's utility function $u_i(x; r_i)$ is not concave for $x < r_i$. This causes the difficulty in the stochastic stability analysis. When utility functions are not concave, $C^-(x)$ may not be monotone decreasing, or $C^+(x)$ may not be monotone increasing. Then, the minimum-cost escape may not always lead the process toward x^*_g.

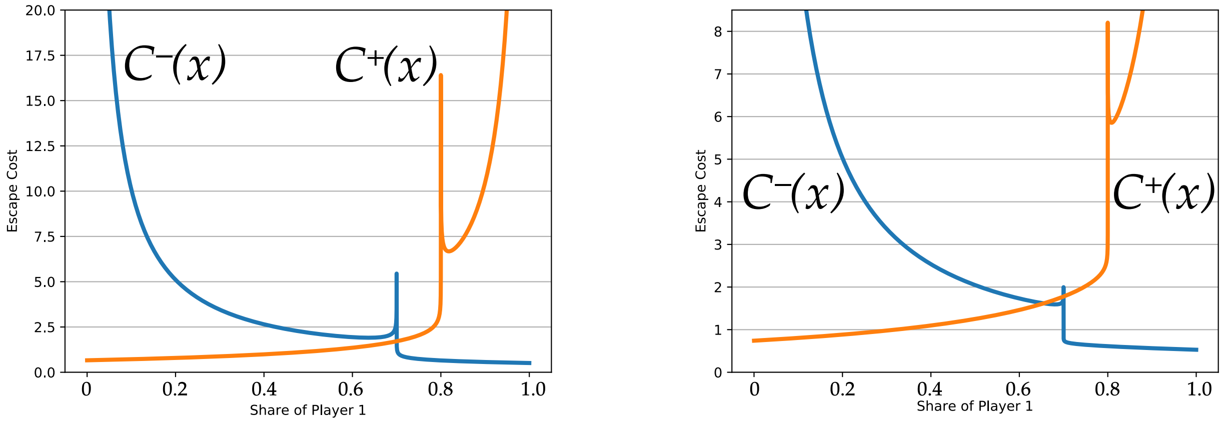
The first condition in Proposition 3, $C^-(x) > C^+(x)$, guarantees that moving toward $Q(x + \delta)$ is the minimum-cost escape from $Q(x)$ for $x \in (0, r_1)$. Similarly to that, the second condition guarantees that moving toward $Q(x - \delta)$ is the minimum-cost escape from $Q(x)$ for $x \in (v - r_2, v)$. The two conditions together ensure that the minimum-cost escape always leads the process toward x^*_g. Similarly to the proof of Theorem 1, we can show that the stochastically stable state is the one that maximizes $c^*_\delta(x)$ in Eq. (7) over $x \in \{\delta, 2\delta, \dots, v - \delta\}$. It converges to x^*_g in the limit of small δ.

Example 5. Suppose the Young's bargaining model with the players' preferences given by Eq. (1). Assume that $v = 1$, $r_1 = 0.7$, and $r_2 = 0.2$. Fig. 5(a) shows approximated $C^-(x)$ and $C^+(x)$ for TK92's estimates, or $\alpha = \beta = 0.88$, $\lambda = 2.25$. This set of parameters satisfies that $C^-(x) > C^+(x)$ for all $x \in (0, r_1)$ and $C^+(x) > C^-(x)$ for all $x \in (v - r_2, v)$. Proposition 3 implies that the intersection between the two curves, which corresponds to the generalized prospect theory Nash bargaining solution, is the stochastically stable division.

Fig. 5(b) shows a case that does not satisfy the condition in Proposition 3. The parameters are $\alpha = 0.97$, $\beta = 0.95$, and $\lambda = 2.25$. Observe that $C^-(x) < C^+(x)$ around $x \in (0.65, 0.7)$, that is, it violates the condition that $C^-(x) > C^+(x)$ for $x \in (0, r_1)$. Proposition 3 does not guarantee that the generalized prospect theory Nash bargaining solution is stochastically stable.

5.3. A review of the literature on experiments

We briefly review experimental studies related to ours, and refer the interested readers to Section 4 of Camerer (2003) for a comprehensive survey on experimental studies on bargaining and to Section 9 of Newton (2018) for a survey of empirical work on evolutionary game theory. There are several studies that conduct experiments on the Nash demand game



(a) $C^-(x)$ and $C^+(x)$ with TK92 estimates

(b) $C^-(x)$ and $C^+(x)$ with $\alpha = .97$ and $\beta = .95$

Fig. 5. $C^-(x)$ and $C^+(x)$ for two sets of parameters.

with an outside option, for example, Binmore et al. (1989), Binmore et al. (1998), Ellingsen and Johannesson (2004), and Feltovich and Swierzbinski (2011). Among them, Binmore et al. (1998) and Feltovich and Swierzbinski (2011) compare observed behavior with the presumably focal points: split-the-difference and deal-me-out (see Section 4.1).

Binmore et al. (1998) conducted experiments on the outside option game, which is a Nash demand game where one player has an outside option to opt out. Feltovich and Swierzbinski (2011) also conducted experiments on the same game with three different pre-play communication treatments. Their experimental results support the *deal-me-out* solution under expected utility theory. They are, however, mainly focused on distinguishing the above two focal points, and they do not have a particular focus on loss aversion or diminishing sensitivity. Our prediction is not far from the *deal-me-out* solution as illustrated in Example 3. It will be of interest to conduct an experiment designed to examine the effects of those behavioral biases.

We note that the predictions of a stochastic stability analysis should be carefully compared with the experimental results. Evolutionary models would probably apply best to the evolution of social conventions, e.g., Young and Burke (2001), and to human cultural evolution, e.g., Kuran and Sandholm (2008), which are thought to occur gradually through players' adjustments on strategies. The predictions may or may not be very suitable for explaining outcomes led by rapid individual learning.

