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A Mal'tsev glance at the fibration $(\ )_0 : Cat\mathbb{E} \to \mathbb{E}$ of internal categories. (English. French summary) Zbl 07475441

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It is familiar [*D. Bourn*, Lect. Notes Math. 1488, 43–62 (1991; Zbl 0756.18007)] that, given any finitely complete category $\mathbb{E}$, the fibration

$$()_0 : Grd\,\mathbb{E} \to \mathbb{E}$$

of internal groupoids in $\mathbb{E}$ is of a strong structural property:

[1] Any fiber $Grd_Y\mathbb{E}$ above an object Y in $\mathbb{E}$ is protomodular, being a Mal'tsev category;

[2] the fiber $Grd_1\mathbb{E}$ above the terminal object 1 is nothing but the category $Gr\,\mathbb{E}$ of internal groups in $\mathbb{E}$.

Nothing comparable did exist for the fibration

$$()_0 : Cat\,\mathbb{E} \to \mathbb{E}$$

of internal categories investigated in [*D. Bourn*, Cah. Topologie Géom. Différ. Catégoriques 29, No. 2, 109–155 (1988; Zbl 0651.18007)]. A new structural aspect of the category Mon of monoids was introduced with the notion of Schreier split epimorphism and the associated notion of partial protomodularity [*D. Bourn* et al., Semigroup Forum 88, No. 3, 739–752 (2014; Zbl 1306.20067); Schreier split epimorphisms in monoids and in semirings. Coimbra: Universidade de Coimbra, Departamento de Matemática (2013; Zbl 1294.18001)].

The principal objective in this paper is to identify a class Σ_Y of split epimorphisms in the fibers $Cat_Y\mathbb{E}$ which would imply partial protomodularity inside them. This is done in §3.1 with the extension of the notion of Schreier split epimorphism to internal categories, leading to the notion of Schreier special category (Definition 4.4) which determines protomodular subcategories of the fibers $Cat_Y\mathbb{E}$.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18A20 Epimorphisms, monomorphisms, special classes of morphisms, null morphisms
18C10 Theories (e.g., algebraic theories), structure, and semantics
18C40 Structured objects in a category (group objects, etc.)
18E13 Protomodular categories, semi-abelian categories, Mal'tsev categories

Keywords:

Mal'tsev and protomodular categories; split epimorphisms; internal categories and groupoids; connected; aspherical and affine groupoids; direction of aspherical affine groupoids; internal weak equivalence

Full Text: Link

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