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On Lie bialgebroid crossed modules. **(English summary)**

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The theory of Lie bialgebras and Poisson Lie groups is due mainly to [V. G. Drinfeld, Dokl. Akad. Nauk SSSR **268** (1983), no. 2, 285–287; MR0688240; M. A. Semenov-Tyan-Shanskiĭ, Funktsional. Anal. i Prilozhen. **17** (1983), no. 4, 17–33; MR0725413]. A significant result is that Lie bialgebras are in one-to-one correspondence with Manin triples. Lie bialgebroids were introduced as a natural generalization of Lie bialgebras in [K. C. H. Mackenzie and P. Xu, Duke Math. J. **73** (1994), no. 2, 415–452; MR1262213], where the one-to-one correspondence between Lie bialgebroids and Manin triples still holds.

The concept of crossed modules was proposed by J. H. C. Whitehead [Ann. of Math. (2) **47** (1946), 806–810; MR0017537]. Lie group crossed modules are in one-to-one correspondence with strict Lie 2-groups [J. C. Baez and A. D. Lauda, Theory Appl. Categ. **12** (2004), 423–491; MR2068521]. Lie algebra crossed modules first appeared in [M. Gerstenhaber, Proc. Nat. Acad. Sci. U.S.A. **51** (1964), 626–629; MR0160807], corresponding to strict Lie 2-algebras [J. C. Baez and A. S. Crans, Theory Appl. Categ. **12** (2004), 492–538; MR2068522]. Lie bialgebra crossed modules were introduced in [Z. Chen, M. Stiénon and P. Xu, J. Geom. Phys. **68** (2013), 59–68; MR3035114], turning out strict Lie 2-bialgebras [C. M. Bai, Y.-H. Sheng and C. Zhu, Comm. Math. Phys. **320** (2013), no. 1, 149–172; MR3046993; O. Kravchenko, Lett. Math. Phys. **81** (2007), no. 1, 19–40; MR2327020]. There is a one-to-one correspondence between connected and simply connected Poisson Lie 2-groups on the one hand and Lie bialgebra crossed modules on the other. Lie algebroid crossed modules were first introduced in [I. Androulidakis, "Crossed modules and the integrability of Lie brackets", preprint, arXiv:math/0501103] as natural generalizations of Lie algebra crossed modules and the infinitesimal counterparts of crossed modules of Lie groupoids.

This paper is concerned with Lie bialgebroid crossed modules. The principal results in this paper go as follows:

- Theorem 3.2 gives an equivalent description of Lie algebroid crossed modules in terms of matched pairs.
- Theorem 3.7 provides an equivalence between Lie algebroid crossed modules and co-quadratic Manin triples.
- Proposition 4.2 clarifies the relationship between Lie algebroid crossed modules and strict split Lie 2-bialgebroids [J. Liu and Y.-H. Sheng, J. Symplectic Geom. **17** (2019), no. 6, 1853–1891; MR4057729].

A synopsis of the paper goes as follows:

- §2 contains a succinct account of standard facts about Lie algebroid crossed modules, Lie algebroid matched pairs and Lie bialgebroids.
- §3 gives the definition of Lie algebroid crossed modules, claiming two main results about them.
- §4 clarifies the equivalence between Lie algebroid crossed modules and strict split Lie 2-bialgebroids.