One of promising areas is to examine individual choice behavior through experiments. Lim and Neary (2016), Mäs and Nax (2016), Alós-Ferrer and Ritschel (2018), and Hwang et al. (2018) conduct such experiments and offer insights into individual choice behavior. In the experimental setting of Hwang et al. (2018), subjects are assigned to two groups. Subjects in one group are randomly mated with subjects in the other group to play a coordination game where off-diagonal payoffs are zero. They report that subjects' strategy choices are biased; deviations from best response behavior are payoff-dependent and have intentional bias (Naidu et al., 2010). It would be interesting to conduct an experiment with our setting and examine if deviations from best response behavior have some particular bias, e.g. the dependence of deviations on the value of an outside option.

6. Concluding remarks

We study an evolutionary model of bargaining where players obey prospect theory. Built upon the stochastic stability analysis, we propose a solution concept incorporating some observed properties of human behavior. From our perspective, studies matching stochastic stability and behavioral economics would be fruitful, and further research will contribute to the literature by unveiling the effects of observed behavioral patterns on extant solution concepts. A similar approach may result in novel solution concepts in other settings. For example, Hwang et al. (2018) consider contract games (Young, 1998) and find that a stochastic stability analysis under the logit choice rule offers a new solution concept, *the Logit bargaining solution*, that differs from extant solutions. A promising area may be multiplayer bargaining settings. As written in the Introduction, there are works on evolutionary dynamics in multiplayer bargaining; Agastya (1997, 1999), Arnold and Schwalbe (2002), Newton (2012), Rozen (2013), Nax (2019), and Sawa (2019) among others. Matching prospect theory with these studies might turn out to be fruitful. We hope that this study will stimulate such studies.

A few questions remain to be answered. First, we have considered the discretized state space. We are not sure whether the equilibrium selection result remains unchanged when we adopt continuous strategy space. One may need to employ a quite different technique to analyze the continuous setting, e.g. Markov chains on general state spaces (see Meyn and Tweedie, 1993 and Newton, 2015 for reference). An approach to examine the influence of continuous state space on equilibrium selection is the double-limit approach (see Binmore et al., 1995; Binmore and Samuelson, 1997; Sandholm, 2010;

Sandholm and Staudigl, 2016; Sandholm and Staudigl, 2018, for example). In our context, the approach could be adopted by reversing the order of limits of δ and ε, i.e. $\lim_{\varepsilon\to 0}\lim_{\delta\to 0}\pi_\varepsilon$. Second, we only consider exogenous outside options. As we see in Section 4.3, the outside options may determine the players' attitude toward bargaining. If we allow players to post their outside option, then it may serve as a strategic self-signaling device that manipulates their actions in the Nash demand game (see, for example, Bénabou and Tirole, 2004 for self-signaling). It would be interesting to consider such more complex strategic settings in future research.

Declaration of Competing Interest

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Appendix A

A1. The computation method

We restate Proposition A.1 in Tröger (2002) for the perturbed dynamic P^ε of our model. It is the main computation method for the proofs of our results in Section A.2. Let $C_0(\omega,\omega')$ be the number of mistakes required for transition (ω,ω'), i.e., $P^\varepsilon_{\omega,\omega'}=\mathcal{O}(\varepsilon^{C_0(\omega,\omega')})$ if $P^\varepsilon_{\omega,\omega'}>0$. Let $C_0(\omega,\omega')=\infty$ if $P^\varepsilon_{\omega,\omega'}=0$. We call $q=(z_1,z_2,\dots,z_h)$ a *path* from z_1 to z_h on a set Z if $z_i\in Z$ for all i and $z_i\neq z_j$ for all $i\neq j$. Let $q=(\omega_1,\dots,\omega_h)$ be a path from ω_1 to ω_h on Ω, and define its cost as

$$C_0(q)=\sum_{i=1}^{h-1}C_0(\omega_i,\omega_{i+1}). \tag{14}$$

Let $\mathcal{Q}_0(\omega,\omega')$ denote the set of paths from ω to ω' on Ω. Define the minimum cost for the process moving from ω to ω' as

$$C_0^*(\omega,\omega')=\min_{q\in\mathcal{Q}_0(\omega,\omega')}C_0(q).$$

Define 0-recurrent set $R_0\subseteq\Omega$ as a set of states with the following properties:

$$C_0^*(\omega,\omega')=0 \qquad \forall\omega,\omega'\in R_0,$$
$$C_0^*(\omega,\omega')>0 \qquad \forall\omega\in R_0 \text{ and } \omega'\in\Omega\setminus R_0.$$

Let $\mathcal{R}_0$ denote the set of 0-recurrent sets. The definition implies that $R_0\in\mathcal{R}_0$ is a recurrent class of the unperturbed dynamic. Define the least escaping cost from $R_0\in\mathcal{R}_0$ as

$$C_0^*(R_0)=\min_{\omega\in R_0}\min_{R_0'\in\mathcal{R}_0\setminus R_0}\min_{\omega'\in R_0'}C_0^*(\omega,\omega'). \tag{15}$$

For $i\geq 1$, we define i-recurrent set R_i and its transition cost C_i in a recursive way. That is, we use the i-recurrent set to define the i+1-recurrent set. Let $\mathcal{R}_i$ be the set of i-recurrent sets for $i\geq 0$. For all $R_{i-1},R'_{i-1}\in\mathcal{R}_{i-1}$, define *transition cost* $C_i:\mathcal{R}_{i-1}\times\mathcal{R}_{i-1}\to\mathbb{R}_+$ as

$$C_i(R_{i-1},R'_{i-1})=0, \qquad \text{for } R_{i-1}=R'_{i-1}$$
$$C_i(R_{i-1},R'_{i-1})=C^*_{i-1}(R_{i-2},R'_{i-2})-1 \qquad \text{for } R_{i-2}\in R_{i-1},\ R'_{i-2}\in R'_{i-1} \text{ with } R_{i-1}\neq R'_{i-1},$$

