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An introduction to diffeology. (English) [Zbl 07425815](#)

Anel, Mathieu (ed.) et al., New spaces in mathematics. Formal and conceptual reflections. Cambridge: Cambridge University Press. 31-82 (2021)

Diffeology began with [*J. M. Souriau*, Lect. Notes Math. 836, 91–128 (1980; [Zbl 0501.58010](#)); in: Feuilletages et quantification géométrique, Journ. lyonnaises Soc. math. France 1983, Sémin. sud-rhodanien Géom. II, 73–119 (1984; [Zbl 0541.58002](#))]. It is so flexible to be applicable not only to manifolds but also to orbifolds by way of example. This paper is an inspiring introduction to diffeology, whose systematic development can be seen in the author's [Diffeology. Providence, RI: American Mathematical Society (AMS) (2013; [Zbl 1269.53003](#))].

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Reviewer: [Hirokazu Nishimura \(Tsukuba\)](#)

MSC:

- [18-XX](#) Category theory; homological algebra
- [58A05](#) Differentiable manifolds, foundations
- [58A40](#) Differential spaces

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