References

1. I. Androulidakis, Crossed modules and the integrability of Lie brackets, preprint (2005), arXiv: math.DG/0501103.
2. J. Baez and A. Crans, Higher-dimensional algebra. VI. Lie 2-algebras, *Theory Appl. Categ.* **12** (2004) 492–538 (electronic). MR2068522
3. J. Baez and A. Lauda, Higher-dimensional algebra V: 2-groups, *Theory Appl. Categ.* **12** (2014) 423–491. MR2068521
4. C. Bai, Y. Sheng and C. Zhu, Lie 2-bialgebras, *Comm. Math. Phys.* **320**(1) (2013) 149–172. MR3046993
5. D. Bashkirov and A. A. Voronov, On homotopy Lie bialgebroids, preprint (2016), arXiv:1612.02026.
6. Z. Chen, M. Stiénon and P. Xu, Weak Lie 2-bialgebras, *J. Geom. Phys.* **68** (2013) 59–68. MR3035114
7. Z. Chen, M. Stiénon and P. Xu, Poisson 2-groups, *J. Differential Geom.* **94**(2) (2013) 209–240. MR3080481
8. M. Dehling, Shifted L_∞-bialgebras, Master thesis, Gottingen University (2011).
9. V. G. Drinfel'd, Hamiltonian structures on Lie groups, Lie bialgebras and the geometric meaning of classical Yang-Baxter equations, Russian, *Dokl. Akad. Nauk SSSR*, **268**(2) (1983) 285–287. MR0688240
10. M. Forrester-Barker, Group objects and internal categories, preprint (2002), arXiv: math.CT/0212065.
11. M. Gerstenhaber, A uniform cohomology theory for algebras, *Proc. Natl. Acad. Sci. USA* **51** (1964) 626–629. MR0160807
12. Y. Kosmann-Schwarzbach, Exact Gerstenhaber algebras and Lie bialgebroids, *Acta Appl. Math.* **41**(1–3) (1995) 153–165. MR1362125
13. O. Kravchenko, Strongly homotopy Lie bialgebras and Lie quasi-bialgebras, *Lett. Math. Phys.* **81**(1) (2007) 19–40. MR2327020
14. C. Laurent-Gengoux and F. Wagemann, Obstruction classes of crossed modules of Lie algebroids and Lie groupoids linked to existence of principal bundles, *Ann. Global Anal. Geom.* **34**(1) (2008) 21–37. MR2415176
15. D. Li-Bland and E. Meinrenken, Courant algebroids and Poisson geometry, *Int. Math. Res. Not.* **11** (2009) 2106–2145. MR2507112
16. J. Liu and Y. Sheng, QP-structures of degree 3 and CLWX 2-algebroids, *J. Symplectic Geom.* **17**(6) (2019) 1853–1891. MR4057729
17. J. Liu and Y. Sheng, Homotopy Poisson algebras, Maurer–Cartan elements and Dirac structures of CLWX 2-algebroids, to appear in *J. Noncommutative Geom.*, DOI:10.4171/JNCG/398. MR4248210
18. Z. Liu, A. Weinstein and P. Xu, Manin triples for Lie bialgebroids, *J. Differential Geom.* **45**(3) (1997) 547–574. MR1472888
19. J. Liu and P. Xu, Exact Lie bialgebroids and Poisson groupoids, *Geom. Funct. Anal.* **6**(1) (1996) 138–145. MR1371234
20. J.-H. Lu, Multiplicative and affine Poisson structures on Lie groups, ProQuest LLC, Ann Arbor, MI, Thesis Ph.D. University of California, Berkeley (1990). MR2685337
21. K. C. H. Mackenzie, General Theory of Lie Groupoids and Lie Algebroids, London Mathematical Society Lecture Note Series, Vol. 213, (Cambridge University Press, Cambridge, 2005). MR2157566
22. K. C. H. Mackenzie and T. Mokri, Locally vacant double Lie groupoids and the integration of matched pairs of Lie algebroids, *Geom. Dedicata* **77**(3) (1999) 317–330.

MR1716681
23. K. C. H. Mackenzie and P. Xu, Lie bialgebroids and Poisson groupoids, *Duke Math. J.* **73**(2) (1994) 415–452. MR1262213
24. R. A. Mehta and M. Zambon, L_∞-algebra actions, *Differential Geom. Appl.* **30**(6) (2012) 576–587. MR2996854
25. T. Mokri, Matched pairs of Lie algebroids, *Glasgow Math. J.* **39**(2) (1997) 167–181. MR1460632
26. D. Roytenberg, Courant algebroids, derived brackets and even symplectic supermanifolds, ProQuest LLC, Ann Arbor, MI, Thesis Ph.D. University of California, Berkeley (1999). MR2699145
27. M. A. Semenov-Tyan-Shanskiĭ, What a classical r-matrix is, *Funkt. Anal. Prilozhen* **17**(4) (1983) 17–33 (Russian). MR0725413
28. Y. Sheng and C. Zhu, Higher extensions of Lie algebroids, *Comm. Contemp. Math.* **19**(3) (2017) 1650034. MR3631929
29. Th. Voronov, L_∞-bialgebroids and homotopy Poisson structures on supermanifolds, preprint (2019), arXiv:1909.04914.
30. J. H. C. Whitehead, Note on a previous paper entitled, On adding relations to homotopy groups, *Ann. Math. (2)* **47** (1946) 806–810. MR0017537
31. P. Xu, Gerstenhaber algebras and BV-algebras in Poisson geometry, *Comm. Math. Phys.* **200**(3) (1999) 545–560. MR1675117
32. C. Zhu, n-groupoids and stacky groupoids, *Int. Math. Res. Not.* **2009**(21) (2009) 4087–4141. MR2549951

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.