where the choices of R_{i-2} and R'_{i-2} are arbitrary. Note that $R_{i-2},R'_{i-2}\in\Omega$ for $i=1$. Since $C^*_{i-1}(R_{i-2},R'_{i-2})\geq 1$, $C_i(R_{i-1},R'_{i-1})\geq 0$ for all $R_{i-1},R'_{i-1}\in\mathcal{R}_{i-1}$. The minimum transition cost $C_i^*:\mathcal{R}_{i-1}\times\mathcal{R}_{i-1}\to\mathbb{R}_+$ is defined as

$$C_i^*(R_{i-1},R'_{i-1})=\min_{q\in\mathcal{Q}_i(R_{i-1},R'_{i-1})}C_i(q),$$

where $\mathcal{Q}_i(R_{i-1},R'_{i-1})$ is the set of paths from R_{i-1} to R'_{i-1} on $\mathcal{R}_{i-1}$ and $C_i(q)$ is defined similarly to (14). Define *i-recurrent set* $R_i\subseteq\mathcal{R}_{i-1}$ as a set with the following properties:

$$C_i^*(R_{i-1},R'_{i-1})=0 \qquad \forall R_{i-1},R'_{i-1}\in R_i,$$
$$C_i^*(R_{i-1},R'_{i-1})>0 \qquad \forall R_{i-1}\in R_i,\ R'_{i-1}\in\mathcal{R}_{i-1}\setminus\{R_i\}$$

where $\mathcal{R}_{i-1}\setminus\{R_i\}$ is the set of $(i-1)$-recurrent sets that are not in R_i. Define

$$C_i^*(R_i)=\min_{R_i'\in\mathcal{R}_i\setminus R_i}\min_{R'_{i-1}\in R_i'}C_i^*(R_{i-1},R'_{i-1}),$$

where R_{i-1} is an arbitrary element of R_i.

Let Θ denote an arbitrary set of nodes and $\theta \in \Theta$. A θ-tree is a collection of edges in Θ such that there is a unique directed path from every $\theta' \neq \theta$ to θ, and there is no cycle. Let $\mathcal{T}_\Theta(\theta)$ denote the set of θ-trees, and define $C : \Theta \times \Theta \to \mathbb{R}_+$. Define

$$\mathcal{M}(\Theta, C) = \arg\min_{\theta \in \Theta} \min_{T \in \mathcal{T}_\Theta(\theta)} \sum_{(\theta', \theta'') \in T} C(\theta', \theta'').$$

Kandori et al. (1993) and Young (1993a) showed that the set of stochastically stable states is given by $\mathcal{M}(\Omega, C_0)$. The following result is due to Proposition A.1 in Tröger (2002). It implies that $\omega \in \mathcal{M}(\Omega, C_0)$ if and only if $\omega \in R_0 \in R_1 \in \ldots \in R_i \in \mathcal{M}(\mathcal{R}_i, C_{i+1})$.

Proposition A.1. *For $i \geq 0$,*

$$\mathcal{M}(\Omega, C_0) = \mathcal{M}(\mathcal{R}_i, C_{i+1}).$$

Note that Proposition 1 of Nöldeke and Samuelson (1993) is equivalent to $\mathcal{M}(\Omega, C_0) = \mathcal{M}(\mathcal{R}_0, C_1)$.

A2. Proofs of the main results

We prove Theorems 1 and 2 and Proposition 1. Let $Q(r_i)$ denote the set of states such that $e_i = O$ is the best response for Player i against any sample from the state and "In" is the best response for the other player against some sample. Let $Q(r_{12})$ denote the set of states such that $e_i = O$ is the best response for all $i \in \{1, 2\}$ against any of their samples. Let $Q(r_{\text{all}}) = Q(r_1) \cup Q(r_2) \cup Q(r_{12})$. In other words, every $\omega \in Q(r_{\text{all}})$ is a convention in which at least one player chooses "Out." Let $x_{\min} = \min\{X_{>r_1}^{<r_2}\}$, $x_{\max} = \max\{X_{>r_1}^{<r_2}\}$, and $\lceil a \rceil$ denote the smallest integer that is greater than or equal to a.

Recall that, in an acyclic game, we can reduce our attention to the set of conventions, i.e., to $\{Q(x) : x \in X_{>r_1}^{<r_2}\} \cup Q(r_{\text{all}})$. We first prove three lemmas, A.1–A.3, which characterize the escaping costs from $Q(x)$ for $x \in X_{>r_1}^{<r_2}$. Lemmas A.4–A.6 exclude $Q(r_{\text{all}})$ from the set of stochastically stable states. Then, we offer a key lemma, A.7, and we prove Theorem 1.

Define

$$c_\delta(x) = \min\{c_\delta^1(x),\ c_\delta^2(x),\ c_\delta^3(x),\ c_\delta^4(x)\}, \tag{16}$$

$$c_\delta^1(x) = \frac{u_2(v - x; r_2) - u_2(0; r_2)}{u_2(v - x_{\min}; r_2) - u_2(0; r_2)} \qquad c_\delta^2(x) = \frac{u_1(x; r_1) - u_1(x - \delta; r_1)}{u_1(x; r_1) - u_1(0; r_1)}$$

$$c_\delta^3(x) = \frac{u_2(v - x; r_2) - u(v - x - \delta; r_2)}{u_2(v - x; r_2) - u_2(0; r_2)} \qquad c_\delta^4(x) = \frac{u_1(x; r_1) - u_1(0; r_1)}{u_1(x_{\max}; r_1) - u_1(0; r_1)}.$$

Lemma A.1 shows that the minimum escaping cost from $Q(x)$ to some other $Q(x')$ for $x, x' \in X_{>r_1}^{<r_2}$ is roughly $kc_\delta(x)$. Lemma A.2 shows that this minimum escaping cost is first increasing and then decreasing, as illustrated in Fig. 4. Lemma A.3 considers the minimum escaping cost from $Q(x)$ to the basin of $Q(r_{\text{all}})$ and shows that the minimum cost to any other conventions should be given by the cost given in Lemma A.1.

