MR4292246 18B25 18B40 18E50 18F10
Funk, Jonathon [Funk, Jonathon R.] (1-CUNYQB-MCS);
Hofstra, Pieter [Hofstra, Pieter J. W.] (3-OTTW-MS)

Locally anisotropic toposes II. (English summary)

Theory Appl. Categ. **37** (2021), *Paper No.* 27, 914–939.

Every Grothendieck topos $\mathcal{E}$ has internal to it a canonical group object called its *isotropy group* [J. R. Funk, P. J. W. Hofstra and B. Steinberg, Theory Appl. Categ. **26** (2012), No. 24, 660–709; MR3065939], which acts canonically on every object of $\mathcal{E}$. We have the isotropy quotient

$$\psi\colon \mathcal{E} \to \mathcal{E}_0$$

where $\mathcal{E}_0$ is the full subcategory of $\mathcal{E}$ on those objects for which the canonical action by the isotropy group is trivial. This paper addresses the following question.

1. What is the nature of the quotient map ψ, and how is $\mathcal{E}$ recovered from its isotropy quotient $\mathcal{E}_0$?

This paper answers the question completely for the class of *locally anisotropic* toposes, where a topos $\mathcal{E}$ is called locally anisotropic if it has a globally supported object U for which $\mathcal{E}/U$ is anisotropic. The present line of inquiry was initiated in [J. R. Funk, P. J. W. Hofstra and S. Khan, Theory Appl. Categ. **33** (2018), Paper No. 20, 537–582; MR3812460], in which it was assumed that the isotropy quotient splits. This paper extends the predecessor without this assumption. The main result is Theorem 5.3, asserting that a locally anisotropic topos $\mathcal{E}$ is equivalent to a topos of group actions $\mathcal{B}(\mathcal{F};\mathbb{G})$ of a connected groupoid $\mathbb{G}$ internal to an anisotropic topos $\mathcal{F}$.

{For Part I see MR3754424.}

Hirokazu Nishimura

References

1. Barr, M. and Paré, R. (1980). Molecular toposes. *J. Pure Appl. Alg.*, 17:127–152. MR0567064
2. Borceux, F. and Janelidze, G. (2001). *Galois Theories*, volume 72 of *Cambridge studies in advanced mathematics*. Cambridge University Press, Cambridge. MR1822890
3. Bunge, M. (2004). Galois groupoids and covering morphisms in topos theory. In *Proceedings of the Fields Institute: Workshop on Descent, Galois Theory and Hopf algebras, Fields Institute Communications, American Mathematical Society*, volume 43, pages 131–162. MR2075584
4. Bunge, M. (2008). Fundamental pushout toposes. *Theory and Applications of Categories*, 20(9):186–214. MR2425549
5. Freyd, P. (1987). All topoi are localic or why permutation models prevail. *J. Pure Appl. Alg.*, 46:49–58. MR0894391
6. Freyd, P. and Yetter, D. (1989). Braided compact closed categories with applications to low dimensional topology. *Advances in Mathematics*, 77:156–182. MR1020583
7. Funk, J. and Hofstra, P. (2018). Locally anisotropic toposes. *J. Pure Appl. Alg.*, 222(6):1251–1286. MR3754424
8. Funk, J., Hofstra, P., and Khan, S. (2018). Higher isotropy. *Theory and Applications of Categories*, 7(1):1–22. MR3812460
9. Funk, J., Hofstra, P., and Steinberg, B. (2012). Isotropy and crossed toposes. *Theory and Applications of Categories*, 26(24):660–709. MR3065939

10. Janelidze, G. (1990). Pure galois theory in categories. *J. of Algebra*, 132:270–286. MR1061480
11. Johnstone, P. T. (2002). *Sketches of an Elephant: A Topos Theory Compendium.* Clarendon Press, Oxford. MR2063092
12. Joyal, A. and Tierney, M. (1984). *An extension of the Galois theory of Grothendieck*, volume Memoirs of the American Mathematical Society 309. American Mathematical Society. MR0756176
13. Lawson, M. V. (1998). *Inverse Semigroups: The Theory of Partial Symmetries.* World Scientific Publishing Co., Singapore. MR1694900

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.