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Modular categories with transitive Galois actions. (English) Zbl 07488571
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Modular categories are spherical braided fusion categories over $\mathbb{C}$ whose braidings are nondegenerate. There have been efforts on classifying modular categories by rank [*P. Bruillard* et al., J. Pure Appl. Algebra 220, No. 6, 2364–2388 (2016; Zbl 1332.18011); Int. Math. Res. Not. 2016, No. 24, 7546–7588 (2016; Zbl 1404.18016); *E. Rowell* et al., Commun. Math. Phys. 292, No. 2, 343–389 (2009; Zbl 1186.18005)], Frobenius-Perron dimension [*P. Bruillard* et al., Can. Math. Bull. 57, No. 4, 721–734 (2014; Zbl 1342.18013); Proc. Am. Math. Soc. 147, No. 1, 21–34 (2019; Zbl 1403.18006)] and Frobenius-Schur exponent [*P. Bruillard* and *E. C. Rowell*, Proc. Am. Math. Soc. 140, No. 4, 1141–1150 (2012; Zbl 1262.18005); *Z. Wan* and *Y. Wang*, Algebra Colloq. 28, No. 1, 39–50 (2021; Zbl 1452.18022)]. There are finitely many modular categories up to equivalence for any given rank [*P. Bruillard* et al., J. Am. Math. Soc. 29, No. 3, 857–881 (2016; Zbl 1344.18008)]. The number of Galois orbits plays prominents roles in most of these papers [*D. Green*, "Classification of rank 6 modular categories with Galois group $\langle(012)(345)\rangle$", Preprint, arXiv:1908.07128; *D. Creamer*, "A computational approach to classifying low rank modular categories", Preprint, arXiv:1912.02269], leading to the idea of classifying modular categories by the number of Galois orbits.

The principal objective in this paper is to study modular categories whose Galois group action on their simple objects are transitive, so that they are called *transitive modular categories*. A synopsis of the paper goes as follows.

- §2 sets up notation, giving a brief review on modular categories.
- §3 defines transitive modular categories, deducing some fundamental properties of them. The prime factorization theorem (Theorem 3.11) claiming that any transitive modular category has a unique factorization into a Deligne product of prime transitive modular categories is established.
- §4 addresses the prime and transitive modular categories obtained from the quantum group categories $\mathcal{C}(\mathfrak{sl}_2, p-2)$ for any odd prime p. It is shown that for any prime $p \geq 5$, every Galois conjugate of $\mathcal{C}(\mathfrak{sl}_2, p-2)^{(0)}$ is prime and transitive.
- §5 studies the modular group representations associated with modular categories. It is shown (Theorem 5.14) that the representations associated with transitive modular categories are irreducible and minimal.
- §6 characterizes the prime transitive modular categories (Theorem 6.4), leading to the complete classification of transitive modular categories (Theorem 6.5).
- §7 introduces transitive super-modular categories, studying them. The authors classify *split* transitive super-modular categories (Theorem 7.4), establishing a unique factorization theorem of transitive super-modular categories (Theorem 7.13). The authors exhibit a family of non-split transitive prime categories over sVec, conjecturing that these are all the s-simple transitive super-modular categories up to Galois conjugate.

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