Lemma A.1. *For every $x \in X_{>r_1}^{<r_2}$, the minimum cost of escaping from $Q(x)$ to a state in the basin of $Q(\hat{x})$ for some other $\hat{x} \in X_{>r_1}^{<r_2}$ is $\lceil kc_\delta(x) \rceil$.*

Proof of Lemma A1. We consider two cases, (a) moving from $Q(x)$ to $Q(\hat{x})$, where $\hat{x} < x$, and (b) moving from $Q(x)$ to $Q(\hat{x})$, where $\hat{x} > x$.

Case (a): Suppose that Player 1 makes i successive mistakes of demanding $\hat{x}$. The process will move from $Q(x)$ to $Q(\hat{x})$ if Player 2 finds that it is profitable to choose $v - \hat{x}$ for some sample, that is,

$$\frac{i}{k} u_2(v - \hat{x}; r_2) + \frac{k - i}{k} u_2(0; r_2) \geq u_2(v - x; r_2) \quad \Leftrightarrow \quad i \geq k \frac{u_2(v - x; r_2) - u_2(0; r_2)}{u_2(v - \hat{x}; r_2) - u_2(0; r_2)}.$$

Over all $\hat{x} \in X_{>r_1}^{<r_2}$ with $\hat{x} < x$, the minimum value of i occurs when $\hat{x} = x_{\min}$ and

$$i = \left\lceil k \frac{u_2(v - x; r_2) - u_2(0; r_2)}{u_2(v - x_{\min}; r_2) - u_2(0; r_2)} \right\rceil = \lceil kc_\delta^1(x) \rceil. \tag{17}$$

The other possibility in Case (a) is that Player 2 makes i successive mistakes of demanding $v - \hat{x}$. Player 1 will find that it is profitable to choose $\hat{x}$ if

$$u_1(\hat{x}; r_1) \geq \left(1 - \frac{i}{k}\right) u_1(x; r_1) + \frac{i}{k} u_1(0; r_1) \quad \Leftrightarrow \quad i \geq k \frac{u_1(x; r_1) - u_1(\hat{x}; r_1)}{u_1(x; r_1) - u_1(0; r_1)}.$$

The minimum value of i occurs when $\hat{x} = x - \delta$ and

$$i = \left\lceil k \frac{u_1(x; r_1) - u_1(x - \delta; r_1)}{u_1(x; r_1) - u_1(0; r_1)} \right\rceil = \lceil kc_\delta^2(x) \rceil. \tag{18}$$

This implies that $\min\{\lceil kc^1_\delta(x)\rceil, \lceil kc^2_\delta(x)\rceil\}$ is the minimum cost of escaping $Q(x)$ to some $Q(\hat{x})$ with $\hat{x} < x$.

Case (b): We omit a detailed proof for this case since it is similar to Case (a). Similarly to (a), we can show that the minimum cost of escaping $Q(x)$ to some $Q(\hat{x})$ with $\hat{x} > x$ is given by $\min\{\lceil kc^3_\delta(x)\rceil, \lceil kc^4_\delta(x)\rceil\}$. Then the claim follows. □

Lemma A.2. *$c^1_\delta(x)$ and $c^2_\delta(x)$ are monotone decreasing. $c^3_\delta(x)$ and $c^4_\delta(x)$ are monotone increasing. Let x^*_δ maximize $c_\delta(\cdot)$ on $X^{<r_2}_{>r_1}$. Then, $c_\delta(x)$ is increasing for $x \le x^*_\delta$ and decreasing for $x \ge x^*_\delta$.*

Proof. We prove the first and second claims. Then the definition of $c_\delta(x)$ in Eq. (16) implies the last claim. It is clear that $c^1_\delta(x)$ is monotone decreasing and that $c^4_\delta(x)$ is monotone increasing. We first show that $c^2_\delta(x)$ is monotone decreasing. Differentiating $c^2_\delta(x)$, we have

$$\frac{\partial}{\partial x}c^2_\delta(x) = \frac{\partial}{\partial x}\left(\frac{u_1(x;r_1) - u_1(x-\delta;r_1)}{u_1(x;r_1) - u_1(0;r_1)}\right)$$
$$= \frac{\overbrace{-u_1(0;r_1)(u_1'(x;r_1) - u_1'(x-\delta;r_1))}^{\text{1st component}}\overbrace{-u_1'(x-\delta;r_1)u_1(x;r_1) + u_1'(x;r_1)u_1(x-\delta;r_1)}^{\text{2nd component}}}{(u_1(x;r_1) - u_1(0;r_1))^2} < 0.$$

Note that

(i) $u_i(0;r_i) < 0$ for $r_i > 0$, (ii) $u_i(x;r_i) > u_i(x-\delta;r_i)$, and
(iii) $u_i'(x;r_i) \le u_i'(x-\delta;r_i)$ for $x > r_i$ (due to diminishing sensitivity).

The first component in the numerator is non-positive due to (i) and (iii). The second component is strictly negative due to (ii) and (iii). Thus, $c^2_\delta(x)$ is monotone decreasing.

Next, we show that $c^3_\delta(x)$ is monotone increasing. Differentiating $c^3_\delta(x)$, we yield

$$\frac{\partial}{\partial x}c^3_\delta(x) = \frac{\partial}{\partial x}\left(\frac{u_2(y;r_2) - u_2(y-\delta;r_2)}{u_2(y;r_2) - u_2(0;r_2)}\right)$$
$$= \frac{\overbrace{-u_2(0;r_2)(u_2'(y-\delta;r_1) - u_2'(y;r_2))}^{\text{1st component}}\overbrace{+u_2'(y-\delta;r_2)u_2(y;r_2) - u_2'(y;r_2)u_2(y-\delta;r_2)}^{\text{2nd component}}}{(u_2(y;r_2) - u_2(0;r_2))^2} > 0,$$

where $y = v - x$. The first component in the numerator is non-negative due to (i) and (iii) and that the second component is strictly positive due to (ii) and (iii). □

