Larizza, Leonardo

On factorisation systems for Ord-enriched categories and categories of partial maps. (English) Zbl 07467853

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This paper presents a new type of factorisation system for **Ord**-categories differing substantially from the lax orthogonal factorisation systems studied in [*M. Manuel Clementino* and *I. López Franco*, Theory Appl. Categ. 35, 1379–1423 (2020; Zbl 1446.18015); *I. López Franco*, Appl. Categ. Struct. 27, No. 5, 463–492 (2019; Zbl 1428.18010)], where **Ord** is the category of partially ordered sets and monotone maps.

A synopsis of the paper goes as follows.

- §1 introduces a new notion of orthogonality.
- §2 studies lax factorisation systems for lax arrow categories that are functorial.
- §3 investigates the conditions under which a lax functorial factorisation system is of an underlying lax weak factorisation sytem, describing the latter.
- §4 presents a class of functorial factorisation systems abiding by the condition above and being equipped with a richer structure close to the one of a monad. The construction parallels in the lax context that of algebraic weak factorisation systems [*M. Grandis* and *W. Tholen*, Arch. Math., Brno 42, No. 4, 397–408 (2006; Zbl 1164.18300); *R. Garner*, Appl. Categ. Struct. 20, No. 2, 103–141 (2012; Zbl 1256.55005); *J. Bourke* and *R. Garner*, J. Pure Appl. Algebra 220, No. 1, 108–147 (2016; Zbl 1327.18004)].
- The second part, dedicated to the study of lax and oplax factorisation systems for categories of parial maps, begins with §5 recalling some useful definitions, notations and properties of categories of partial maps, some of which can be found in [*M. P. Fiore*, Axiomatic domain theory in categories of partial maps. Cambridge: Cambridge Univ. Press (1996; Zbl 0979.68549); *E. Robinson* and *G. Rosolini*, Inf. Comput. 79, No. 2, 95–130 (1988; Zbl 0656.18001)].
- §6 provides a general construction of a lax functorial weak factorisation system for any category of partial maps equipped with the general paritial order $\leq$.
- §7 exhibits the close link between factorisation systems on a category $\mathcal{C}$ and oplax factorisation systems on $\mathcal{P}(\mathcal{C})$.
- §8 discusses a process of obtaining lax and oplax weak factorisation systems for pointed categories of partial maps.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18A20	Epimorphisms, monomorphisms, special classes of morphisms, null morphisms
18A32	Factorization systems, substructures, quotient structures, congruences, amalgams
18B10	Categories of spans/cospans, relations, or partial maps
18B35	Preorders, orders, domains and lattices (viewed as categories)
18D20	Enriched categories (over closed or monoidal categories)

Keywords:

factorization systems; lax arrow categories; **Ord**-enriched categories; partial maps

Full Text: Link

References:

[1] References [Adámek et al., 2020] Adámek, J. Herrlich, H., Rosický, J., and Tholen, W. (2002). Weak factorization systems and topological functors. Appl. Categ. Structures, 10(3):237-249.

Bourke and Garner, 2016Bourke, J. and Garner, R. (2016). Algebraic weak factorisation systems I: accessible awfs. J. Pure Appl. Algebra, 220(1):108-147.

Bunge, 1974Bunge, M. C. (1974). Coherent extensions and relational algebras. Trans. Amer. Math. Soc., 197:355-390. · Zbl 0358.18004

Clementino and López Franco, 2016Clementino, M. M. and López Franco, I. (2016). Lax orthogonal factorisation systems. Adv. Math., 302:458-528.

Clementino and López Franco, 2020Clementino, M. M. and López Franco, I. (2020). Lax orthogonal factorisations in ordered structures. Theory Appl. Categ., 35(36):1379-1423.

Cockett et al. 2021Cockett, R., Cruttwell, G., Gallagher, J., and Pronk, D. (2021). Latent factorization systems for restriction categories. In preparation.

[2] Fiore, 1995] Fiore, M. P. (1995). Order-enrichment for categories of partial maps. Math. Structures Comput. Sci., 5(4):533-562.

[3] Fiore, 2004] Fiore, M. P. (2004). Axiomatic domain theory in categories of partial maps, volume 14. Cambridge University Press.

Freyd and Kelly, 1972Freyd, P. J. and Kelly, G. M. (1972). Categories of continuous functors, I. J. Pure Appl. Algebra, 2(3):169-191.

[4] Garner, 2009] Garner, R. (2009). Understanding the small object argument. Appl. Categ. Structures, 17(3):247-285.

Grandis and Tholen, 2006Grandis, M. and Tholen, W. (2006). Natural weak factoriza-tion systems. Arch. Math., 42(4):397-408.

López Franco, 2019López Franco, I. (2019). Cofibrantly generated lax orthogonal fac-torisation systems. Appl. Categ. Structures, 27:no.5 463-492. · Zbl 1428.18010

Riehl, 2011Riehl, E. (2011). Algebraic model structures. New York J. Math., 17(173-231):27.

Robinson and Rosolini, 1988Robinson, E. and Rosolini, G. (1988). Categories of partial maps. Information and computation, 79(2):95-130.

Walker, 2020Walker, C. (30th June 2020). Characterization of lax orthogonal factoriza-tion systems. Algebra, Logic and Topology Seminar (CMUC).

[5] Gabriella Böhm, Wigner Research Centre for Physics: bohm.gabriella (at) wigner.mta.hu Maria Manuel Clementino, Universidade de Coimbra: mmc@mat.uc.pt Valeria de Paiva, Nuance Communications Inc: valeria.depaiva@gmail.com Richard Garner, Macquarie University: richard.garner@mq.edu.au Ezra Getzler, Northwestern University: getzler (at) northwestern(dot)edu

[6] Dirk Hofmann, Universidade de Aveiro: dirk@ua.pt Pieter Hofstra, Université d' Ottawa: phofstra (at) uottawa.ca Anders Kock, University of Aarhus: kock@math.au.dk Joachim Kock, Universitat Autònoma de Barcelona: kock (at) mat.uab.cat Stephen Lack, Macquarie University: steve.lack@mq.edu.au Tom Leinster, University of Edinburgh: Tom.Leinster@ed.ac.uk Matias Menni, Conicet and Universidad Nacional de La Plata, Argentina: matias.menni@gmail.com Ieke Moerdijk, Utrecht University: i.moerdijk@uu.nl Susan Niefield, Union College: niefiels@union.edu

[7] Kate Ponto, University of Kentucky: kate.ponto (at) uky.edu Robert Rosebrugh, Mount Allison University: rrosebrugh@mta.ca Jiří Rosický, Masaryk University: rosicky@math.muni.cz Giuseppe Rosolini, Università di Genova: rosolini@disi.unige.it Michael Shulman, University of San Diego: shulman@sandiego.edu Alex Simpson, University of Ljubljana: Alex.Simpson@fmf.uni-lj.si James Stasheff, University of North Carolina: jds@math.upenn.edu

[8] Tim Van der Linden, Université catholique de Louvain: tim.vanderlinden@uclouvain.be