For Lemma A.3, define

$$\hat{c}_\delta(x) = \min\{c^5_\delta(x), c^6_\delta(x)\}, \tag{19}$$
$$c^5_\delta(x) = 1 + \frac{u_1(x;r_1)}{u_1(x;r_1) - u_1(0;r_1)}, \quad c^6_\delta(x) = 1 + \frac{u_2(v-x;r_2)}{u_2(v-x;r_2) - u_2(0;r_2)}.$$

Lemma A.3. *For every $x \in X^{<r_2}_{>r_1}$, the minimum cost of escaping from $Q(x)$ to a state in the basin of $Q(r_{\text{all}})$ is at least $\lceil k\hat{c}_\delta(x)\rceil$. Furthermore, $c_\delta(x) \le \hat{c}_\delta(x)$, where $c_\delta(x)$ is given by Eq. (16).*

Proof of Lemma A3. Fix $Q(x)$ for some $x \in X^{<r_2}_{>r_1}$. We first consider the minimum number of mistakes that can potentially switch to $Q(r_1)$. Suppose that Player 2 successively demands $v - \delta$ by mistake.[15] Player 1 will find that $e_1 = O$ is optimal for any sample if there are i successive records of Player 2 demanding $v - \delta$ such that

$$u_1(r_1;r_1) \ge \frac{m-i}{k}u_1(x;r_1) + \frac{k-(m-i)}{k}u_1(0;r_1)$$
$$\Leftrightarrow\ i \ge (m-k) + k\frac{u_1(x;r_1)}{u_1(x;r_1) - u_1(0;r_1)}. \tag{20}$$

Note that if Inequality (20) holds, Player 1 will choose "Out" even with a sample including $(m-i)$ instances of $v - x$, which gives the highest utility of "In" among all samples. $c^5_\delta(x)$ is given by Inequality (20) together with $m \ge 2k$.

Similarly to that, we can show that the minimum number of mistakes that can switch from $Q(x)$ to $Q(r_2)$ is bounded below by $\lceil kc^6_\delta(x)\rceil$. For switching to $Q(r_{12})$, the minimum number of mistakes is at least the sum of the minimum numbers of mistakes to switch to $Q(r_1)$ and to switch to $Q(r_2)$. This is because "Out" must be optimal for both players in $Q(r_{12})$. Thus, the minimum escaping cost to $Q(r_1) \cup Q(r_2) \cup Q(r_{12})$ is the minimum of $c^5_\delta(x)$ and $c^6_\delta(x)$.

Finally, observe that $c^2_\delta(x) \le c^5_\delta(x)$ and $c^3_\delta(x) \le c^6_\delta(x)$, where $c^2_\delta(x)$ and $c^3_\delta(x)$ are defined in Eq. (16). This proves that $c_\delta(x) \le \hat{c}_\delta(x)$. □

[15] Any demand greater than $v - r_1$ will work. $v - \delta$ is one such demand.

Next, we prove that states in $Q(r_{\text{all}})$ are never stochastically stable. Lemma A.3 implies that the minimum cost of escaping from $Q(x)$ to any $\omega \in Q(r_{all})$ is at least $k+1$ (≥ 3). Then, it suffices to show that every $\omega \in Q(r_{\text{all}})$ has a sequence of transitions toward $Q(x)$ for some $x \in X^{<r_2}_{>r_1}$ such that the cost of each transition is at most two. Lemma A.4 shows that every $\omega \in Q(r_{\text{all}})$ has a sequence of transitions toward some coordinated state. Lemma A.5 shows that every coordinated state in $Q(r_{\text{all}})$ has a sequence of transitions toward $Q(x)$ for $x \in X^{<r_2}_{>r_1}$. Together with A.4 and A.5, Lemma A.6 proves the claim.

Lemma A.4. *For any $\omega_1 \in Q(r_{\text{all}})$, there exists a sequence of states $(\omega_2, \ldots, \omega_h)$ such that $\omega_i \in Q(r_{\text{all}})$ for $1 \leq i \leq h$, ω_h is coordinated on $(x, \nu - x)$ for some $x \in X$, and $1 \leq C_0(\omega_i, \omega_{i+1}) \leq 2$ for $1 \leq i \leq h-1$.*

Proof of Lemma A4. We prove the claim for each of $Q(r_1)$ and $Q(r_{12})$. We omit the proof for $Q(r_2)$, since it is similar to $Q(r_1)$. Suppose that $\omega_1 \in Q(r_1)$. Let

$$\omega_1 = \{(x^1, y^1), \ldots, (x^m, y^m)\}, \tag{21}$$

$$\omega_i = \{\underbrace{(\delta, \nu-\delta), \ldots, (\delta, \nu-\delta)}_{(i-1)\text{ records}}, (x^1, y^1), \ldots, (x^{m-i+1}, y^{m-i+1})\} \qquad \text{for } 2 \leq i \leq m+1. \tag{22}$$

Note that $\omega_i \in Q(r_1)$ for $2 \leq i \leq m+1$. This is because the new entries of Player 2's demands are $\nu - \delta$, i.e., the largest possible claim. For Player 1, any sample of ω_i is no better than any sample of ω_1. Observe that $C_0(\omega_i, \omega_{i+1}) = 1$ if $BR_2(K_2) = (I, x-\delta)$ for some $K_2(\omega_i)$, and $C_0(\omega_i, \omega_{i+1}) = 2$ otherwise. This proves the claim for $Q(r_1)$.

Next, suppose that $\omega_1 \in Q(r_{12})$. Again, write ω_1 as Eq. (21) and consider a sequence of states given by (22). Note that $\omega_i \in Q(r_1) \cup Q(r_{12})$ for $2 \leq i \leq m+1$. Player 2's new demands are $\nu - \delta$, which prevent "In" from being optimal for Player 1. Player 2 may find "In" optimal in some period because Player 1's entries are δ, the smallest possible claim. In such a period, the state should be in $Q(r_1)$. Observe that $C_0(\omega_i, \omega_{i+1}) = 1$ if $BR_2(K_2) = (I, \nu-\delta)$ for some $K_2(\omega_i)$, and $C_0(\omega_i, \omega_{i+1}) = 2$ otherwise. □

Note that $\omega \notin Q(r_{12})$ if ω is coordinated. ω_{m+1} in the proof of Lemma A.4 is not in $Q(r_{12})$ even if the state starts with $\omega_1 \in Q(r_{12})$.

Lemma A.5. *If $\omega_1 \in Q(r_j)$ for $j \in \{1, 2\}$ is coordinated on $(x, \nu - x)$, there exists a sequence $(\omega_2, \ldots, \omega_h)$ such that ω_h is coordinated on $(\hat{x}, \nu - \hat{x})$ for some $\hat{x} \in X^{<r_2}_{>r_1}$ and $C_0(\omega_i, \omega_{i+1}) \leq 1$ for $1 \leq i \leq h-1$. Moreover, there exists i^{**} such that $\omega_i \in Q(r_j)$ for $i < i^{**}$ and $C_0(\omega_i, \omega_{i+1}) = 0$ for $i \geq i^{**}$.*

Proof. We prove the claim for $\omega_1 \in Q(r_1)$. The proof for $Q(r_2)$ is similar. Fix $\hat{x} \in X^{<r_2}_{>r_1}$. Let

$$\omega_i = \{\underbrace{(\hat{x}, \nu-x), \ldots, (\hat{x}, \nu-x)}_{i-1\text{ records}}, \underbrace{(x, \nu-x), \ldots, (x, \nu-x)}_{m-i+1\text{ records}}\} \qquad \text{for } 2 \leq i \leq i^*, \tag{23}$$

where i^* is the minimum number such that there exists at least one sample from ω_{i^*} for which $\nu - \hat{x}$ is the best response for Player 2. There is a positive probability that Player 1 makes i^*-1 successive mistakes of playing $(I, \hat{x})$ from period 1 through i^*-1. This realizes ω_i in (23). Let, for $i^*+1 \leq i \leq i^*+m$,

$$\omega_i = \{\underbrace{(\hat{x}, \nu-\hat{x}), \ldots, (\hat{x}, \nu-\hat{x})}_{i-i^*\text{ records}}, \underbrace{(\hat{x}, \nu-x), \ldots, (\hat{x}, \nu-x)}_{i^*-1\text{ records}}, \underbrace{(x, \nu-x), \ldots, (x, \nu-x)}_{m-i+1\text{ recorrds}}\}, \tag{24}$$

where "j records" means zero records if $j \leq 0$. Let i^{**} be the minimum i such that there exists at least one sample from ω_i for which $(I, \hat{x})$ is the best response for Player 1. Note that $i^*+1 \leq i^{**} \leq i^*+m$ holds and that $\omega_i \in Q(r_1)$ for $i < i^{**}$.

Suppose that Player 1 makes successive mistakes of $(I, \hat{x})$ during periods i^* and $i^{**}-1$. Player 2 will choose $(I, \nu-\hat{x})$ during those periods if she draws (i^*-1) records of $\hat{x}$. Observe that $C_0(\omega_i, \omega_{i+1}) = 1$ for $i < i^{**}$ and that $C_0(\omega_i, \omega_{i+1}) = 0$ for $i^{**} \leq i \leq i^*+m$ in the sequence of states given by (23) and (24). The proof is complete by letting $h = i^*+m$. □

The next lemma excludes $Q(r_{\text{all}})$ from the stochastically stable states.

Lemma A.6. *Any $\omega \in Q(r_{\text{all}})$ is not stochastically stable for all $k \geq 2$.*

Proof. Using Proposition A.1, we show that $Q(r_{\text{all}}) \ni \omega \notin R_2$ for all $R_2 \in \mathcal{R}_2$. Lemma A.3 implies that at least three mutations are required to escape from $Q(\hat{x})$ to $Q(r_{\text{all}})$ for all $\hat{x} \in X^{<r_2}_{>r_1}$. Then, it suffices to show that, for any $\omega \in Q(r_{\text{all}})$, $C_2(\omega, Q(x)) = 0$ for some $x \in X^{<r_2}_{>r_1}$. Fix $\omega_1 \in Q(r_{\text{all}})$. Lemmas A.4 and A.5 imply that there exists a sequence of states $(\omega_1, \ldots, \omega_h)$ such that $\omega_h = Q(x_h)$ for some $x_h \in X^{<r_2}_{>r_1}$ and there exists i^* for which

$$\begin{aligned} &\omega_i \in Q(r_{\text{all}}) \quad \text{and} \quad 1 \leq C_0(\omega_i, \omega_{i+1}) \leq 2 && \forall i < i^*, \\ &C_0(\omega_i, \omega_{i+1}) = 0 && \forall i^* \leq i \leq h. \end{aligned}$$

This implies that $C_2^*(\omega_1, Q(x')) = 0$ for some $Q(x') \in R_2 \in \mathcal{R}_2$.[16] Since Lemma A.3 implies that $C_2^*(Q(x'), \omega_1) > 0$, ω_1 is never stochastically stable. the claim follows. □

Now, we are ready to prove Theorem 1. We first prove Lemma A.7, which shows that only the escapes to adjacent conventions, i.e., from $Q(x)$ to $Q(x-\delta)$ or $Q(x+\delta)$, matter.

Lemma A.7. *If δ is sufficiently small, then, for every $x \in X_{>r_1}^{<r_2}$, the minimum cost of escaping from $Q(x)$ to $Q(x')$ for some $x' \neq x$ is $\lceil kc_\delta^*(x) \rceil$, where*

$$c_\delta^*(x) = \min\left\{ \frac{u_1(x;r_1) - u_1(x-\delta;r_1)}{u_1(x;r_1) - u_1(0;r_1)}, \frac{u_2(v-x;r_2) - u_2(v-x-\delta;r_2)}{u_2(v-x;r_2) - u_2(0;r_2)} \right\}. \tag{25}$$

Proof of Lemma A7. We use the same notations as in Eq. (16). Let

$$f(x) = \left\lceil \frac{c_\delta^1(x)}{\delta} \right\rceil \wedge \left\lceil \frac{c_\delta^2(x)}{\delta} \right\rceil \wedge \left\lceil \frac{c_\delta^3(x)}{\delta} \right\rceil \wedge \left\lceil \frac{c_\delta^4(x)}{\delta} \right\rceil.$$

Recall that $c_\delta(x) = \min\{c_\delta^1(x), c_\delta^2(x), c_\delta^3(x), c_\delta^4(x)\}$ and that Lemma A.3 implies that $kc_\delta(x)$ gives the minimum escaping cost from $Q(x)$ to any other convention. It is clear that $f(x) = \lceil c_\delta^i(x)/\delta \rceil$ if $c_\delta(x) = c_\delta^i(x)$ for some $1 \le i \le 4$. Observe that $\frac{c_\delta^1(x)}{\delta}$ and $\frac{c_\delta^4(x)}{\delta}$ are unbounded as δ approaches zero and the other two are bounded:

$$\lim_{\delta \to 0} \frac{c_\delta^2(x)}{\delta} = \frac{u_1'(x;r_1)}{u_1(x;r_1) - u_1(0;r_1)}, \qquad \lim_{\delta \to 0} \frac{c_\delta^3(x)}{\delta} = \frac{u_2'(v-x;r_2)}{u_2(v-x;r_2) - u_2(0;r_2)}.$$

Thus, for all sufficiently small δ, $c_\delta(x) = \min\{c_\delta^2(x), c_\delta^3(x)\} = c_\delta^*(x)$. The claim follows. □

Proof of Theorem 1. First, we prove the "if" part. Lemma A.6 together with Proposition A.1 implies that we can restrict our attention to $\{Q(x) : x \in X_{>r_1}^{<r_2}\}$ and the cost minimization problem of trees over $\mathcal{R}_2$. Lemma A.7 implies that the minimum cost of escaping from $Q(x)$ is given by $\lceil k \min\{c_\delta^2(x), c_\delta^3(x)\} \rceil$ for all sufficiently small δ, i.e., the least costly escape from $Q(x)$ is a sequence of transitions toward an adjacent convention. This further implies that if $Q(x), Q(x'') \in R_2$ for $x < x''$, then $Q(x') \in R_2$ for all x' with $x < x' < x''$.[17]

Let $R_2(x)$ denote $R_2 \in \mathcal{R}_2$ with $x = \min_{x':Q(x') \in R_2} |x_\delta^* - x'|$. Then, $\mathcal{R}_2$ is written as $\mathcal{R}_2 = \{R_2(x_1), \ldots, R_2(x_{l-1}), R_2(x_l), R_2(x_{l+1}), \ldots, R_2(x_L)\}$, where $x_1 < x_2 < \cdots < x_L$ and $x_l = x_\delta^*$ for some $l \in \{1, \ldots, L\}$.[18] Construct a tree T_δ having root $R_2(x_\delta^*)$ on $\mathcal{R}_2$ in the following way:

(i) construct the directed edge $(R_2(x_i), R_2(x_{i+1}))$ if $i < l$,
(ii) construct the directed edge $(R_2(x_i), R_2(x_{i-1}))$ if $i > l$.

Step (i) places a minimum-cost edge for $x < x_\delta^*$. To see this, recall that $c_\delta^2(x)$ is the cost of transition $(Q(x), Q(x-\delta))$ and $c_\delta^3(x)$ is the cost of transition $(Q(x), Q(x+\delta))$. With Lemma A.2, which shows that $c_\delta^2(x)$ is monotone decreasing and $c_\delta^3(x)$ is monotone increasing, the definition of x_δ^* implies that $\mathrm{sgn}(c_\delta^3(x) - c_\delta^2(x)) = \mathrm{sgn}(x - x_\delta^*)$. Similarly, Step (ii) places a minimum-cost edge for $x > x_\delta^*$. Since all of the edges are minimum cost edges for sufficiently small δ, T_δ constructed above has the least cost among all $Q(x)$-trees for all $x \in X_{>r_1}^{<r_2}$ on $\mathcal{R}_2$.

For the "only if" part, observe that the cost of an x'-tree for any $x' \neq x_\delta^*$ must amount to at least the cost of T_δ constructed above plus $(c_\delta^*(x_\delta^*) - c_\delta^*(x'))$. Observe also that for $\delta > 0$ and all sufficiently large $k > 0$, $c_\delta^*(x_\delta^*) \ge c_\delta^*(x') + 1$ for all $x' \in X_{>r_1}^{<r_2} \setminus \{x_\delta^*\}$. Then, the cost of T_δ is strictly lower than the cost of any x'-tree for any $x' \neq x_\delta^*$. □

Proof of Theorem 2. The proof consists of two steps. We characterize the prospect theory Nash bargaining solution in Step 1. Then, we show the convergence in Step 2.

Step 1: We solve the maximization problem given by Eq. (6). Differentiating the RHS of the equation in terms of x, we obtain the first order condition:

$$u_1'(x^*;r_1)(u_2(v-x^*;r_2) - u_2(0;r_2)) - u_2'(v-x^*;r_2)(u_1(x^*;r_1) - u_1(0;r_1)) = 0. \tag{26}$$

The second order condition is

$$\begin{aligned} &u_1''(x^*;r_1)(u_2(v-x^*;r_2) - u_2(0;r_2)) - u_1'(x^*;r_1)u_2'(v-x^*;r_2) \\ &\quad + u_2''(v-x^*;r_2)(u_1(x^*;r_1) - u_1(0;r_1)) - u_2'(v-x^*;r_2)u_1'(x^*;r_1) < 0. \end{aligned}$$

It is straightforward to show that the second order condition holds by using the facts that $u_i' > 0$ and $u_i'' \le 0$ for $r_1 < x < v - r_2$ and $i = 1, 2$.

[16] This R_2 satisfies either $Q(x_h) \in R_2$ or $C_2^*(Q(x_h), Q(x')) = 0$ for some $Q(x') \in R_2$.

[17] This is because the least cost path from $Q(x')$ must be toward $Q(x)$ or $Q(x'')$. Then, $C_2^*(Q(x), Q(x'')) = 0$ and $C_2^*(Q(x''), Q(x)) = 0$ must imply that either $C_i^*(Q(x'), Q(x)) = 0$ or $C_i^*(Q(x'), Q(x'')) = 0$ for some $i \le 2$.

[18] Since it is most costly to escape from x_δ^*, x_δ^* should be in some R_2.

Step 2: We show that x^*_δ satisfies Eq. (26) in the limit. Let

$$f^2_\delta(x) = \frac{1}{\delta}\frac{u_1(x;r_1) - u_1(x-\delta;r_1)}{u_1(x;r_1) - u_1(0;r_1)}, \qquad f^3_\delta(x) = \frac{1}{\delta}\frac{u_2(v-x;r_2) - u_2(v-x-\delta;r_2)}{u_2(v-x;r_2) - u_2(0;r_2)}.$$

Together with Eq. (7), $c^*_\delta(x)$ can be written as $c^*_\delta(x) = \delta \min\{f^2_\delta(x), f^3_\delta(x)\}$. Note that $f^2_\delta(x)$ is decreasing and $f^3_\delta(x)$ is increasing over x. Assuming that both functions have continuous supports, $c^*_\delta(x)$ is maximized at $\hat{x}$ such that $f^2_\delta(\hat{x}) = f^3_\delta(\hat{x})$. Observe that

$$\lim_{\delta\to 0} f^2_\delta(x) = \frac{u'_1(x;r_1)}{u_1(x;r_1) - u_1(0;r_1)}, \qquad \lim_{\delta\to 0} f^3_\delta(x) = \frac{u'_2(v-x;r_2)}{u_2(v-x;r_2) - u_2(0;r_2)}.$$

The claim follows from Theorem 1 and Eq. (26). □

Proof of Proposition 1.. If $r_1 \geq v/2$, let $r^*(r_2) = r_2$. According to Eq. (6), it is clear that the claim holds for this case. In the remainder of the proof, we prove the claim for $r_1 < v/2$.

Let for all $i \in \{1, 2\}$ and $j \in \{2, 3, \dots\}$,

$$du^+_i(x) = \frac{\mathrm{d}u^+_i(x)}{\mathrm{d}x}, \qquad d^j u^+(x) = \frac{\mathrm{d}^j u^+_i(x)}{\mathrm{d}x^j}, \qquad \partial_r u^-_i(0;r) = \frac{\partial u^-_i(0;r)}{\partial r}, \qquad \partial^j_r u^-_i(0;r) = \frac{\partial^j u^-_i(0;r)}{\partial r^j}.$$

Suppose a sequence of $\{k\}$ such that $k\delta$ approaches a constant as δ approaches zero, i.e., $\lim_{\delta\to 0} k\delta = k_\delta > 0$. Note that if $Q(x)$ is (generically) stochastically stable, $\pi_0(Q(x)) > 0$ for all sufficiently large k in the sequence. Define the cost $C_-(\cdot)$ as follows:

$$C_-(x;r_1) = \lim_{\delta\to 0} C(x, x-\delta) = k_\delta \frac{du^+_1(x-r_1)}{u^+_1(x-r_1) + u^-_1(0;r_1)} > 0.$$

This is the cost of escaping from $Q(x)$ to $Q(x-\delta)$ as δ approaches zero. Similarly, define the cost of escaping from $Q(x)$ to $Q(x+\delta)$ in the small δ limit as

$$C_+(x;r_2) = \lim_{\delta\to 0} C(x, x+\delta) = k_\delta \frac{du^+_2(v-x-r_2)}{u^+_2(v-x-r_2) + u^-_2(0;r_2)} > 0.$$

The limiting condition of the first derivative guarantees that $C_-(v/2;r_1) > C_+(v/2;r_2)$ for all r_1 sufficiently close to $v/2$; then the intersection between $C_-(x;r_1)$ and $C_+(x;r_2)$ is strictly greater than $v/2$, i.e. $x^*_\delta > v/2$. This implies that $r^*(r_2) < v/2$.

Next, observe that

$$\frac{\partial C_-(x;r_1)}{\partial r_1} = \frac{X}{(u^+_1(x-r_1) + u^-_1(0;r_1))^2},$$

where

$$X = -d^2u^+_1(x-r_1)\{u^+_1(x-r_1) + u^-_1(0;r_1)\} - du^+_1(x-r_1)\{-du^+_1(x-r_1) + \partial_r u^-_1(0;r_1)\}.$$

Observe that

$$\frac{\partial X}{\partial r_1} = \underbrace{d^3u^+_1(x-r_1)\{u^+_1(x-r_1) + u^-_1(0;r_1)\}}_{\text{strictly positve}} + \underbrace{du^+_1(x-r_1)(-d^2u^+_1(x-r_1) - \partial^2_r u^-_1(0;r_1))}_{\text{non-negative}}.$$

The positive condition of the third-order derivative guarantees that $\frac{\partial C_-(x;r_1)}{\partial r_1}$ is increasing over r_1. If $\frac{\partial C_-(x;r_1)}{\partial r_1} \geq 0$ for $r_1 = r_2$, then $C_-(v/2;r_1)$ is monotone increasing for $r_1 \geq r_2$. We let $r^*(r_2) = r_2$, and the claim holds. If $\frac{\partial C_-(x;r_1)}{\partial r_1} < 0$ for $r_1 = r_2$, then $C_-(v/2;r_1)$ is first monotone decreasing over r_1, next attains the minimum value, and is monotone increasing thereafter. Let $r^*(r_2) = \mathrm{argmax}\{r_1 : C_-(v/2;r_1) = C_+(v/2;r_2),\ r_1 > r_2\}$. Note that such $r^*(r_2)$ exists because $\frac{\partial C_-(x;r_1)}{\partial r_1}$ is strictly increasing. □

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